Copyright © 1971 American Telephone and Telegraph Company
THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 50, No. 7, September, 1971
Printed in U.S.A.

Multiple-Path Fading on Line-of-Sight Microwave Radio Systems as a Function of Path Length and Frequency

By C. L. RUTHROFF

(Manuscript received March 22, 1971)

Transmission over line-of-sight microwave radio paths is analyzed with the aid of a piece-wise linear approximation of the atmospheric index of refraction. The simple model is adequate; it predicts published experimental results.

A short path is defined on which no deep fading can occur and the maximum length of such a path is estimated from measured data for New Jersey. Expressions are presented for the worst-case amplitude-frequency response and for the maximum echo delay for short paths. It is shown that if the normal Fresnel-zone clearance is maintained on short paths, no fading will occur due to substandard conditions of propagation.

W. T. Barnett's result is also predicted from this model: the distribution of attenuation on long paths is a function of L^3/λ where L is the path length and λ is the free space wavelength.

The distribution of deep fades on long paths is predicted by this model to have the same slope as the Rayleigh distribution, the slope normally found in measurements of attenuation distributions on long paths.

I. INTRODUCTION

Terrestrial and satellite radio systems for common carrier applications at frequencies above 10 GHz have been proposed recently.[1,2] For terrestrial systems the lengths of transmission paths may be limited to a few kilometers by the attenuation caused by rain.[3–5] A similar limitation holds for satellite systems—for that part of the transmission path which is contained in earth's atmosphere.

For both terrestrial and satellite systems, it is anticipated that economies can be achieved if the information can be transmitted as sequences of short pulses. But O. E. DeLange has shown that pulses with durations of a few nanoseconds can suffer serious degradation during transmission over a 22.8-mile path at a frequency of 4 GHz; in his experiment, the degradation occurred during conditions of frequency-selective fading.[6,7] For the proposed systems, this result raises an important question—for the path lengths of interest, can pulses of a few nanoseconds duration be transmitted without suffering intolerable degradation due to frequency-selective fading? No answer to this question has been found in the literature; one of the purposes of this paper is to answer it.

In pioneering work on the propagation of microwaves, A. B. Crawford, DeLange, W. C. Jakes, and W. M. Sharpless have shown that severe frequency-selective fading on line-of-sight transmission paths can be explained in terms of multiple-path transmission.[6–9] Their results have been confirmed by others and it is generally accepted that the mechanism of multiple-path transmission is responsible for most severe frequency-selective fading.[10–15]

In this paper, fading due to multiple-path transmission and fading due to substandard refraction are discussed with emphasis on short transmission paths. In the model used, signal power from the transmitter travels to the receiver over two or more paths.[13] Multiple-paths are caused by a layer of atmosphere above the transmission path which has a negative gradient in the index of refraction. Generally the energy traveling over the separate paths will undergo unequal phase shifts; if the received signal, which is the vector sum of the signals received from these paths, is reduced substantially below its free-space value by this mechanism, multiple-path fading is said to occur.

Multiple-path fading, then, is a function of the magnitude of the gradient in the index of refraction of the refracting layer. It is reasonable to assume that there is a maximum value of this gradient which occurs in a climatic region. Based on this assumption, it will be shown that a path length L_0 exists such that for any path of length $L \leqq L_0$—called a short path—deep multiple-path fading cannot occur. The length L_0 is estimated for New Jersey from transmission measurements made by Crawford and Jakes.[7] For example, at 30 GHz, $L_0 \approx$ 4.8 km in New Jersey.

Expressions are also derived for the frequency selectivity, maximum echo delay, and the path clearance required to eliminate fading due to substandard refraction.

Fading on long paths is also discussed and it is shown that the path attenuation distribution is a function of L^3/λ where L is the path length, and λ is the wavelength. This agrees with Barnett's result which is based on extensive measurements. The correct slope of the distributions of deep fades on long paths is also predicted from this model.

II. REFRACTION IN A SINGLE LAYER

Most severe selective fading occurs during clear summer nights when temperature inversions and associated meteorological effects produce negative gradients in the index of refraction of the atmosphere.[7,10,13] Figure 1 illustrates a simple profile of the index of refraction which can produce two or three transmission paths between transmitter and receiver; two transmission paths are shown.[7] As is usual, the index of refraction is assumed to change only in the vertical dimension. From geometric optics, the position of the elevated negative gradient in the index of refraction allows a direct ray between trans-

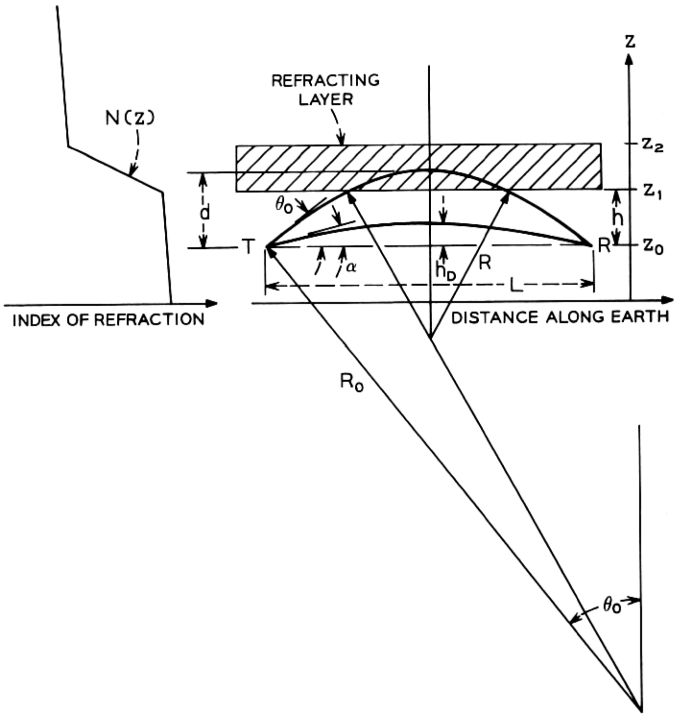

Fig. 1—Refraction from a single layer.

mitter and receiver as shown in Fig. 1, and causes another ray, launched upward at a small angle, θ_0, to be refracted into the receiver.

The magnitudes of the gradients in the index of refraction are small and only those rays which are launched at small angles to the horizontal can be refracted into the receiver. For small angles it has been shown that if the gradient in the index of refraction is constant the refracted ray follows the arc of a circle with radius R, where[13]

$$\frac{1}{R} \approx -\frac{dn(z)}{dz} = -10^{-6}\frac{dN(z)}{dz}. \tag{1}$$

The height above earth is z, $n(z)$ is the index of refraction, and $N(z) \equiv [n(z) - 1]10^6$. The dispersion in the index of refraction is known to be small so we assume $n(z)$ and R are independent of frequency.[13]

The model of Fig. 1 is a simple approximation to the actual variations in the index of refraction which occur on transmission paths. The model is not used because it is simple but because it does all that can be asked of a model—the results predicted from it agree with the results measured in corresponding experiments:

(*i*) Either one, two, or three discrete rays are predicted by the model and all of these combinations of rays have been observed.[7] Four or more discrete rays have not been observed.

