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THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 50, No. 7, September, 1971
Printed in U.S.A.

Data Transmission by Combined AM and PM

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(Manuscript received March 2, 1971)

This paper presents theoretical analysis and experimental verification of the performance of a digital data modem which uses combined amplitude and phase modulation (AM–PM). The theoretical model assumes operation over the bandlimited additive Gaussian channel. The receiver used in the experiment, and for which theoretical results are presented, uses an envelope detector in parallel with a phase detector to recover the data. The criteria of error rate and communication efficiency (measured in bits per cycle of bandwidth) as functions of average signal-to-noise ratio (S/N) are used to make comparisons with other modulation schemes. The theory predicts a performance from 1 to 4 dB poorer than what can be realized from single sideband (SSB) modulation. We present experimental results which indicate excellent agreement with the theory.

I. INTRODUCTION

Theoretical investigations of the performance of combined amplitude and phase modulation systems have been reported on by C. R. Cahn[1] (1960), J. C. Hancock and R. W. Lucky[2] (1960), Lucky and Hancock[3] (1962), and more recently by M. Leiter and M. P. Talbot[4] (1967). These investigators found that, on the additive white Gaussian channel, combined AM and PM yields a better error rate at fixed signal-to-noise ratio than PM or AM alone when the number of total levels is $\geqq 8$.

Here we rigorously establish these claims for a practical system. Instead of using linear matched filter detection (Ref. 2), we use differential or coherent phase detection to estimate the phase and a conventional envelope detector to estimate the amplitude. While theoretically optimum signal locations in two-dimensional space are generally not quantized but subject only to fixed power limitation, we consider practical signal sets which are quantized.

The questions we answer in this paper are the following: Given an N-level system and the facility to use L amplitudes and M phases such that $N = LM$, what is the value of L, and consequently M, such that the error rate is smallest for a fixed amount of S/N when signaling over the bandlimited additive Gaussian channel? How does this combined modulation scheme compare with only PM or AM at fixed S/N? How efficient is this modulation scheme in terms of attainable bits per cycle of bandwidth at fixed error rate? And finally how close can these theoretical results be realized in the laboratory with actual hardware?

II. THE MATHEMATICAL MODEL

We represent the transmitted signal by the time series

$$S(t) = \sum_n (V + Ab_n) \cos [\omega_0 t + \theta_n] g(t - nT) \tag{1}$$

where $b_n = \pm(2k - 1)$, $k = 1, 2, \cdots, L/2$, for all n are the amplitude symbols and $\{\theta_n\}$ is a set of M equally spaced phases. The signal $g(t)$ is a bandlimited ideal pulse such that $g(nT) = 1$, when $n = 0$ and $g(nT) = 0$ for all other n. For the calculations that follow we take $g(t) = \sin[(\pi t)/T]/[(\pi t)/T]$ without loss of generality. The positive constants V, A, and ω_0 are so far arbitrary. The total amount of information carried by $S(t)$ is $1/T$ $(\log_2 L + \log_2 M) = 1/T$ $(\log_2 N)$ b/s, $N = ML$. We regard the amplitude symbols $\{b_n\}$ and the phase symbols $\{\theta_n\}$ as independent random variables. By introducing a uniformly distributed phase in the argument of the cos $(\cdot)$ function in equation (1) and also randomizing the epoch of $g(t)$ renders the signal $S(t)$ stationary. A straightforward calculation* reveals that the total average power in $S(t)$ is

$$P_S = \langle S^2(t) \rangle = \left(\frac{V^2}{2} + \frac{A^2}{2} \frac{L^2 - 1}{3} \right) \tag{2}$$

where $\langle \cdot \rangle$ denotes the ensemble average.

For the added noise we use the standard representation

$$W(t) = x(t) \cos \omega_0 t + y(t) \sin \omega_0 t$$

where $x(t)$ and $y(t)$ are stationary zero mean independent baseband Gaussian processes possessing identical variance σ^2.

The data receiver processes the sum signal $S(t) + W(t)$ to obtain estimates of $\{b_n\}$ and $\{\theta_n\}$. Reception is accomplished in two parallel

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* Detailed calculations of power in data signals may be found in Ref. 5, pp. 51–55.

detectors. These detectors will be denoted respectively by E—the envelope detector and ϕ—the phase detector.

The E detector operates by first computing the envelope of $S(t) + W(t)$ and then synchronously sampling the result at every T seconds. The ϕ detector computes the phase of signal plus noise (see Ref. 5 for a detailed discussion of phase modulation and demodulation) which is then synchronously sampled to derive estimates of $\{\theta_n\}$. In all these operations we assume that correct timing information is available at the receiver.

We begin by analyzing the E detector first. The envelope of the sum signal is

$$E^2(t) = [\sum_n (V + Ab_n) \cos \theta_n g(t - nT) + x(t)]^2 + [\sum_n (V + Ab_n) \sin \theta_n g(t - nT) + y(t)]^2 \tag{3}$$

and a sample taken at $t = 0$ gives

$$E^2 = \{(V + Ab_0) \cos \theta_0 + x\}^2 + \{(V + Ab_0) \sin \theta_0 + y\}^2 \tag{4}$$

where we have used some obvious notational shorthands. In order that a genuine envelope exist we must have $V + Ab_0 > 0$ for all values of b_0. This results in the condition

$$V > (L - 1)A \tag{5}$$

since max $b_0 = (L - 1)A$. Our task in the next section will be to evaluate the error rate in the E detector from estimates of the probability distribution of E^2.

The ϕ detector computes either the phase differential of $S(t) + W(t)$ or the actual phase. As long as $S(t)$ has a nonvanishing envelope these measurements are unambiguous.

III. ERROR RATE ANALYSIS

The error rate of the E detector is determined from the conditional tail probabilities of E^2 [equation (4)]

$$F_\alpha = P_r[E^2 \leqq \alpha^2 \mid \bar{x}, \bar{y}]$$

and

$$F_\beta = P_r[E^2 \geqq \beta^2 \mid \bar{x}, \bar{y}]$$

where

$$\bar{x} = (V + Ab_0) \cos \theta_0$$

and

$$\bar{y} = (V + Ab_0) \sin \theta_0 .$$

It is difficult to analyze the exact representation of these probabilities and therefore we resort to exponentially tight upper bounds.

