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Some Comparisons of Load and Loss Data With Current Teletraffic Theory*

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Data are presented and compared with current traffic loss theory in three trunking areas: high-usage groups; full (nonalternate routed) groups; and final groups. Both single hour and average busy season busy hour load-loss comparisons are made. Methods of estimating offered loads from carried loads, and from the proportion of calls blocked, are considered. The blocking which results from offering nonrandom traffic to overflow groups is discussed. Modifications in the theory commonly applied are indicated in each case when necessary to obtain satisfactory agreement with observed values.

I. INTRODUCTION

The ultimate test of theory is comparison with reality. The procurement of reliable and reproducible traffic data is relatively difficult since little control can be exercised over the character and level of the input to the load carrying system without invalidating the genuineness of the input. There is then a continuing need to compare the commonly used load-loss relations with observations made under a variety of trunking configurations and load characteristics. Data are presented here and compared with loss theory in three trunking areas: high-usage groups; full (nonalternate routed) groups; and final groups. The various findings are summarized at the end of the paper.

II. HIGH-USAGE GROUP STUDIES

2.1 *Loads and Losses on High-Usage Groups*

It is commonly assumed that the requirements of a "random" or Poisson input are met until originating traffic has passed through a restrictive group of paths or switches. It would be difficult and cer-

* Substantially as presented at the Sixth International Teletraffic Congress, Munich, 1970.

tainly impractical in most operational situations to check such an assumption by examining the call arrival instants or to analyze their interarrival times, as well as make a corresponding study of their service times. Since in any event exact conformity with the theoretical assumptions could not be found, the question would remain as to the relative adequacy with which they were met.

The traffic engineer's usual wish is to describe the blocking which will occur in real-life situations when a given average load is offered to a group of paths or switches. It will generally suffice then to compare observed load versus loss relationships with those theoretically derived, rather than attempt an assessment of the agreement of more basic requirements.

The simplest trunking situation occurs when a parcel of traffic arising from an indefinite (but potentially large) number of customers is offered to a full access high-usage trunk group. A high-usage group is presumed here to receive calls quite directly from customers without earlier significant constriction and serves them immediately if paths are available. If, however, all of the high-usage trunks are busy, arriving calls are alternate routed to other groups in the hierarchy; and negligibly few calls return to the high-usage group for retrial. Thus the conditions for the application of Erlang's loss formula are apparently well satisfied. How well do observations made in actual exchanges agree with this theory?

Data of the above character have been obtained on many high-usage groups, both interlocal and intertoll. For making comparisons, hourly readings of three recording registers are commonly taken:

(*i*) Number of calls offered, m
(*ii*) Number of calls blocked, n
(*iii*) Average number of calls seen on trunks during the period by switches-in-use counts taken every 100 seconds, giving an estimate of the average load carried in erlangs, ℓ.

Observations made by A. Descloux[1] indicate that a substantially unbiased estimate a of the hourly offered load is obtained from

$$a = \ell \Big/ \left(1 - \frac{n}{m}\right). \tag{1}$$

Typical results of plotting proportion blocked, n/m, versus load offered a are shown in Figs. 1, 2, and 3. Figures 1 and 2 are for interlocal groups studied at an Arlington, Massachusetts, No. 5 crossbar office, and at

Kildare (Chicago), a No. 1 crossbar office. Figure 3 data are on intertoll groups at Memphis, Tennessee, a sectional center in the toll hierarchy.

We know that over a long period an equilibrium load of a erlangs will show a significant hour-to-hour variability in offered loads.[2] Also, when such variable loads are offered to a trunk group, we should expect the individual hour losses to tend to fall on the convex side of an arithmetic plot of the long-run theoretical load-loss curve, since when aggregated their average loss should just meet the theory. To examine this phe-