(*ii*) Frequency-sweep measurements, which heretofore have been approximated by more than three and as many as eleven distinct signals, are easily approximated by three distinct signals if time variations are assumed in the height, thickness, and index of refraction of the refracting layer.[7,10]

(*iii*) Barnett has determined the dependence of attenuation distributions on path length and wavelength from measurements on many paths at several frequencies; the same result is predicted by this model.

(*iv*) The correct slope of deep-fading distributions on long paths is also predicted from this model.

2.1 *Phase-Shift Along a Ray—General Case*

The geometry for a single refracting layer is shown in Fig. 1. The direct ray has radius R_0 and an angle of departure $\alpha \approx L/2R_0$. A refracted ray leaves the transmitter at angle θ_0 and follows the arc of a circle with radius R_0 until the ray reaches height Z_1. At this point the slope of the index of refraction changes and in the region $Z_1 \leqq z \leqq Z_2$ the ray follows an arc of a circle with radius R. When the ray passes into the region $Z_0 \leqq z \leqq Z_1$, it again follows an arc of a circle with radius R_0 and arrives at the receiver at the angle θ_0 above the horizontal axis.

The phase shift, ϕ, between the transmitter and receiver is given by

$$\phi = \frac{2\pi}{\lambda} \int n(z)\, ds, \tag{2}$$

where, $n(z)$ is the index of refraction,
λ is the free-space wavelength, and
ds is the element of length along the ray.

The integral in (2) is the optical length of the ray.

It is possible for a refracted ray to rise above the refracting layer and still reach the receiver. To include this case we use the geometry of Fig. 2 to derive the phase shift along a refracted ray. There are three regions to consider.

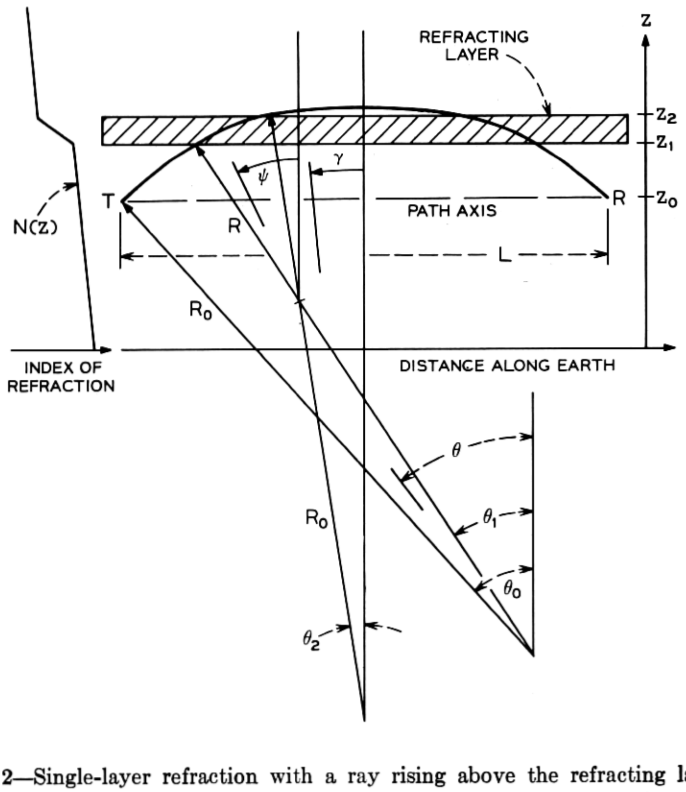

Fig. 2—Single-layer refraction with a ray rising above the refracting layer.

Region I

In the region below Z_1, $n = n_0 - m_0 z$, and a point on the ray is given by $z = Z_0 + R_0(\cos\theta - \cos\theta_0)$, $\theta_1 \leqq \theta \leqq \theta_0$. Substituting for z in n and noting that $dn/dz = -m_0 = -1/R_0$,

$$n = n_0 - m_0 Z_0 - \cos\theta + \cos\theta_0, \qquad \theta_1 \leqq \theta \leqq \theta_0. \tag{3}$$

The element of length along the arc in this region is $ds = R_0 d\theta$.

Region II

In the refracting layer, $n = n_0 - m_0 Z_1 - m(z - Z_1)$, and a point on the ray is given by $z = Z_1 + R(\cos\psi - \cos\theta_1)$, $0 \leqq \psi \leqq \theta_1$. From Fig. 2, $\cos\theta_1 = \cos\theta_0 + (Z_1 - Z_0)/R_0$. Substituting for $\cos\theta_1$ in z, for z in n and noting that $dn/dz = -m = -1/R$,

$$n = n_0 - m_0 Z_0 - \cos\psi + \cos\theta_0, \qquad \theta_2 \leqq \psi \leqq \theta_1. \tag{4}$$

In the refracting layer, $ds = R d\psi$.

Region III

Above the refracting layer, $n = n_0 - m_0 z + m_0(Z_2 - Z_1) - m(Z_2 - Z_1)$ and a point on the ray is given by $z = Z_2 + R_0(\cos\gamma - \cos\theta_2)$, $0 \leqq \gamma \leqq \theta_2$. From Fig. 2, $\cos\theta_2 = \cos\theta_0 + (Z_1 - Z_0)/R_0 + (Z_2 - Z_1)/R$. Substituting for $\cos\theta_2$ in z, for z in n and noting that $dn/dz = -m_0 = -1/R_0$,

$$n = n_0 - m_0 Z_0 - \cos\gamma + \cos\theta_0, \qquad 0 \leqq \gamma \leqq \theta_2. \tag{5}$$

In this region $ds = R_0 d\gamma$.