The lower tail probability is upper bounded by

$$F_\alpha = \int_0^\infty u(\alpha^2 - E^2)\, dP(E^2) \leqq \int_0^\infty e^{\lambda\alpha^2} e^{-\lambda E^2}\, dP(E^2)$$

$$= e^{\lambda\alpha^2} M_{E^2}(-\lambda), \qquad \lambda \geqq 0 \tag{6}$$

where $u(\cdot)$ is the unit step function, P is the probability distribution of E^2, and $M_{E^2}(\lambda) = \langle e^{\lambda E^2} \rangle$ is the moment generating function of E^2. In a like manner an upper bound to the upper tail probability is

$$F_\beta \leqq e^{-\beta^2\lambda} M_{E^2}(\lambda). \tag{7}$$

Since E^2 [equation (4)] is the sum of squares of two independent Gaussian random variables with means $\bar{x}$, $\bar{y}$ and identical variance σ^2 the moment generating function is readily calculated (see Ref. 5, p. 270). The result is

$$M_{E^2}(\lambda) = \frac{1}{1 - 2\lambda\sigma^2} \exp \left\{ \lambda \frac{\bar{x}^2 + \bar{y}^2}{1 - 2\lambda\sigma^2} \right\} \tag{8}$$

where

$$\bar{x}^2 + \bar{y}^2 = (V + Ab_0)^2 = a^2.$$

Using equation (8) in equations (6) and (7) gives

$$F_\alpha \leqq \frac{1}{1 + 2\lambda\sigma^2} \exp E_+(\lambda),$$

$$F_\beta \leqq \frac{1}{1 - 2\lambda\sigma^2} \exp E_-(\lambda) \tag{9}$$

where

$$E_+(\lambda) = \lambda\alpha^2 - \lambda \frac{a^2}{1 + 2\lambda\sigma^2}$$

and

$$E_-(\lambda) = -\lambda\beta^2 + \lambda \frac{a^2}{1 - 2\lambda\sigma^2}.$$

The upper bounds just derived hold for only positive λ. As is usual in these calculations there exists an optimum λ for each of these bounds which makes them exponentially tightest. To find these λ's we set the derivative of E_+ and E_- to zero to obtain:

$$\frac{d}{d\lambda} E_+(\lambda) = 0 \rightarrow \frac{-a^2 + \alpha^2}{4\alpha^2} + \lambda\sigma^2 + (\lambda\sigma^2)^2 = 0$$

yielding the two roots

$$\lambda = \frac{1}{2\sigma^2}\left(-1 \pm \frac{a}{\alpha}\right).$$

For a positive root to exist it is necessary that $a^2 > \alpha^2$ (a physically reasonable condition) and the optimum λ is therefore

$$\sigma^2\lambda = \frac{1}{2}\left(\frac{a}{\alpha} - 1\right).$$

When this value of λ is substituted into (9) we get the final result

$$F_\alpha \leqq \frac{\alpha}{a} \exp\left\{-\frac{1}{2\sigma^2}(a - \alpha)^2\right\}. \tag{10}$$

Similarly when $d/d\lambda\ E_-(\lambda)$ is set to zero we obtain two roots for λ namely $\lambda\sigma^2 = 1/2\ (1 \pm a/\beta)$. Here the physically reasonable condition is that $a/\beta < 1$ giving rise to two positive roots. When both of these roots are substituted into $E_-(\lambda)$ it is found that $\lambda\sigma^2 = 1/2\ (1 - a/\beta)$ yields the smallest value. Using this root in equation (9) results in the final answer for an upper bound on the upper tail probability.

$$F_\beta \leqq \frac{\beta}{a} \exp\left\{-\frac{1}{2\sigma^2}(\beta - a)^2\right\}. \tag{11}$$

We now have all the ingredients to perform error rate calculations.

Symbol detection is obtained by comparing the received envelope to fixed thresholds. The received envelope in the absence of noise is $V + Ab_0 = V \pm A(2k - 1)$, $k = 0, 1, 2, \cdots, L/2$ and the thresholds are set to $V \pm 2Ak$. We see from equations (10) and (11) that the exponents of both upper and lower tail probabilities depend on the difference between the received amplitude value and the threshold. The maximum of the difference is simply A. Consequently when only adjacent errors are considered, the error rate is upper bounded by

$$P_E \leqq C \exp\left\{-\frac{A^2}{2\sigma^2}\right\} \tag{12}$$

where the constant C is of minor importance. More general techniques[5] for studying the behavior of tail probabilities of arbitrary Gaussian quadratic forms show that equation (12) is also an asymptotic formula, approaching the actual error rate as $\sigma^2 \to 0$. We therefore use $\exp(-A^2/2\sigma^2)$ as a good estimate for the error rate in subsequent calculations.

The probability of error for phase detectors is well known (see Ref. 5). The explicit formula for differential phase detection is

$$P_{\phi d} = \exp\left\{-\frac{D^2}{2\sigma^2} 2\sin^2\frac{\pi}{2M}\right\} \tag{13}$$

while for coherent phase detection

$$P_{\phi c} = \exp\left\{-\frac{D^2}{2\sigma^2}\sin^2\frac{\pi}{M}\right\} \tag{14}$$

holds where D is the minimum value of the envelope in the absence of noise.

$$D = V - A(L - 1). \tag{15}$$

It is reasonable to require that the error rates in the E and ϕ detectors be exponentially identical. This requirement leads to the condition

$$A^2 = \begin{cases} 2(V - AL + A)^2 \sin^2\dfrac{\pi}{2M}, & \text{differential detection (D. D.)} \\ (V - AL + A)^2 \sin^2\dfrac{\pi}{M}, & \text{coherent detection (C. D.).} \end{cases} \tag{16}$$

Solving for V explicitly gives

$$V = \begin{matrix} Af_d(L, M), & \text{D.D.} \\ Af_c(L, M), & \text{C.D.} \end{matrix} \tag{17}$$

where

$$f_d(L, M) = L - 1 + \frac{1}{\sqrt{2}}\frac{1}{\left|\sin\dfrac{\pi}{2M}\right|}, \quad \text{D.D.}$$

and

$$f_c(L, M) = L - 1 + \frac{1}{\left|\sin\dfrac{\pi}{M}\right|}, \quad \text{C.D.}$$

These formulas indicate how the power is to be partitioned between the signal that carries phase information and the signal that carries the amplitude information and consequently determine the threshold levels in the E detector.

Using equation (17) in equation (2) we obtain a formula for the total average transmitted power as a function of the number of amplitude levels L and the number of phases M

$$P_S = \frac{A^2}{2}\left[f_i^2(L, M) + \frac{L^2 - 1}{3}\right] \tag{18}$$

where f_i is either f_d or f_c depending on the type of phase detector employed.