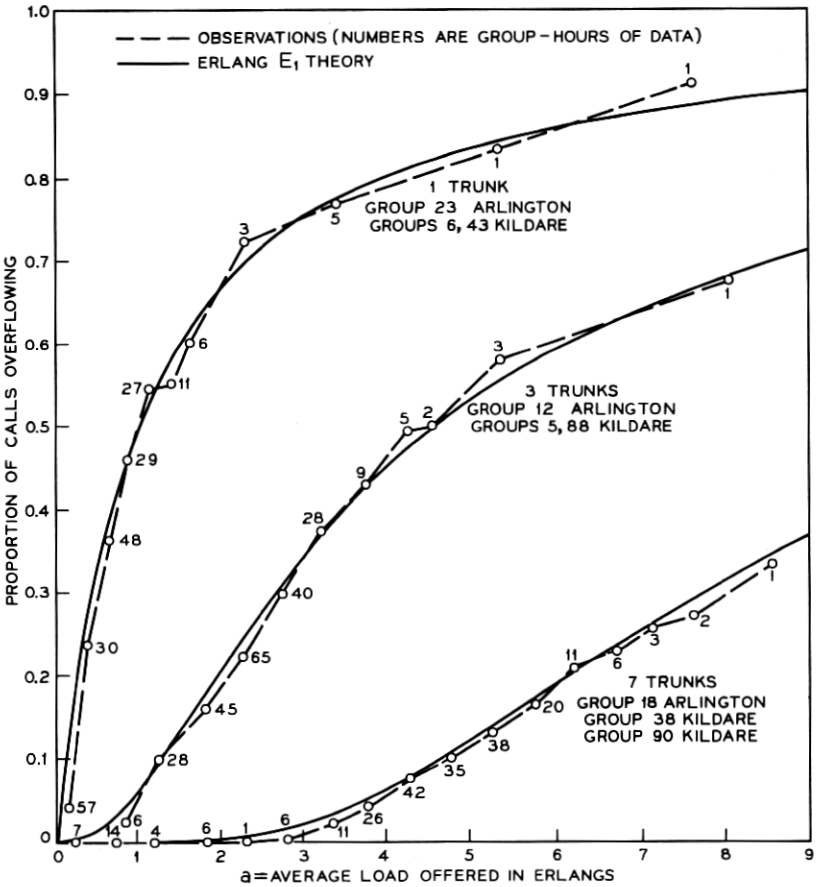

Fig. 1—Comparison of Erlang's loss theory with observations on interlocal high-usage trunk groups: busy hour, Arlington, 29 days; and busiest 3 hours, Kildare, 20 days, 1958.

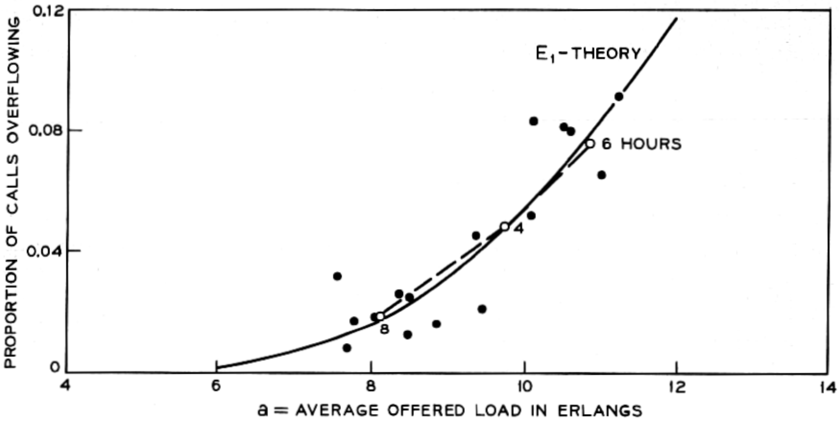

Fig. 2—Comparison of Erlang's loss theory with observations on interlocal high-usage trunks: Kildare group No. 12, 14 trunks, 11–12 A.M., 18 days, 1958. (• single hour. ○ aggregated hourly values.)

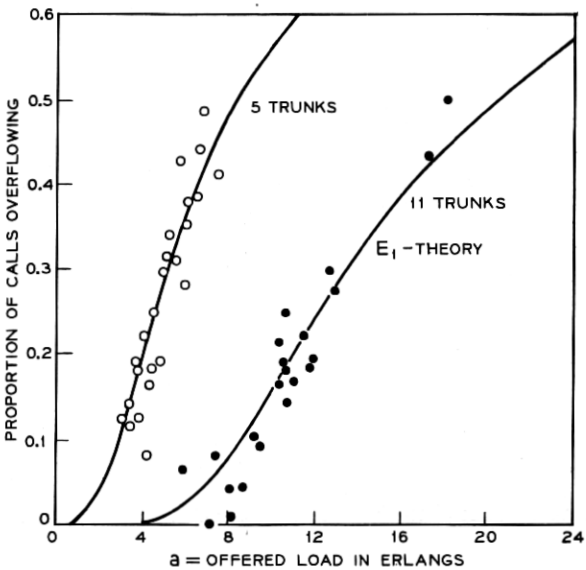

Fig. 3—Comparison of Erlang's loss theory with observations on intertoll high-usage trunk groups, Memphis, 1957–1958: 5 trunks to Milwaukee, 11 trunks to Cleveland.