Since $m_0 Z_0 \ll n_0$, it will be omitted from (3), (4), and (5). By substituting these equations into (2) the optical length of the refracted ray can be written.

$$\begin{aligned}\frac{\lambda\phi}{2\pi} \approx\ & 2\int_{\theta_1}^{\theta_0} (n_0 - \cos\theta + \cos\theta_0) R_0\, d\theta \\ & + 2\int_{\theta_2}^{\theta_1} (n_0 - \cos\psi + \cos\theta_0) R\, d\psi \\ & + 2\int_0^{\theta_2} (n_0 - \cos\gamma + \cos\theta_0) R_0\, d\gamma \end{aligned} \tag{6}$$

$$\begin{aligned} \approx\ & 2R_0[(n_0 + \cos\theta_0)(\theta_0 - \theta_1) - \sin\theta_0 + \sin\theta_1)] \\ & + 2R[(n_0 + \cos\theta_0)(\theta_1 - \theta_2) - \sin\theta_1 + \sin\theta_2] \\ & + 2R_0[(n_0 + \cos\theta_0)\theta_2 - \sin\theta_2]. \end{aligned} \tag{7}$$

Only those rays which reach the receiver are of interest. For a ray to be refracted into the receiver the geometry of Fig. 2 requires that the following two relations be satisfied.

$$R_0 \sin \theta_0 - (R_0 - R)(\sin \theta_1 - \sin \theta_2) = L/2, \tag{8}$$

$$0 \leqq \theta_2 \leqq \theta_1 \leqq \theta_0 . \tag{9}$$

The launch angle, θ_0, and the angles θ_1 and θ_2 are related to the height, h, of the refracting layer above the path axis and its vertical thickness, v, by the following equations.

$$h \equiv Z_1 - Z_0 = R_0 \cos \theta_1 - R_0 \cos \theta_0 , \quad \text{and} \tag{10}$$

$$v \equiv Z_2 - Z_1 = R \cos \theta_2 - R \cos \theta_1 . \tag{11}$$

In principle, θ_0, θ_1 and θ_2 can be found from (8), (10), and (11) in terms of the path length, gradient of the index of refraction, and the height and thickness of the refracting layer. These values of θ_0, θ_1 and θ_2 can then be substituted into (7) to determine the optical length of the refracted ray. Our purpose is better served, however, by specializing the solution.

2.2 *Phase-Shift Along a Ray—Special Case*

The worst case of interference, that is, the largest difference in phase between the direct and refracted rays, occurs when the layer it thick enough that no ray which reaches the receiver rises above the layer. This can be demonstrated by the usual methods and the proof is not reproduced here.

The rest of this paper will be concerned primarily with the worst case interference, for which $\theta_2 = 0$.

When $\theta_2 = 0$, (8) and (9) become

$$R_0 \sin \theta_2 - (R_0 - R) \sin \theta_1 = L/2, \quad \text{and} \tag{12}$$

$$0 \leqq \theta_1 \leqq \theta_0 . \tag{13}$$

The limits on the launch angle, θ_0, are found by substituting (13) into (12).

$$L/2R_0 \leqq \sin \theta_0 \leqq L/2R. \tag{14}$$

We introduce a normalizing parameter k such that

$$k \equiv \frac{2R_0}{L} \sin \theta_0 , \quad \text{then, from} \quad (14), \qquad 1 \leqq k \leqq R_0/R. \tag{15}$$

Now,

$$\theta_0 \approx \frac{kL}{2R_0} + \frac{1}{6}\left(\frac{kL}{2R_0}\right)^3, \tag{16a}$$

$$\cos\theta_0 \approx 1 - \frac{1}{2}\left(\frac{kL}{2R_0}\right)^2, \quad \text{and} \tag{16b}$$

$$\theta_1 \approx \frac{L(k-1)}{2(R_0 - R)} + \frac{1}{6}\left(\frac{L(k-1)}{2(R_0 - R)}\right)^3. \tag{16c}$$

With (14), (15), and (16) and recalling that $\theta_2 = 0$, the optical length of the refracted ray is

$$\frac{\lambda\phi_R}{2\pi} \approx n_0 L + \frac{L^3}{24R_0^2}\left[(n_0+1)k^3 - \frac{(n_0+1)(k-1)^3}{(1-R/R_0)^2} - 3k^2\right]. \tag{17}$$

The index of refraction $n_0 \approx 1$, so (17) can be written

$$\frac{\lambda\phi_R}{2\pi} \approx L + \frac{L^3}{24R_0^2}\left[2k^3 - \frac{2(k-1)^3}{(1-R/R_0)^2} - 3k^2\right], \tag{18}$$

for

$$1 \leqq k \leqq R_0/R.$$

The interference between the direct and refracted rays will be determined from (18). First, however, we need expressions for the departure angle and the distance a ray rises above the path as functions of the height of the refracting layer.

2.3 *The Relation Between the Angle of Departure and the Height of the Refracting Layer*

The distance of the refracting layer above the path axis is $h = Z_1 - Z_0$. From Fig. 1,

$$h = R_0(\cos\theta_1 - \cos\theta_0).$$

It is convenient to express h in terms of the maximum distance, h_D, that the direct ray rises above the path axis,

$$h_D = R_0(1 - \cos\alpha).$$

Substituting from (16) and noting that $\alpha \approx L/2R_0$,

$$h_D \approx L^2/8R_0, \quad \text{and} \tag{19}$$

$$\frac{h}{h_D} \approx k^2 - \frac{(k-1)^2}{(1-R/R_0)^2}. \tag{20}$$

2.4 *The Distance a Ray Rises above the Path*

From Fig. 1, the maximum distance, d, that a ray rises above the path axis is

$$d = R(1 - \cos \theta_1) + h.$$

Substituting from (16), (19), and (20),

$$\frac{d}{h_D} \approx k^2 - \frac{(k-1)^2}{(1 - R/R_0)}. \tag{21}$$

2.5 *The Optical Length of the Direct Ray*

If $\theta_1 = 0$ the refracting layer is too high to affect transmission. In this case, transmission is normal and a single ray, launched at an angle $\alpha = \sin^{-1} L/2R_0$, will reach the receiver. This is the direct ray and its optical length is given by setting $\theta_1 = \theta_2 = 0$ and $\theta_0 = \alpha = \sin^{-1} L/2R_0 \approx L/2R_0 + 1/6(L/2R_0)^3$ in (7).

$$\frac{\lambda \phi_D}{2\pi} \approx n_0 L + \frac{(n_0 - 2)}{24R_0^2} L^3. \tag{22}$$

The index of refraction $n_0 \approx 1$ and (22) can be written

$$\frac{\lambda \phi_D}{2\pi} \approx L - \frac{L^3}{24R_0^2}. \tag{23}$$

2.6 *Three-Path Transmission*

The height of the refracting layer above the path axis, the maximum height that a refracted ray rises above the path axis, and the phase shifts, ϕ_R, along the refracted rays are shown in Fig. 3 as functions of k for $R_0/R = 4$. The abscissa is approximately proportional to the angle of departure of a ray because θ_0 is small and $k = (2R_0/L) \sin \theta_0$. The curves in Fig. 3 were computed from (18), (20), and (21).

From Fig. 3 it can be seen that either one, two, or three rays can reach the receiver depending upon the height of the refracting layer; several examples are shown in Fig. 4. In Fig. 4a the layer height is 2.2 h_D. In addition to the direct ray, two rays are refracted into the receiver. As the layer height decreases the vertical separation between the two refracted rays increases as indicated in Figure 4b.