IV. THEORETICAL COMPARISONS

We now wish to compare the probability of error exponents of the mixed modulation system with that of pure phase and pure amplitude modulation at fixed signal-to-noise ratio. For the mixed modulation system we first express $A^2/2$ as a function of the power using equation (18) and then substitute into equation (12). Neglecting the constant coefficient we obtain

$$P_{E\phi} \sim \exp\left\{-\frac{P_S}{\sigma^2}\frac{1}{g_i^2(L, M)}\right\} \tag{19}$$

where

$$g_i^2(L, M) = f_i^2(L, M) + \frac{L^2 - 1}{3}.$$

When only phase modulation is used with total number of levels $N = ML$, the error rates become

$$P_\phi \sim \begin{cases} \exp\left\{-\frac{P_S}{\sigma^2}2\sin^2\frac{\pi}{2ML}\right\}, & D.D. \\ \exp\left\{-\frac{P_S}{\sigma^2}\sin^2\frac{\pi}{ML}\right\}, & \text{C.D.} \end{cases} \tag{20}$$

When only amplitude modulation* is used, $V = 0$ and $\{\theta_n = 0\}$ in equation (1), the error rate for $LM = N$ levels is[5]

$$P_a \sim \exp\left\{-\frac{P_S}{\sigma^2}\frac{3}{(LM)^2 - 1}\right\}. \tag{21}$$

We have thus far ignored the front-end filter which precedes the detectors. Since the data spectrum is flat across the band, no distortion

* We assume that coherent demodulation is used.

will result if the transfer function of the front-end filter is also flat. We therefore use the characteristic

$$H(f) = \begin{cases} 1, & f_0 - \dfrac{1}{2T} \leqq |f| \leqq f_0 + \dfrac{1}{2T} \\ 0, & \text{elsewhere} \end{cases} \tag{22}$$

where $\omega_0 = 2\pi f_0$.

Assuming that the added noise has a flat double-sided spectral density N_0, the variance σ^2 is

$$\sigma^2 = \frac{2N_0}{T} \tag{23}$$

and the common factor in all the error-rate expressions becomes

$$\Lambda = \frac{P_S}{\sigma^2} = \frac{TP_S}{2N_0} = \frac{\text{energy/bit}}{\text{watts/cycle}}.$$

The factors multiplying Λ in equations (19), (20), and (21) are all equal to or less than unity and therefore may be regarded as a degradation over binary systems. (These factors reduce to unity for binary systems.)

Comparisons between these systems can conveniently be made by noting the relative sizes of these coefficients as functions of L and M such that $N = LM$. Toward this end the following definitions are made.

$$\begin{aligned} D_{E\phi i} &= 10 \log_{10} g_i^2(L, N), \qquad i = c \quad \text{or} \quad d \\ D_\phi &= -10 \log_{10} \left(2 \sin^2 \frac{\pi}{2N}\right) \\ D_{\phi c} &= -10 \log_{10} \left(\sin \frac{\pi}{N}\right) \\ D_a &= 10 \log_{10} \left(\frac{N^2 - 1}{3}\right). \end{aligned} \tag{24}$$

A natural question which now arises is the following: for fixed N is there an L which minimizes $D_{E\phi i}(L, N)$? The answer is yes and the comparisons we present are based on this optimum choice of L — the number of amplitude levels.

Our first results are summarized in Table I for typical values of N. For instance, in a 4-level system all phase modulation yields better performance than a combined system since D_ϕ is less than $D_{E\phi i}$ by 1.7

TABLE I—TABULATION OF PERTINENT PARAMETERS

N	Optimum L (For combined systems only)	Optimum M (For combined systems only)	$D_{E\phi d}$ in dB	$D_{E\phi c}$ in dB	D_{ϕ} in dB	D_{a} in dB	$D_{\phi c}$ in dB
4	2	2	7.0	7.0	5.3	7.0	3.0
8	2	4	9.6	8.3	11.2	13.2	8.3
16	2	8	13.5	11.5	17.3	19.3	14.2
32	4	8	16.9	15.6	23.2	25.3	20.2
64	4	16	20.4	18.5	29.2	31.3	26.2
128	8	16	23.5	22.2	35.2	37.4	32.2

dB. There is no advantage here in splitting the levels. On the other hand for an 8-level system, 2 amplitudes and 4 phases appears preferable than just 8 phases or 8 amplitudes and the advantages gained are 1.6 dB when 8 phases are used and 3.6 dB when 8 amplitudes are used. It is always advantageous to mix the levels when $N \geqq 8$. The advantage becomes more pronounced as N becomes large.

We now turn attention to the communication efficiency of the various modulation schemes considered. The figure of merit used is the attainable number of bits per cycle for a given signal-to-noise ratio and fixed error rate. The rate R in bits per cycle attained by these systems is

$$R = \frac{\log_2 N}{TB} = \frac{\rho}{B} \log_2 N \tag{25}$$

where ρ is the symbol signaling rate and B is the required bandwidth. For ideal double-sideband systems such as PM and AM-PM, ρ/B is equal to unity while for baseband and single sideband systems, ρ/B is equal to two. We therefore have

$$\begin{aligned} (R)_{\text{DSB}} &= \log_2 N \quad \text{bits/cycle} \\ (R)_{\text{SSB}} &= 2 \log_2 N \quad \text{bits/cycle.} \end{aligned} \tag{26}$$

Since we are comparing systems operating over unequal bandwidths, it is appropriate to give an alternative interpretation of the constant

$$\Lambda = \frac{P_S}{2N_0} = \frac{P_S}{2N_0\rho}. \tag{27}$$

It is tempting to call Λ the signal-to-noise ratio. However the quantity $2N_0\rho$ is not necessarily the noise power in the physical bandwidth. The harmful noise power can only be determined once a relationship between the signaling rate and the physical bandwidth is given. For double sideband systems Λ is in fact the actual signal-to-noise ratio, since $\rho = B$. However for baseband and SSB, $\Lambda = \frac{1}{2}(\mathrm{S/N})$. We dwell on this point because often the parameter Λ is used to make comparisons and it is very easy to overlook the 3-dB factor when what is often of interest is the actual S/N. For example, we see from Table I that 4-level AM is 1.7 dB worse than 4-level PM. We must bear in mind however that the AM system for which the figures are given in the table are double sideband AM. If, however, the comparison is sought for baseband AM or SSB-AM then 3 dB more must be added to the 7 dB.