nomenon under controlled conditions two simulations were run with a designed offer of 1.50 erlangs comprised of 180-second exponential holding time calls. The actual average offer observed was $\bar{a} = 1.485$ erlangs to 1 trunk for 400 hours, and 1.409 erlangs to 4 trunks for 100 hours. The results are displayed on Fig. 4. The individual hour losses, while conforming generally to the shape of the theoretical curve, appear to be slightly more linearly disposed. As longer summary periods are used, the observed values would be expected to approach the Erlang loss curve. This is seen clearly in the simulation results. A slight semblance of this tendency is seen on Figs. 1 thru 3, but it is considerably obscured by other fluctuations.

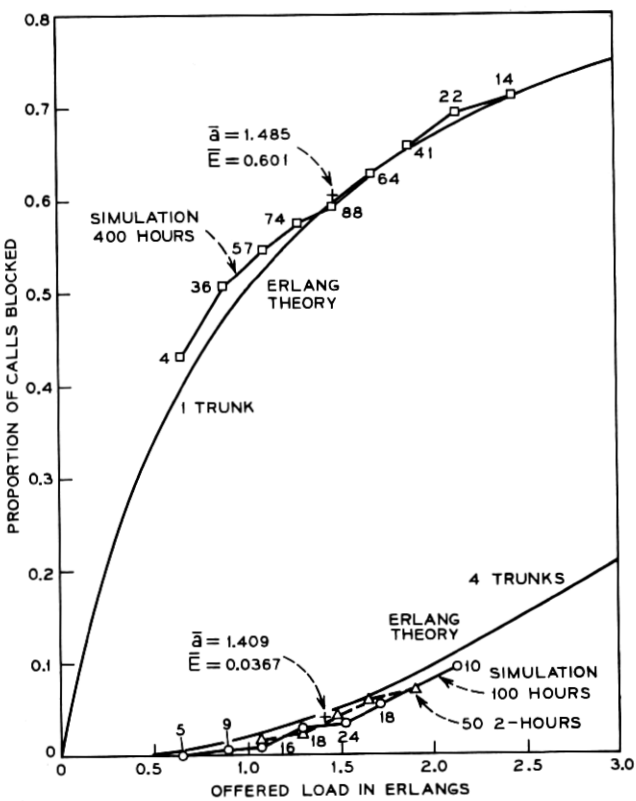

Fig. 4—Load vs loss simulation, with equilibrium offered loads, results summarized by 1- and 2-hour periods. (Numbers at points show hours of data averaged; $\bar{a}$ and $\bar{E}$ are mean values for all hours.)

We are directed to the conclusion that although Erlang's loss theory is properly applied only to a system operating in statistical equilibrium over a long period of time, for engineering purposes it is suitable for describing conversation type traffic load versus loss results summarized by periods as short as one hour (summary period $\geqq 20$ average holding times).

2.2 *Estimation of Hourly Offered Loads from Switch Counts*

It is customary to furnish only a switch counting device on high-usage trunk groups, and to depend on the accuracy of Erlang's load-loss relation,

$$E_{1,x}(a) = \frac{a^x/x!}{1 + a + a^2/2! + \cdots + a^x/x!},$$

to estimate the corresponding offered load by solving for a in

$$\ell = a[1 - E_{1,x}(a)] \tag{2}$$

where x is the number of trunks in the group. Descloux[1] has shown that a certain amount of bias occurs by this method. The reason for this is made clear by examining Fig. 5.

Figure 5a shows the loads offered, as estimated by equation (1), to three 1-trunk groups during the 3 busiest hours of the day for 20 business days. The load carried (= occupancy here), as determined by switch counting, is plotted as the ordinate. The Erlang relation is drawn as the solid line.