When the height of the layer equals the height of the direct ray, $h = h_D$ and the direct ray coincides with one refracted ray; two rays reach the receiver. And finally, when the layer height is less than h_D, only one ray, refracted from the layer, reaches the receiver as shown in Figure 4d.

The direct ray exists when the height of the layer equals or exceeds h_D, that is, when $h \geqq h_D$. As illustrated in Figure 4c, the direct ray and one refracted ray coincide when $h = h_D$. This occurs when $k = 1$ for the refracted ray.

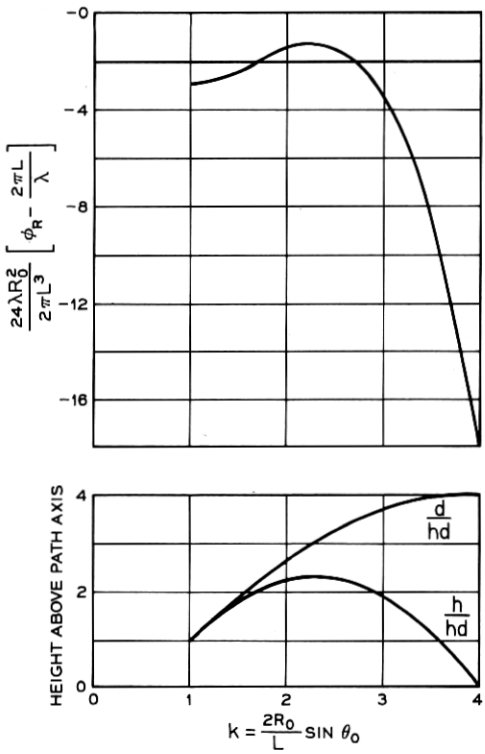

Fig. 3—Phase and height relations in single-layer refraction for $R_0 = 4R$.

III. SHORT PATHS

It has been known by many people for a long time, and it is intuitively obvious, that if a path is short enough, no deep fading will occur. Apparently no quantitative analysis of short paths has been published; therefore, we present a definition of a short path, estimates of short path lengths in New Jersey for several frequencies of interest, and an analysis of transmission over short paths.

3.1 *Definition of a Short Path*

The largest phase difference between rays occurs when $h/h_D = 1$. In this case the direct ray and one refracted ray coincide, reducing the number of rays to two. The corresponding values of k are

$$k_1 = 1, \quad \text{and}$$

$$k_2 = \frac{1 + (1 - R/R_0)^2}{1 - (1 - R/R_0)^2}.$$

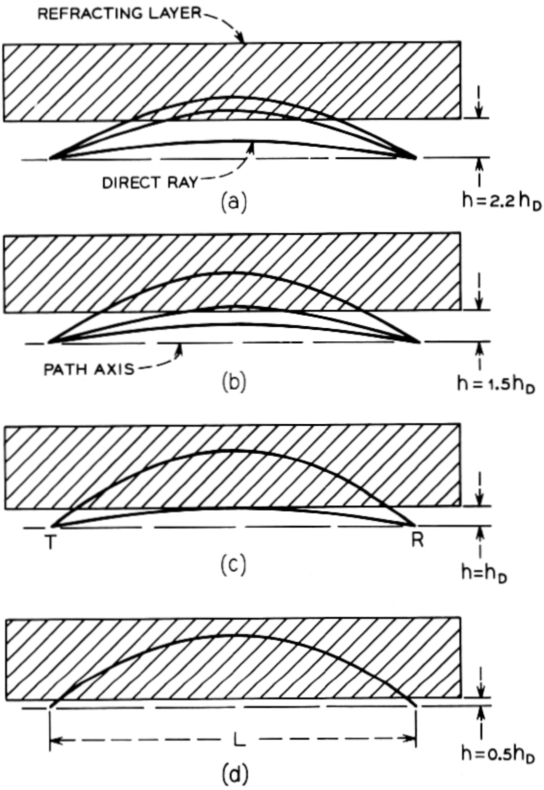

Fig. 4—Single-layer refraction.

The phase difference between these two rays is $\beta \equiv \phi_D - \phi_R$. From (18) and (23),

$$\frac{\lambda\beta}{2\pi} \equiv \frac{\lambda}{2\pi}(\phi_D - \phi_R) \approx \frac{L^3}{24R^2}\frac{(1 - R/R_0)^4}{(1 - R/2R_0)^2}. \tag{24}$$

The direct and refracted rays arrive at the receiver out of phase as shown in Fig. 5. The total received signal, E_R, is the vector sum of the signals received along the two rays.

$$\mathbf{E_R} = \mathbf{E}_0 + \mathbf{E}\mathbf{e}^{j\beta},$$

where $\beta \equiv \phi_D - \phi_R$. The magnitude of the received signal is found to be

$$|E_R| = [E_0^2 + 2EE_0 \cos\beta + E^2]^{\frac{1}{2}}. \tag{25}$$

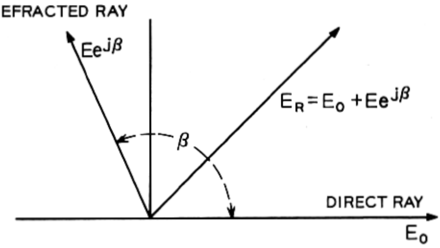

Fig. 5—Phase relations between the signals received via the direct and refracted rays.

A deep fade occurs if $|E_R| \ll E_0$, where E_0 is the amplitude of the received signal during normal transmission.

It may be seen from (25) that deep fading cannot occur unless $\cos \beta$ is negative. Now suppose that the path length is chosen so that, for the maximum negative gradient which occurs in the index of refraction, $|\beta|$ does not exceed a maximum value $|\beta_{\max}| \leqq \pi$. Then the deepest fade—the minimum value of $|E_R|$—can be computed from (25). For instance, if $|\beta_{\max}| \leqq \pi/2$ the minimum value of $|E_R|$ is E_0 and occurs when $E = 0$, and if $|\beta_{\max}| > \pi/2$ the minimum value of $|E_R|$ is given by $|E_R|_{\min} = E_0 \sin |\beta_{\max}|$ which occurs when $E = -E_0 \cos |\beta_{\max}|$. The minimum received signal amplitude is plotted in Fig. 6 as a function of the maximum difference in phase between the two rays.

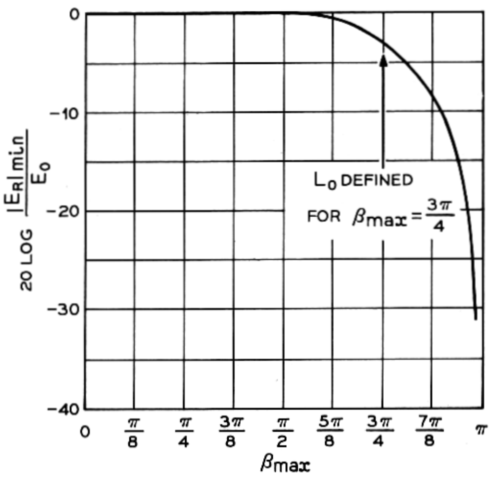

Fig. 6—The minimum amplitude of the received signal as a function of the maximum phase angle between the two received rays.