We see from equations (20) or (21) that in order to maintain a fixed error rate in PM and AM the signal-to-noise ratio must vary as the square of the number of levels when the latter is large. When S/N is expressed in dB it then is directly proportional to the logarithm of the number of levels and hence to the efficiency expressed in bits per cycle. Plots of efficiency vs S/N in dB for AM and PM will therefore appear linear with identical slopes. (The intercepts may be different.) A similar argument applies to SSB modulation but the resulting slope is twice as large compared with PM and AM. This is evident from the fact that in SSB twice as many bits per cycle can be attained as in PM or double sideband AM with the same number of levels.

We shall now show that the efficiency of combined AM and PM as a function of S/N expressed in dB is also linear and follows the same slope as the efficiency of SSB. In order to prove this claim we must demonstrate that in combined AM and PM S/N varies linearly with the number of levels to maintain a fix error rate. This variation can be deduced by considering the asymptotic behavior of $g_i^2(L, N)$ [equation (19)]. Disregarding all multiplicative factors of L and N and dropping the additive constants we see that

$$\begin{aligned} g_i^2(L, N) &\sim \left(L + \frac{N}{L}\right)^2 + L^2 \\ &\sim L^2 + N + N^2L^{-2}. \end{aligned} \tag{28}$$

The optimum L is proportional to $\sqrt{N}$ since

$$\frac{\partial g_i^2(L, N)}{\partial L} \sim L - \frac{N^2}{L^3} = 0 \tag{29}$$

implies that

$$g_i^2(N) \sim N. \tag{30}$$

The various relationships using similar calculations as above are briefly summarized below

SSB or AM Baseband

$$(\mathrm{S/N})_{\mathrm{dB}} \sim 20 \log_{10} N = \frac{20}{\log_2 10} \log_2 N$$

and since

$$(R) \text{ bits/cycle} = 2 \log_2 N$$

we have that

$$R \sim \frac{\log_2 10}{10} (\mathrm{S/N})_{\mathrm{dB}} .$$

Combined AM and PM

$$(\mathrm{S/N})_{\mathrm{dB}} \sim 10 \log_{10} N = \frac{10}{\log_2 10} \log_2 N$$

and since

$$(R) \text{ bits/cycle} = \log_2 N$$

we have that

$$R \sim \frac{\log_2 10}{10} (\mathrm{S/N})_{\mathrm{dB}} .$$

PM and Double Sideband AM

$$(\mathrm{S/N})_{\mathrm{dB}} \sim 20 \log_{10} N = \frac{20}{\log_2 10} \log_2 N$$

and since

$$(R) \text{ bits/cycle} = \log_2 N$$

we have that

$$R \sim \frac{\log_2 10}{20} (\mathrm{S/N})_{\mathrm{dB}} .$$

To complete the picture we find from Shannon's capacity formula that

$$\begin{aligned}(R)_{\mathrm{capacity}} &\sim \log_2 (\mathrm{S/N}) \\ &= \tfrac{1}{3}(\mathrm{S/N})_{\mathrm{dB}}\end{aligned} \qquad (31)$$

and since $\frac{1}{3}$ is very close to $\frac{1}{10} \log_2 10$ we see that SSB and combined AM and PM follow the capacity slope.

It now remains to uncover how far these curves are from one another. The results appear in Fig. 1. A most interesting picture is revealed; combined AM and PM is indeed only 3–4 dB worse than SSB when differential phase detection is used. When coherent phase detection is employed only 1–2 dB is given up.

V. SYSTEM IMPLEMENTATION

We now discuss the implementation of a combined AM-PM modem which was used to evaluate the performance of the various signal formats analyzed in the previous sections. For convenience a basic signaling rate of 1200 symbols per second was used operating over an ideal linear channel in the frequency range of 600-3000 Hz. The signaling elements were designed to have raised cosine shapes with 100-percent excess bandwidth.[5] In order to optimize performance in the presence of additive Gaussian noise, signal shaping was divided between transmitter and receiver.

The transmitter was designed to have the capability of generating the signal vectors shown in Fig. 2. For noncoherent (differential) demodulation of PM, the generated signal phases were spaced at odd

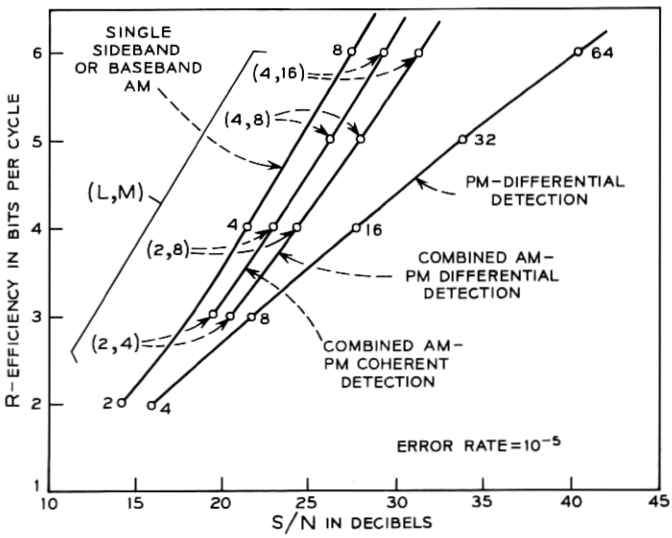


Fig. 1—Theoretical efficiency vs S/N for various modulation systems.

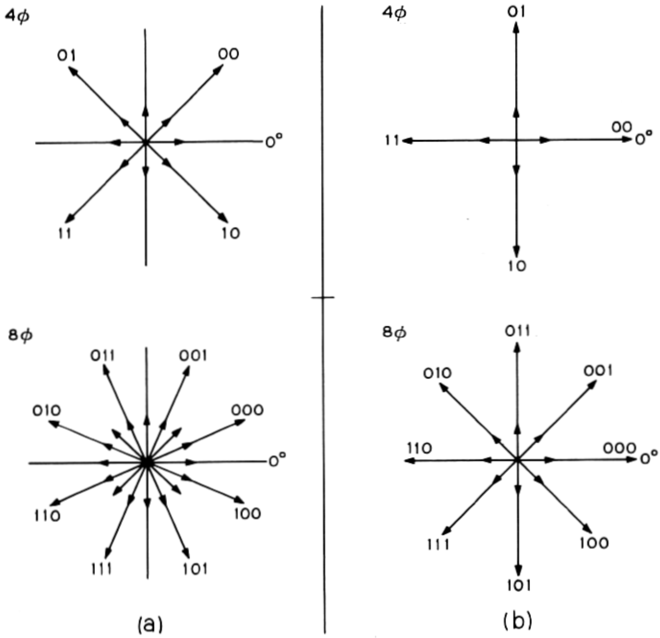


Fig. 2—Tail-to-head phase shifts: (a) noncoherent (differential) demodulation of PM; (b) coherent demodulation of PM.

multiples of 45 degrees and 22-1/2 degrees for 4ϕ and 8ϕ, respectively. For coherent PM demodulation, the spacing was multiples of 90 degrees and 45 degrees, respectively.