The points comprise a correlation scatter diagram. The classical regression line of "y on x," that is, the average value of carried load for a selected average offer, would be expected to agree with the Erlang loss relation, since the latter is derived on the basis of offered load as the "independent variable." The "y on x" regression line generated from the data is indeed seen on Fig. 5b (dashed broken line) to conform nicely with the Erlang theory.

Unless there were perfect correlation, the "x on y" regression line could not be expected to agree with Erlang's theory, and this is corroborated by the observed regression (solid broken) line on Fig. 5b. At lower than median occupancies (and offered loads) this regression line lies slightly below the Erlang theory, and for larger than median occupancies, it lies consistently above. Thus when estimating offered loads from carried loads by Erlang's loss formula [equation (2)] we should expect the values to be smaller than true for lower than average occupancies, and greater than true for larger than average occu-

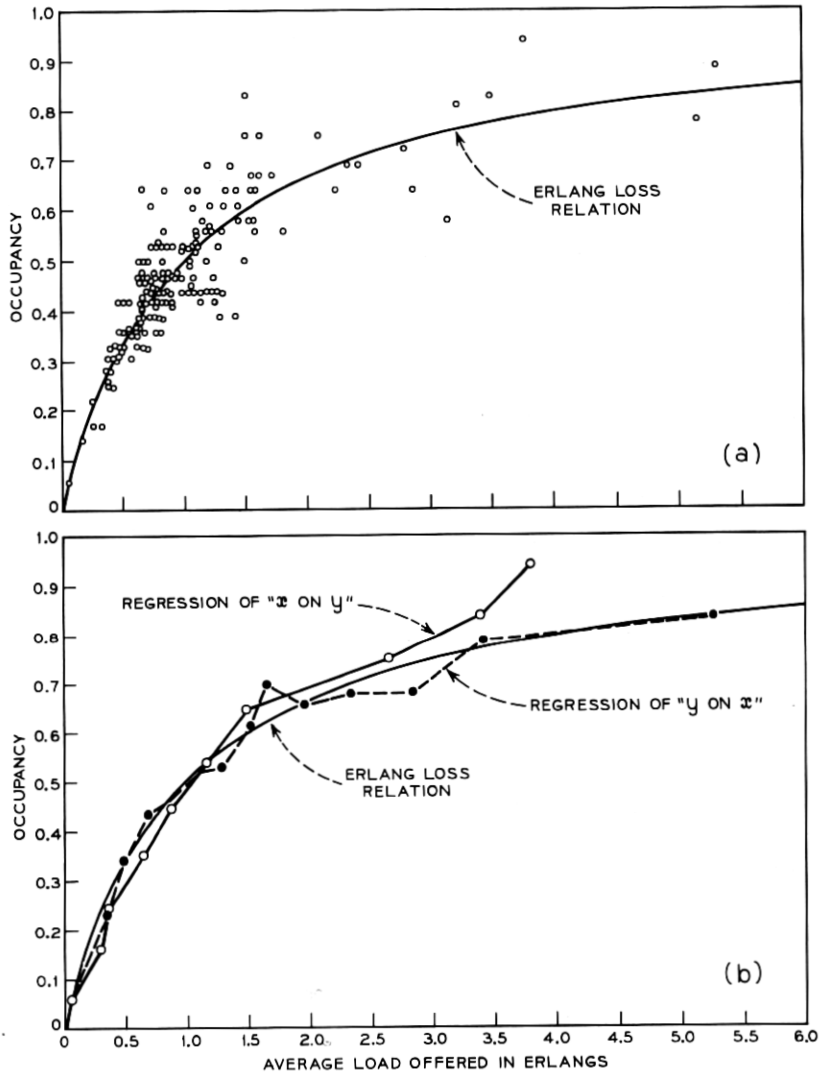

Fig. 5—Carried vs offered loads, data on three 1-trunk high-usage groups, Kildare, 1958, 3 hours per day, 20 business days. (a) scatter diagram, (b) observed regression lines.

pancies. For example, for one trunk at an observed load carried (occupancy) of 0.75 erlang, Erlang theory would predict an offered load of 3.0 erlangs, while the observed regression line would more nearly suggest an offer of 2.64 erlangs, a difference of 14 percent.