We now define a path length, L_0, such that for any path of length $L \leqq L_0$, there is no deep fading. L_0 is chosen arbitrarily as the path length for which $|\beta_{max}| = (3\pi)/4$ radians. The deepest fade which can occur on a path of length L_0 is 3 dB as may be seen in Fig. 6. L_0 can be computed from (24) with $\beta = (3\pi)/4$ radians.

$$L_0^3 = 9\lambda R^2 \frac{(1 - R/2R_0)^2}{(1 - R/R_0)^4}. \tag{26}$$

The definition has been made for the largest phase difference which can occur between the direct ray and a refracted ray. This situation occurs when the layer is tangent to the direct ray in which case only two rays exist. If the layer rises a third ray will be refracted into the receiver. The maximum phase difference between any two rays will be less than $3\pi/4$ radians in this situation and the maximum fade will be less than 3 dB. It should be noted that no restriction has been put on the magnitude of the signal received via the refracted rays; for a short path the maximum fade is 3 dB regardless of the magnitude of the sum of the refracted rays as illustrated in Fig. 7.

3.2 *An Estimate of* L_0

Crawford and Jakes, and Sharpless, measured angles of arrival for several years on a 22.8-mile transmission path at a frequency of 24 GHz.[7-9] The maximum difference in the angle of arrival between rays arriving at the receiver was 0.65 degree, the rays arriving at angles of 0.05, 0.35, and 0.7 degree above the normal angle of arrival. From this data, L_0 has been computed for several frequencies of interest and listed in Table I. The details of the computations are given in the

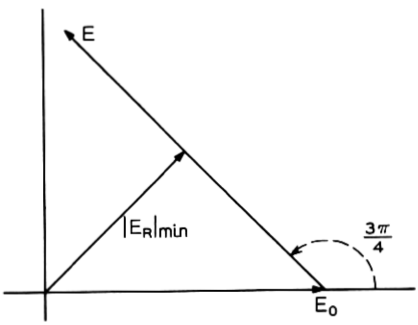

Fig. 7—Phase diagram showing V_{min} independent of refracted signal amplitude E for a short path where $\beta_{max} = 3\pi/4$.

TABLE I—THE LENGTH OF SHORT PATHS IN NEW JERSEY

Frequency in GHz	λ in cm	L_0 in km
4	7.5	9.37
6	5.0	8.20
10	3.0	6.93
20	1.5	5.5
30	1.0	4.8
60	0.5	3.8

Appendix. A reasonable interpretation of this result is that for path lengths less than those given in the table, in similar climatic regions, a deep fade would be a rare event. Paths of length $L \leqq L_0$ will be called short paths.

Since the values of L_0 in Table I were computed from data taken on a 22.8-mile path in New Jersey at 24 GHz, they should be regarded as approximate. More accurate values can be determined from measurements on shorter paths.

3.3 *The Slope of the Amplitude as a Function of Frequency*

Recalling that β is a function of frequency it follows from (25) that the magnitude of the received signal is also dependent upon frequency. Now,

$$\frac{d \ln |E_R|}{d\beta} = \frac{1}{|E_R|} \frac{d|E_R|}{d\beta}$$

and performing the indicated operations using (25), we get

$$\frac{d|E_R|}{|E_R|} = -\left[\frac{(E/E_0) \sin \beta}{1 + 2(E/E_0) \cos \beta + (E/E_0)^2}\right] d\beta.$$

The maximum value of the quantity in the brackets for a path of length L_0 occurs for $E = E_0$ and is

$$\left.\frac{d|E_R|}{|E_R|}\right|_{\max} = -\frac{d\beta}{2(\sqrt{2} - 1)}.$$

Now, after substituting $d\beta = \beta df/f$ we have

$$\left.\frac{d|E_R|}{|E_R|}\right|_{\max} = -\frac{\beta}{2(\sqrt{2} - 1)} \frac{df}{f}. \tag{27}$$

Example:

For a path of length L_0, $\beta = (3\pi)/4$ and (27) becomes

$$\left.\frac{d|E_R|}{|E_R|}\right|_{\max} = -\frac{3\pi}{8(\sqrt{2} - 1)} \frac{df}{f} = -2.85 \frac{df}{f}.$$

For a 500-MHz bandwidth at 20 GHz, the maximum change in amplitude across the band is approximately 7 percent or 0.6 dB.

3.4 *The Maximum Echo Delay*

The received signal is the sum of the signals transmitted over the direct and refracted paths. From the definition of a short path the maximum phase difference between the received signals is $\beta_{max} = (3\pi)/4$ radians corresponding to a difference in path length $\Delta L = \frac{3}{8}\lambda$. The maximum time delay τ is

$$\tau = \frac{\Delta L}{c} = \frac{3}{8f} \text{ seconds.} \tag{28}$$

At a frequency of 20 GHz the maximum echo delay on a short path is 0.01875 nanosecond. Pulses with a duration of one nanosecond would suffer little degradation on a short path at this frequency.

IV. LONG PATHS

Radio systems at 4 and 6 GHz normally have paths much longer than L_0. Consider a path at 6 GHz of length $L = 5L_0 \approx 41$ km. For the same negative gradient of the index of refraction used to derive L_0, the path difference between the direct and refracted rays is

$$\beta = \frac{3\pi}{4}\left(\frac{L}{L_0}\right)^3 = \frac{3\pi}{4} \times 125 \text{ radians.}$$

On the phase plane the phase difference between the direct and refracted waves corresponds to 46 complete revolutions plus an angle of $7\pi/4$ radians. This situation is illustrated in Fig. 8. Small changes in the

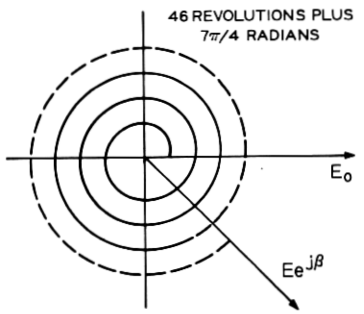

Fig. 8—The phase between signals received via the direct and refracted rays on a 41-km path at 6 GHz for the gradient in the index of refraction corresponding to $L_0 = 8.20$ km.

gradient of the index of refraction will cause the vector $\mathbf{E}e^{j\beta}$ to make a complete revolution—and pass through one of the phases $(2n - 1)\pi$, $n = \pm 1, \pm 2, \cdots$, where it is possible for deep fading to occur. Similarly, changes in frequency can cause $\mathbf{E}e^{j\beta}$ to rotate through large angles, a mechanism reflected in the frequency selectivity of this type of fading on long paths.