Generation of the amplitude levels was accomplished in a balanced amplitude modulator according to the formulas given in equation (17) and repeated on Fig. 3.

The receiver is shown in block diagram form in Fig. 4. The receiving filter eliminates out-of-band noise and provides signal shaping. Amplitude and phase demodulation was accomplished in separate parallel paths.

The phase detector was designed to operate in either a coherent mode or in a differentially coherent mode. For differential demodulation the signal from the receiving filter was multiplied with delayed and phase-shifted versions of itself. Low-pass filtering of these products produced in-phase (I) and quadrature (Q) baseband eye patterns. Analog arithmetic performed on I and Q at the sampling instants yielded the recovered data which was fed out in serial form. For coherent demodulation, everything was done in the same manner except that a coherent

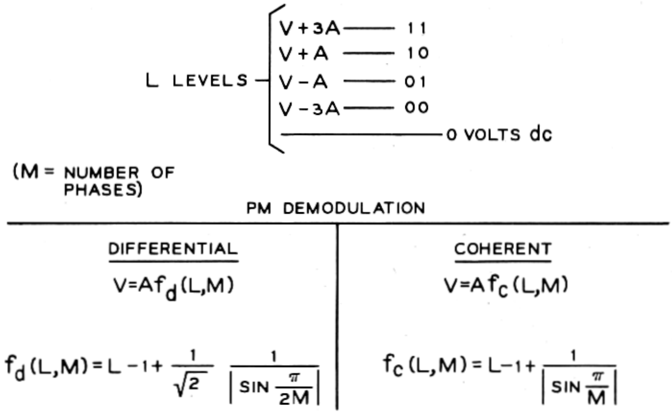


Fig. 3—Definition of amplitude levels.

local carrier signal was used to multiply the phase-shifted delay line outputs.

Amplitude demodulation was performed by extracting the envelope of the signal from the receiving filter. The coded information contained in the envelope (E) was obtained by slicing the envelope and folded envelope. The AM data was then fed out in serial form.

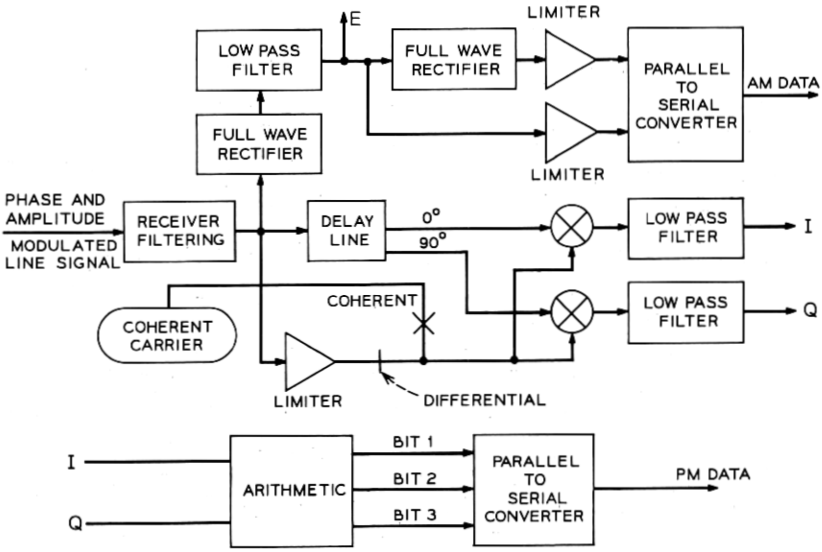


Fig. 4—Receiver block diagram.

Figures 5 thru 8 show baseband eye patterns of some of the signal formats generated for the experiments. Also shown are polar plots, generated by attaching the I and Q signals to the horizontal and vertical inputs of an oscilloscope. Z-axis intensification at the sampling instant produced the points in signal space.[6] Note that the coherent polar plots are rotated 45 degrees from this position in Fig. 2*b*. This 45-degree rotation eliminates a zero-voltage level which would otherwise appear in the eye patterns.

VI. EXPERIMENTAL RESULTS

The configuration shown in Fig. 9 was used to evaluate the error performance of various signal formats in the presence of additive band-limited Gaussian noise. The various signal formats generated in the experiments are summarized below:

4ϕ	Differential (D) Coherent (C)
8ϕ	D, C
$2L$, 4ϕ	D, C
$2L$, 8ϕ	D, C
$4L$, 8ϕ	D, C

Since timing jitter and phase off-set are not included in the theory, timing phase and coherent carrier phase were adjusted manually at the receiver.

The experimental results are shown in Fig. 10. These results were used to generate the efficiency curve of Fig. 11. The theoretical results are again repeated (properly scaled) on this curve.

Correspondence between predicted and measured performance is seen to be quite good. For example, the predicted difference in performance between 8ϕ differential and (4ϕ, $2L$) differential is 1.6 dB. The measured difference is found to be 1.5 dB.

Another indicator of how closely the implementation and the theoretical model match can be seen from Fig. 10. The measured performance of 4ϕ differential departs from theory by only 0.8 dB.

VII. ACKNOWLEDGMENT

We wish to acknowledge R. J. Tracy for making significant design contributions to the experimental modem.