An appropriate theoretical expression for the "x on y" regression in this circumstance is difficult to generate, depending partially as it does on the day-to-day distribution of the offered loads, and this last becomes a matter of observation of customer characteristics. Since the regression lines may be decidedly nonlinear, the theoretical approaches of simple correlation theory are inadequate. In the face of such difficulties, one reverts to the reduction of data taken in the field for practical estimating results.

Figure 6 shows the ratio of two estimates of the offered load, a_2/a_1, versus observed occupancy of several 1-trunk groups. Here a_1 is the assumed unbiased estimate from equation (1), and a_2 is from equation (2), the latter making use only of the carried load measurement. In spite of the rather wide differences in loading among the five groups shown, the a_2/a_1 ratios are quite similar and one would not have too much difficulty in drawing a central ratio line through the field for general correction of a_2 values.

Figure 7 shows a similar chart for three trunks, indicating the ten-

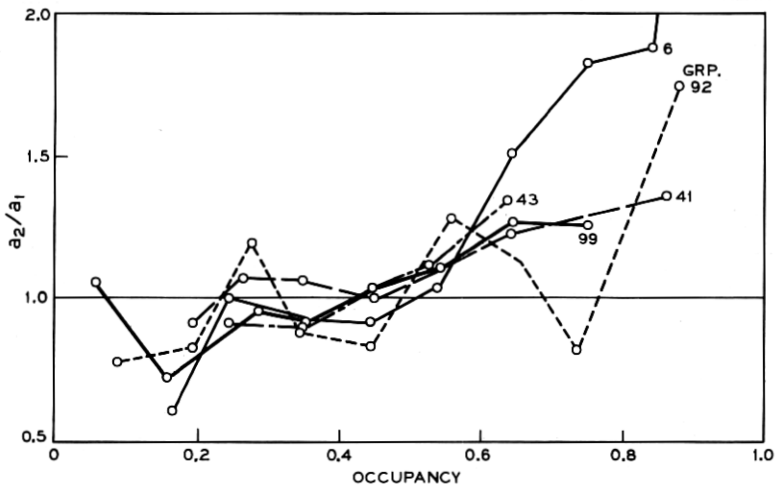

Fig. 6—Ratio of estimates of single hour loads offered to 1 trunk. Estimate a_1 is from switch and call counts, estimate a_2 from switch count and Erlang's loss formula.

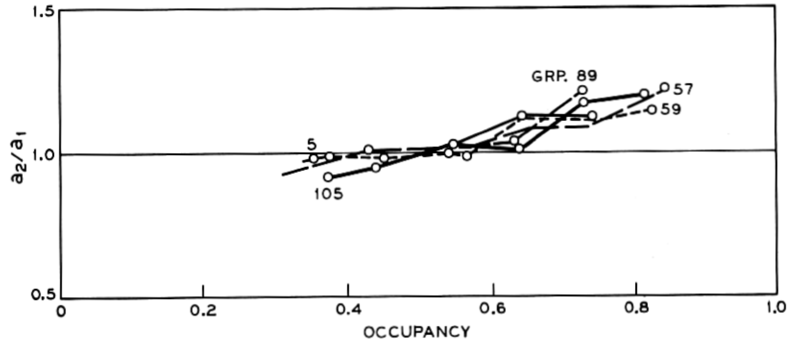

Fig. 7—Ratio of estimates of single hour loads offered to 3 trunks. Estimate a_1 is from switch and call counts, estimate a_2 from switch count and Erlang's loss formula.

dency for the ratios to approach 1.0 with increasing trunk group size. Figure 8 shows a rough summary of the occupancies at which the ratios would exceed 1.05 with various-sized trunk groups. Thus one might conclude, for example, that estimates by Erlang's loss formula of hourly offered loads on 12-trunk groups would not be acceptable