4.1 *Effect of Small Changes in the Gradient of the Index of Refraction on Interference*

From (1) and (24) the fractional change in β as a function of the fractional change in the gradient of the index of refraction can be derived.

$$\frac{d\beta}{\beta} = \frac{2}{(1 - R/R_0)(1 - R/2R_0)} \frac{d(dN/dz)}{dN/dz}. \tag{29}$$

For the 6-GHz, 41-km path, the fractional change in dN/dz required to produce a rotation of $\mathbf{E}e^{j\beta}$ through 2π radians is, for $R_0/R = 4$,

$$\frac{d(dN/dz)}{dN/dz} = 0.7 \text{ percent}.$$

Thus, small changes in the gradient of the index of refraction can produce large changes in the amplitude of the received signal on long transmission paths.

4.2 *Effect of Changes in Frequency on Interference*

Taking the derivative of β with respect to frequency in (24), we get $d\beta = \beta df/f$. For the 6-GHz, 41-km path, the change in frequency required to cause a change in β of 2π radians is

$$df = f\frac{d\beta}{\beta} = \frac{6 \times 10^3 \times 2\pi}{3\pi \times 125/4} = 128 \text{ MHz}.$$

During such fading conditions a frequency-sweep experiment would show minima separated by approximately 128 MHz.

Swept-frequency measurements have been reported in the literature and the results have been approximated by adjusting the amplitudes and phases of a number of sinusoidal signals to obtain a good fit to the experimental amplitude-frequency response.[7,10] Each sinusoid represented a discrete ray with constant amplitude and delay. In all cases, more than three sinusoids—rays—were required to obtain a satisfactory fit to the measured data.

The apparent contradiction between these results and the three-ray theory disappears when time-variations in the height, the thickness, and the gradient of the index of refraction of the refracting layer are included. All of the approximations were made with the tacit assumption that the optical lengths of the rays remained constant during the time of a single sweep. This assumption does not hold in practice; the best evidence of this is the sequence of amplitude-frequency responses shown in Fig. 10 of Ref. 7. Each frequency-sweep took one second and sweeps are shown at ten-second intervals. The substantial changes in response which occur in ten seconds imply that non-negligible changes in response may occur during the one-second sweep time. Instead of postulating additional rays to approximate the measured frequency-swept response of the path, it is reasonable to postulate appropriate time variations in the height, the thickness, and the gradient of the index of refraction of the refracting layer.

There is another argument in support of this view. Four or more distinct rays have never been observed in measurements of angle-of arrival although two-ray or three-ray transmission has been observed on many occasions.[7] And while transmission with four or more rays is conceivable—for instance, with two layers like those in Fig. 2—the incidence of such conditions is apparently much less than the incidence of two-path or three-path transmission.

4.3 *The Onset of Fading on Long Paths*

It is reasonable to suppose that the negative gradient of the index of refraction develops slowly on clear summer nights. At some time a second ray may form and interfere with the direct ray at the receiver. For a small gradient the received signal may increase initially since β is small. As the negative gradient increases in magnitude the angle β will increase and cause the received signal to decrease when β approaches π radians. The angle β will continue to increase as the magnitude of the gradient increases. When β is large, rapid fading may occur due to small variations in the magnitude of the gradient as described previously. Small changes in the height or thickness of the layer may have a similar effect.

Now, smaller inversions of the index of refraction occur more often than do larger ones. Therefore, for a relatively small inversion, a long path may have fading whereas a shorter path may not. We may expect, then, that the fading distribution is a function of β and hence, of path length L and wavelength λ.

4.4 *Distribution of Attenuation as a Function of Path Length and Frequency*

The attenuation distribution is usually written as the probability—fraction of time—that the received signal amplitude, $|E_R|$, is less than a specified value, V_0.

$$\text{ATTENUATION DISTRIBUTION} \equiv P(|E_{\mathbf{R}}|/E_0 \leqq V_0/E_0). \tag{30}$$

In the present model, the amplitudes of the signals received via the refracted rays are nominally equal to the amplitude of the signal received in the absence of the refracting layer. Since the probability of three rays of equal amplitude combining to form a deep fade is much less than the probability of two rays of equal amplitude combining to form a deep fade, we conclude that the distribution of attenuation is dominated by two-ray interference. The received signal is written

$$E_{\mathbf{R}}/E_0 = 1 + (E/E_0)e^{i\beta},$$

where, from (18) and (23),

$$\beta = \phi_D - \phi_R \approx \frac{2\pi L^3}{24\lambda R_0^2}\left[2k^3 - \frac{2(k-1)^3}{(1 - R/R_0)^2} - 3k^2 + 1\right].$$

By virtue of (1), (11), (15), (16), and (20), β can be written

$$\beta \approx \frac{2\pi L^3}{24\lambda R_0^2} F(dN/dz, h, v). \tag{31}$$

Although E and E_0 are nominally equal, there are always small variations in their magnitudes due to small inhomogeneities in the atmosphere. The ratio E/E_0 may therefore be expected to have a rather narrow probability density function which has a mean value of unity and which changes very little in the interval $1 - V_0/E_0 \leqq E/E_0 \leqq 1 + V_0/E_0$, where $V_0 \ll E_0$. On long paths the model suggests that E/E_0 and β are statistically independent. Then,

$$P(|E_R|/E_0 \leqq V_0/E_0) = \iint p_1(E/E_0)p_2(\beta)\, d\beta\, d(E/E_0)$$

and the integral is evaluated over a circle of radius V_0/E_0 centered at the origin of the phase plane.

For deep fades, $V_0 \ll E_0$ and the distribution can be written

$$\begin{aligned} P(|E_R|/E_0 \leqq V_0/E_0) &= \int p_1(E/E_0)\, d(E/E_0) \int p_2(\beta)\, d\beta \\ &\approx P(1 - V_0/E_0 \leqq E/E_0 \leqq 1 + V_0/E_0) \\ &\quad \cdot \sum_{n=0}^{\infty} P((2n+1)\pi - V_0/E_0 \leqq \beta \leqq (2n+1)\pi + V_0/E_0) \\ &\approx p_1(1)\frac{2V_0}{E_0} \sum_{n=0}^{\infty} P\Big((2n+1)\pi - V_0/E_0 \\ &\quad \leqq \frac{2\pi L^3}{24\lambda R_0^2} F(dN/dz, h, v) \leqq (2n+1)\pi + V_0/E_0\Big). \end{aligned} \tag{32}$$

The probability density function of $F(dN/dz, h, v)$ is not a function of path length or frequency, hence, the relationship in (32) agrees with Barnett's results which were obtained from measured distributions on long paths; the attenuation distribution is independent of path length, L, and wavelength, λ, as long as the ratio L^3/λ remains constant.[16]

Most of the time the sum in (32) is zero. During periods of heavy fading on long paths β may be regarded as being uniformly distributed in the interval $0 \leqq \beta \leqq 2\pi$. In this case, (32) may be written

$$P(| E_R |/E_0 \leqq V_0/E_0) \propto \frac{2}{\pi} p_1(1)\left(\frac{V_0}{E_0}\right)^2. \tag{33}$$

The slope of the distribution in (33) is the same as that of the Rayleigh distribution normally resulting in measurements of deep fading on long paths.