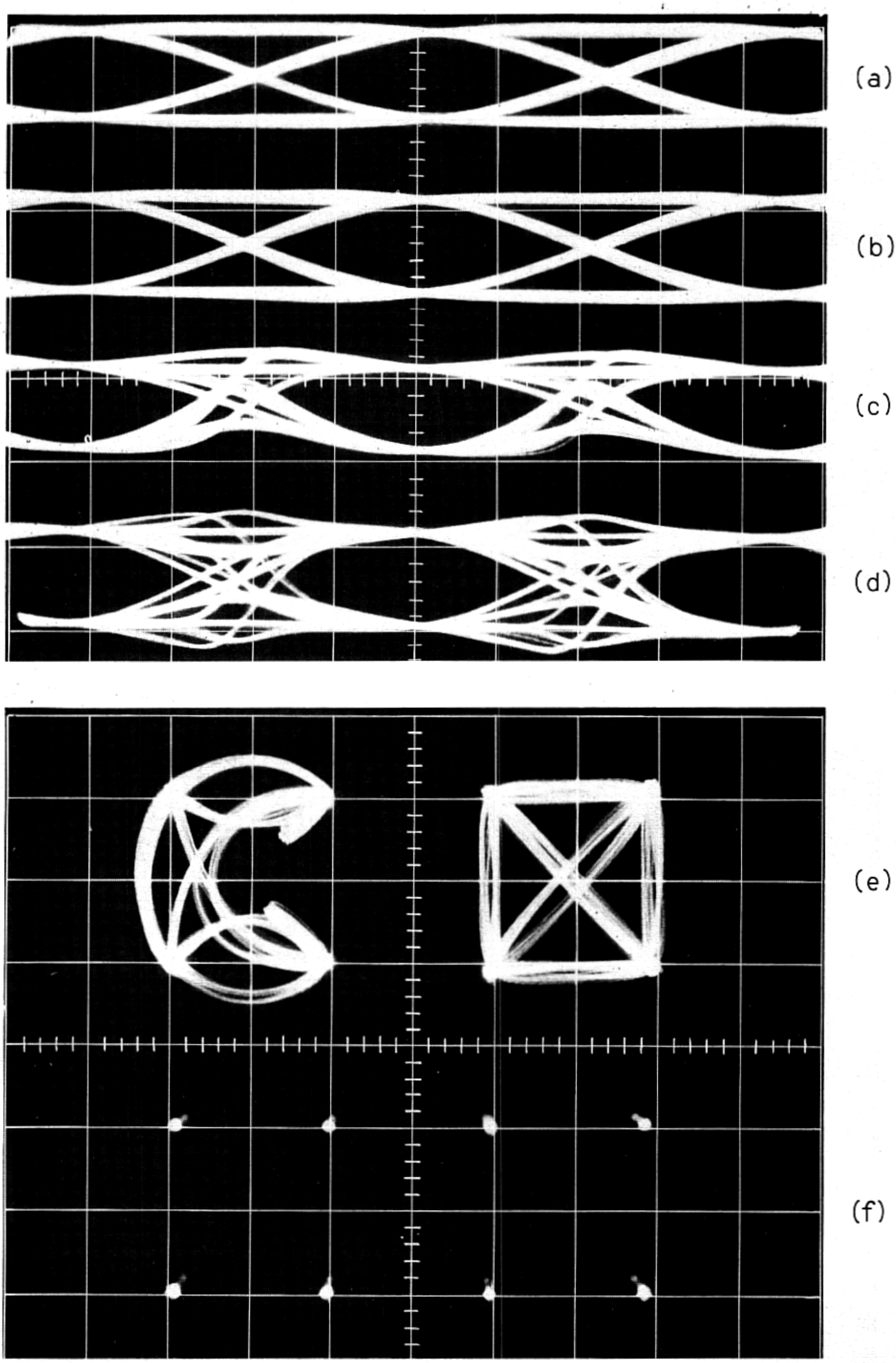


Fig. 5—Four-phase eye patterns: (a) Q coherent; (b) I coherent; (c) Q differential; (d) I differential; (e) polar plots, l. to r., differential, coherent; (f) polar plots intensified at sample time.

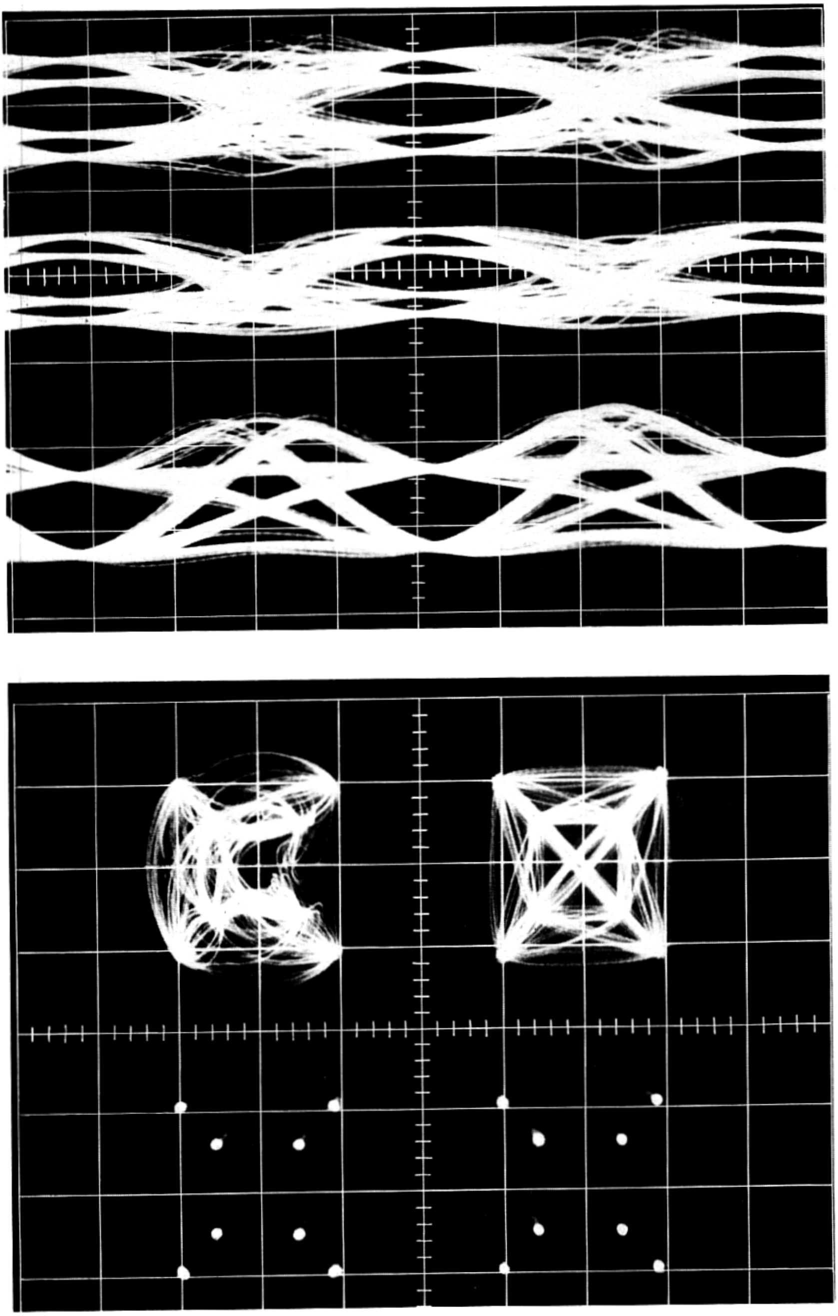
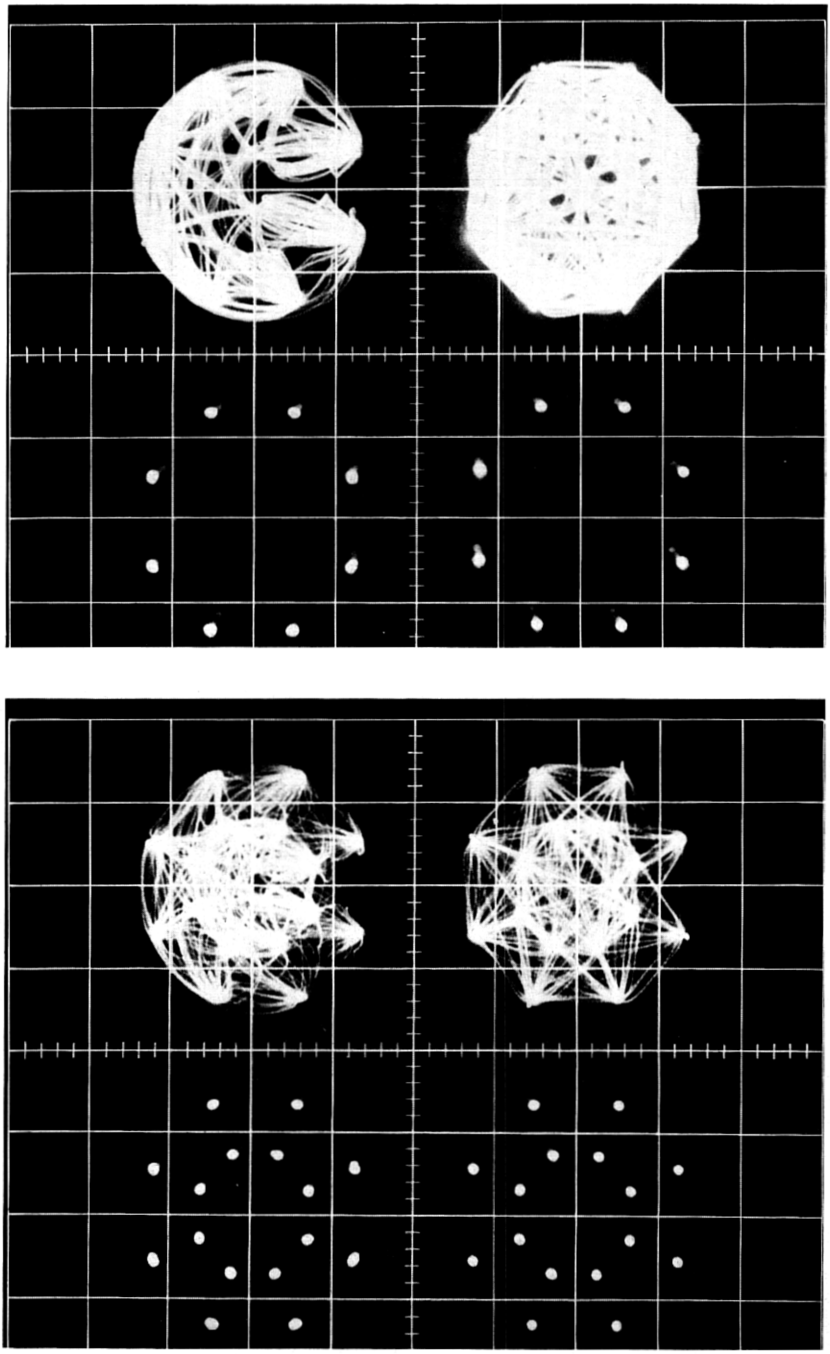


Fig. 6—Four-phase, two-level eye patterns: (a) I differential; (b) Q differential; (c) AM envelope; (d) polar plots, l. to r., differential, coherent; (e) polar plots intensified at sample time.



(a)

(b)

Fig. 7—(a) Eight-phase eye patterns; (b) Eight-phase, two-level eye patterns.