V. THE EFFECT OF SUBSTANDARD CONDITIONS ON SHORT PATHS

On long transmission paths fading can also be caused by an unusually large positive gradient in the index of refraction of the atmosphere.[7,13] During such conditions the rays from the transmitter bend away from the earth. If the gradient is large enough, a ray which is tangent to the earth in the direction of the receiver will be bent upward enough to miss the receiving antenna. This means that no direct ray will reach the receiver; the receiver is therefore in the diffraction region and the received signal amplitude will be substantially reduced below the level received when the conditions of propagation are normal. This type of fading is said to be caused by substandard refraction.[7,13]

We assume that the gradient in index of refraction is in the vertical direction only and extends over the length of the path. The radius of curvature of a ray is computed as described in Section II; in this case the center of curvature is above the transmission path. The geometry is shown in Fig. 9a, and the substandard gradient is illustrated in Fig. 9b. As long as the gradient in the index of refraction results in a bending radius $R \geqq R_{\min}$, a ray can reach the receiver. If the gradient increases further, the bending radius is reduced and there is no ray which can reach the receiver; the ray with radius R_x in Fig. 9a illustrates this condition.

The geometry is shown in Fig. 10 for the tangent and normal rays for a flat earth approximation. The clearance between the direct ray and the surface of the earth is d, and from the geometry of Fig. 10.

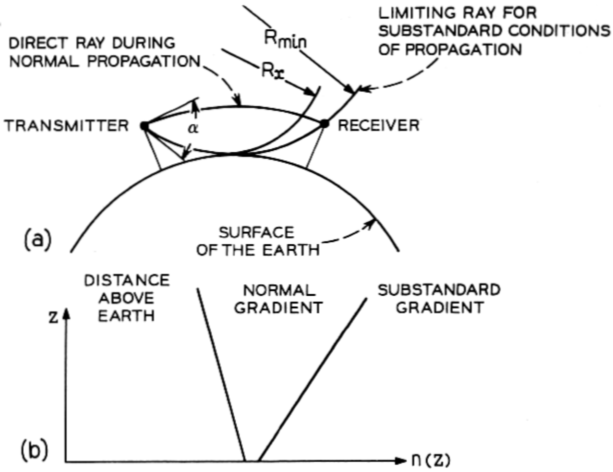

Fig. 9—(a) Ray bending during substandard conditions. (b) The index of refraction of the atmosphere during normal and substandard conditions.

$$d \approx \frac{L\theta_{max}}{4} \approx \frac{L^2}{8R_{min}}. \tag{34}$$

If the smallest value of R_{min} can be determined, a path clearance, d, can be computed from (34) such that no fading of this type is possible on a path whose clearance exceeds d. It is not possible to guarantee a value of R_{min} but a useful estimate can be made.

Crawford and Jakes observed no such fading on a 22.8-mile path during several years of measurement.[7] The path clearance was 280 feet. The radius of the limiting ray for this path, computed from (34), is $R_{min} \approx 2 \times 10^8$ cm. Since no fading due to substandard refraction was observed, a ray with a radius this small probably did not occur. For any path, in a similar climatic region, with a clearance d, com-

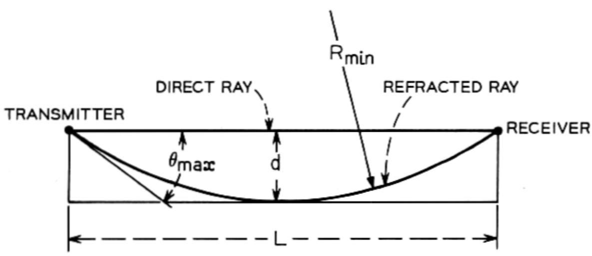

Fig. 10—Ray bending during conditions of substandard propagation.

puted from (34) with $R_{min} = 2 \times 10^8$ cm, such a fade would be a rare event.

Since no fading due to substandard refraction was observed the radius $R_{min} = 2 \times 10^8$ cm may be conservative. However, Crawford and Jakes did observe such fading on another path of length 12.6 miles and with a clearance of 63 feet.[7] Fading due to substandard refraction on this path could occur for rays with radii less than 2.84×10^8 cm. Evidently rays with radii less than 2.84×10^8 cm did occur but there were no rays with radii less than 2×10^8 cm. The use of $R_{min} = 2 \times 10^8$ cm to determine path clearances in similar climates is therefore not unduly pessimistic.

Path clearances, d_0, sufficient to eliminate fading due to substandard refraction on short paths can be computed from (34) using the lengths of short paths in Table I. These are shown in Table II.

It is of interest that d_0, for short paths, is always less than $\sqrt{\lambda L_0}/2$. Therefore, if a short path meets the usual Fresnel-zone clearance objective of $\sqrt{\lambda L_0}/2$, such a fade would be a rare event.

VI. CONCLUSION

Negative gradients in the index of refraction of the atmosphere are believed to be the cause of most severe fading on line-of-sight transmission paths operated at microwave frequencies. It has been shown in this paper that if the gradient has a maximum value, there is a length of transmission path, L_0, below which there is no deep multiple-path fading. Paths of length less than L_0 are called short paths. This path length has been estimated for New Jersey from measurements, reported by Crawford and Jakes, of the angle-of-arrival of microwave energy during conditions of severe multiple-path fading. The path length L_0 depends upon the maximum difference in the angles-of-arrival measured, and the largest difference recorded in

TABLE II—CLEARANCE REQUIRED TO PREVENT FADING DUE TO SUBSTANDARD REFRACTION ON SHORT PATHS

Frequency in GHz	λ in cm	L_0 in km	d_0 in cm
4	7.5	9.37	548
6	5.0	8.20	420
10	3.0	6.93	300
20	1.5	5.5	189
30	1.0	4.8	144
60	0.5	3.8	90.4

several years of measurement was used to compute L_0 at several frequencies of interest. These are shown in Table I. A reasonable interpretation of this result is that for short paths a deep fade would rarely, if ever, occur.

It is also shown that for short paths—paths shorter than L_0—the transmission is insensitive to bandwidth; pulses with durations of the order of one nanosecond will not suffer severe degradation on short paths.