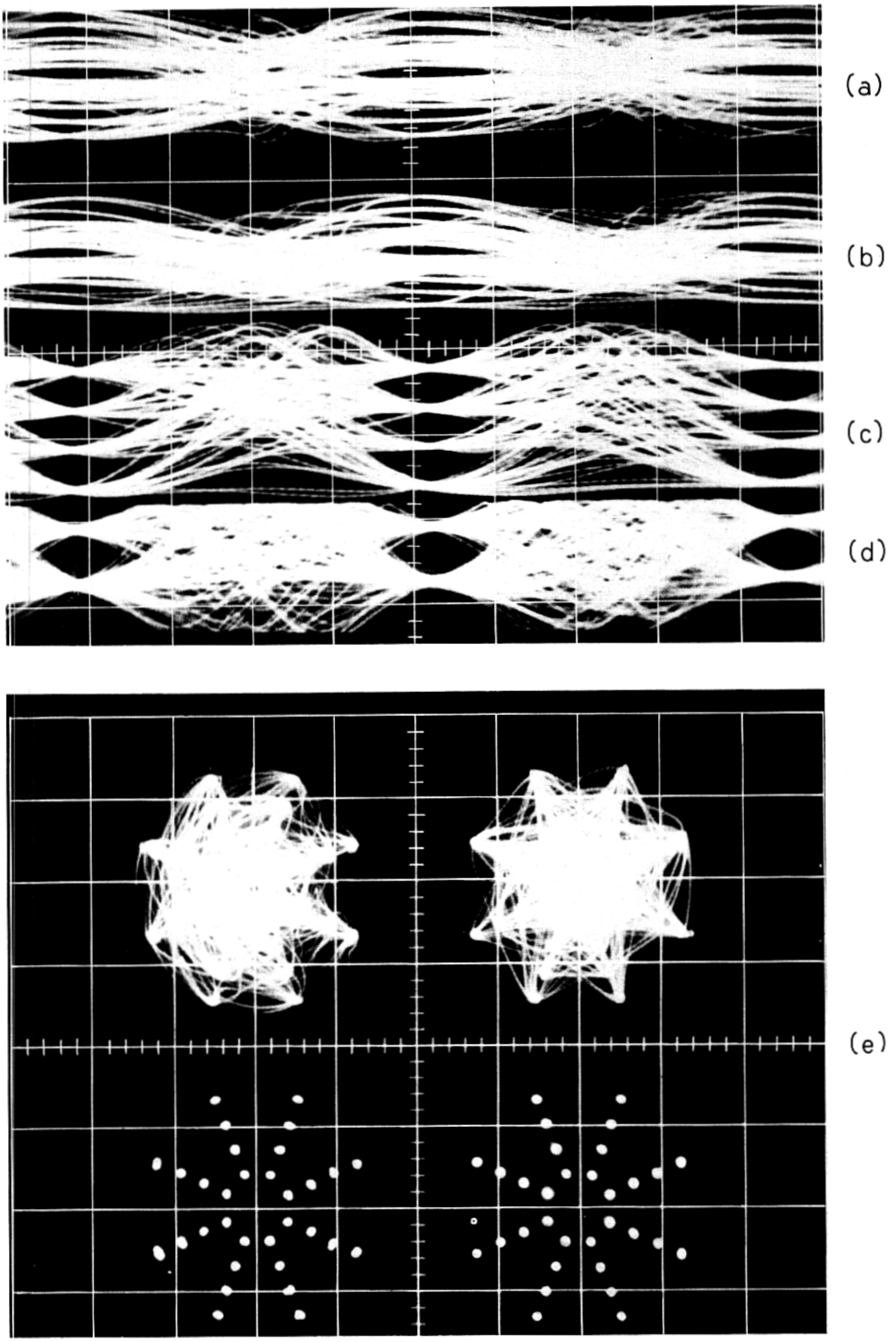


Fig. 8—Eight-phase, four-level eye patterns: (a) I differential; (b) Q differential; (c) AM envelope; (d) folded envelope; (e) polar plots.

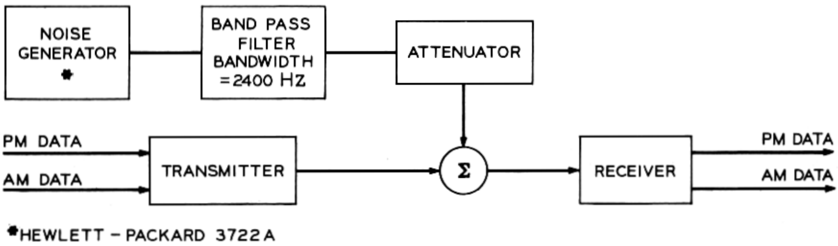


Fig. 9—Experimental configuration.

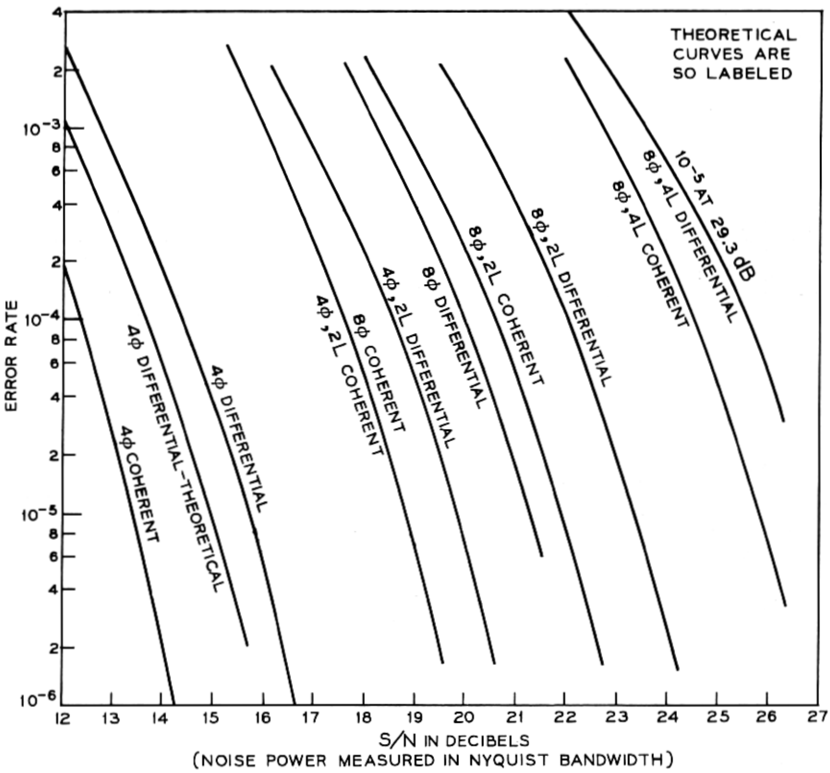


Fig. 10—Performance curves.

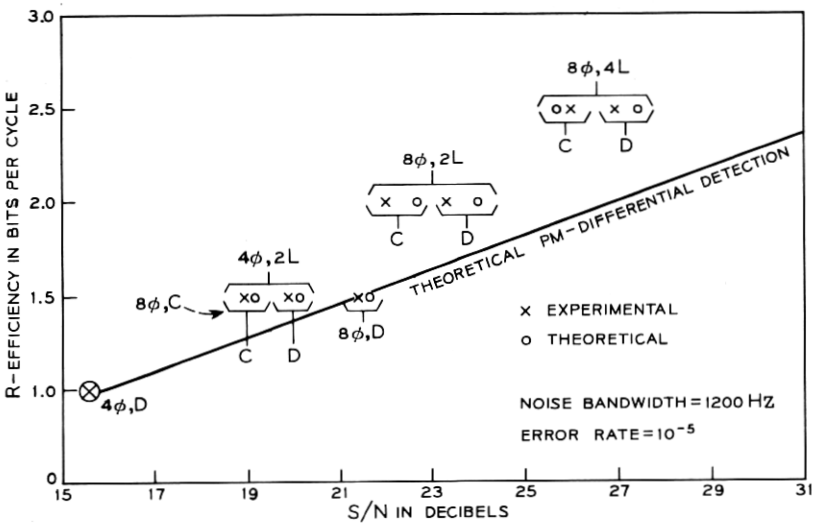


Fig. 11—Performance comparisons for noise impairment.

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