The distribution of attenuation of fading on long paths is predicted from this model to be independent of the path length, L, and the wavelength, λ, as long as the ratio L^3/λ remains constant. This result is in agreement with Barnett's result which was obtained by other means.

The frequency-selectivity of long paths was also derived from the model. Using the values of L_0 presented in Table I, the minimum frequency separation between adjacent minima in a frequency-sweep measurement on a long path can be computed. The frequency separation between adjacent minima is inversely proportional to the third power of the path length.

The lengths of short paths were computed from measurements made in New Jersey and probably apply to regions with similar fading experience. The negative gradients in the index of refraction are functions of meterological parameters including temperature and humidity, and these may vary substantially in different climates. We may expect, for instance, that path length L_0 may be different in desert regions than it is in New Jersey.

VII. ACKNOWLEDGMENTS

This work was done in response to E. F. O'Neill's question about the transmission of short pulses over short-hop radio systems. The estimates of the lengths of short paths in New Jersey could not have been made without the experiments of A. B. Crawford and W. C. Jakes—experiments which were beautifully conceived, executed, and reported. Finally, I am grateful to T. L. Osborne, V. K. Prabhu, M. V. Schneider, and L. C. Tillotson for their comments and advice.

APPENDIX

In the experiments of Crawford and Jakes, the maximum difference in the angle of arrival between rays arriving at the receiver was 0.65 degree. This event occurred in the month of August and three rays were present, arriving at angles of 0.05, 0.35, and 0.7 degree above the normal angle of arrival.

The index of refraction at the surface of the earth, N_s, and its gradient dN_s/dz, vary from season to season as described by B. R. Bean and E. J. Dutton.[17] In our notation,

$$-\frac{dN_s}{dz} = 7.32\, e^{(0.005577 N_s)}, \tag{35}$$

where the unit of height is a kilometer. In the month of August, $N_s \approx 360$ in New Jersey. From (1) and (35),

$$\frac{dN_s}{dz} = -54.5N \text{ units/km}, \quad \text{and}$$

$$R_0 = 1.83 \times 10^9 \text{ cm}. \tag{36}$$

We assume that the normal angle of arrival during non-fading conditions is determined by this value of R_0. In addition to seasonal variations in N_s and dN_s/dz, there are diurnal variations.[17] It is assumed here that such a variation increased the angle of arrival of the direct ray by 0.05 degree for the event described above. A new value of R_0, corresponding to the new angle of arrival will now be computed.

The angle of arrival of the direct ray during nonfading conditions is computed using (36),

$$\alpha_0 = \frac{L}{2R_0} = 0.001 \text{ radian} = 0.0573 \text{ degree}.$$

During the fading event, α_0 increased to a new value

$$\alpha_1 = 0.0573 + 0.05 = 0.1073 \text{ degree}.$$

The new radius of curvature is $R_0 = L/(2\alpha_1) = 0.98 \times 10^9$ cm. The angles of arrival of the two refracted rays are

$$\theta_1 = 0.35 + 0.0573 = 0.4073 \text{ degree} \quad \text{and}$$

$$\theta_2 = 0.7 + 0.0573 = 0.7573 \text{ degree}.$$

From (16a),

$$k_1 \approx \theta_1/\alpha_1 = 3.8, \quad \text{and}$$

$$k_2 \approx \theta_2/\alpha_1 = 7.05.$$

These values of k occur for the same layer, that is, for the same h/h_D. Then, from (20),

$$k_2^2 - \frac{(k_2 - 1)^2}{(1 - R/R_0)^2} = k_1^2 - \frac{(k_1 - 1)^2}{(1 - R/R_0)^2}.$$

Solving for $(1 - R/R_0)^2$,

$$(1 - R/R_0)^2 = \frac{k_1 + k_2 - 2}{k_1 + k_2} = 0.815,$$

for which

$$R_0/R = 10.3.$$

After substitution into (23) we get,

$$L_0 = 4.8\lambda^{\frac{1}{3}} \text{ kilometers, } \lambda \text{ in cm.}$$

REFERENCES

1. Tillotson, L. C., "Use of Frequencies above 10 GHz for Common Carrier Applications," B.S.T.J., *48,* No. 6 (July–August 1969), pp. 1563–1576.
2. Tillotson, L. C., "A Model of a Domestic Satellite Communication System," B.S.T.J., *47,* No. 10 (December 1968), pp. 2111–2137.
3. Hathaway, S. D., and Evans, H. W., "Radio Attenuation at 11 KMC," B.S.T.J., *38,* No. 1 (January 1959), pp. 73–97.
4. Semplak, R. A., and Turrin, R. H., "Some Measurements of Attenuation by Rainfall at 18.5 GHz," B.S.T.J., *48,* No. 6 (July–August 1969), pp. 1745–1756.
5. Ruthroff, C. L., "Rain Attenuation and Radio Path Design," B.S.T.J., *49,* No. 1 (January 1970), pp. 121–135.
6. DeLange, O. E., "Propagation Studies at Microwave Frequencies by Means of Very Short Pulses," B.S.T.J., *31,* No. 1 (January 1952), pp. 91–103.
7. Crawford, A. B., and Jakes, W. C., "Selective Fading of Microwaves," B.S.T.J., *31,* No. 1 (January 1952), pp. 68–90.
8. Sharpless, W. M., "Measurement of the Angle of Arrival of Microwaves," Proc. IRE, *34,* No. 11 (November 1946), pp. 837–845.
9. Crawford, A. B., "Further Observations of the Angle of Arrival of Microwaves," Proc. IRE, *34,* No. 11 (November 1946), pp. 845–848.
10. Kaylor, R. L., "A Statistical Study of Selective Fading of Super-High Frequency Radio Signals," B.S.T.J., *32,* No. 9 (September 1953), pp. 1187–1202.
11. Durkee, A. L., "Results of Microwave Propagation Tests on a 40-Mile Overland Path," Proc. IRE, *36,* No. 2 (February 1948), pp. 197–205.
12. Friis, H. T., "Microwave Repeater Research," B.S.T.J., *27,* No. 2 (April 1948), pp. 183–246.
13. Kerr, D. E., *Propagation of Short Radio Waves,* New York: McGraw-Hill, 1951.
14. Desirant, M., and Michiels, J. L., *Electromagnetic Wave Propagation,* New York: Academic Press, 1960, pp. 471–478.
15. Saxton, J. A., *Radio-Wave Propagation in the Troposphere,* New York: Elseviere Publishing Co., 1962, pp. 3–15.
16. Barnett, W. T., unpublished work.
17. Bean, B. R., and Dutton, E. J., *Radio Meteorology,* National Bureau of Standards Monograph 92, Washington: U. S. Government Printing Office, March 1, 1966.