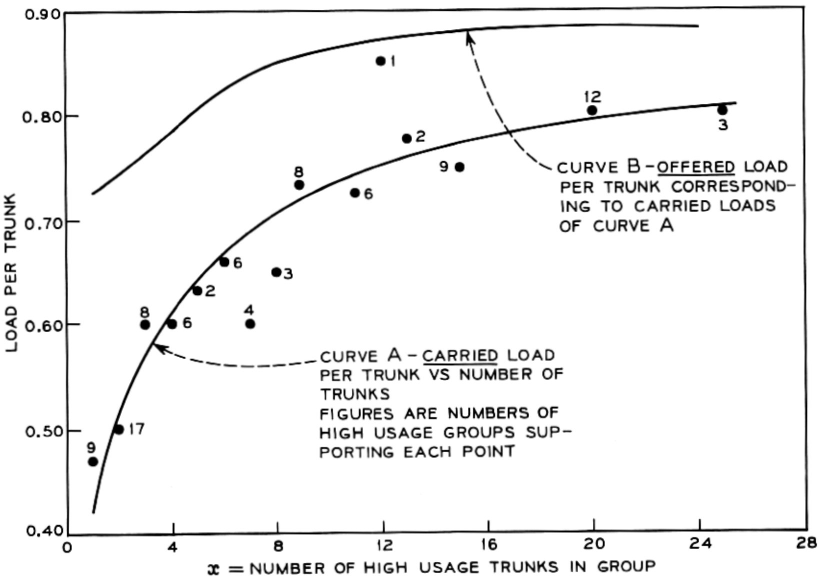

Fig. 8—Median occupancy (load per trunk) beyond which ratio a_2/a_1 for single hours exceeds 1.05 (approx.). Kildare data for 3 morning hours.

without correction (that is they would over-estimate by more than 5 percent) for occupancies greater than 0.75.

2.3 *Estimation of Average Overflow Load from Average Offered Load During a Busy Season*

When an offered load a varies from hour to hour in excess of the amount expected in 1-hour segments of a longtime load in statistical equilibrium,[2] the average overflow $\bar{\alpha}$ over a period of time will exceed that estimated by entering the Erlang loss relation with $\bar{a}$, that is, $\bar{\alpha} > \bar{a} \cdot E_{1,x}(\bar{a})$. This result is caused by the always concave upwards shape of the offered load, overflow load curves. The amount of such excess will depend strongly on the magnitude and character of the offered loads; in any event as a gets larger, the effect decreases for a given size of trunk group.

Numerous studies have shown that day-to-day busy hour variations in the busy season tend to follow a Type III Pearson distribution,

$$\theta(a) = Ka^h e^{-ca}, \tag{3}$$

in which the constants are determined from the mean $\bar{a}$ and variance v of the data, as

$$h = \bar{a}^2/v - 1$$

$$c = \bar{a}/v$$

and K is the normalizing coefficient, $c^{h+1}/\Gamma(h + 1)$. Typical is the example shown in Fig. 9 for a 9-trunk interlocal group in which $\bar{a} = 5.72$, $\text{Var}(a) = 1.61$, yielding $h = 19.20$, $c = 3.55$.

Again, there is found a considerable correlation between the day-to-day variance and the mean of such a distribution. Figure 10 shows on a log-log plot the field of variances versus means of loads offered during 3 hours each day for 20 days on 72 high-usage interlocal groups at Kildare office. Summaries at other exchanges in the United States confirm a similar association of variances and means. The general line of regression of variance on mean for the corresponding scatter diagrams is shown by the solid line, whose equation is approximately

$$\text{Var}\ (a) \doteq 0.31\bar{a}. \tag{4}$$

(A dashed line has also been drawn to approximate the major axis of the elliptical pattern of points. Its equation is

$$\text{Var}\ (a) = 0.13\bar{a}^{1.58}. \tag{5}$$

It will be referred to in a later section.)

If the Erlang loss formula can be used to estimate the proportion of calls which overflow a high-usage group during a single hour, the average load overflowing, $\bar{\alpha}$, over a series of hours is then calculable from

$$\bar{\alpha} = \int_{a=0}^{\infty} aE_{1,x}(a)\theta(a)\, da. \tag{6}$$

Similarly, the day-to-day variance of the overflow loads is determined from

$$\text{Var}(\alpha) = \int_{a=0}^{\infty} [aE_{1,x}(a)]^2\theta(a)\, da - \bar{\alpha}^2. \tag{7}$$

Curves have been calculated by numerical integration, using the regression relation of Fig. 10, which give the ratio of $\bar{\alpha}$ to $\alpha_{\bar{a}}$, the latter being the overflow corresponding to an offer of $\bar{a}$, found according to

$$\alpha_{\bar{a}} = \bar{a}E_{1,x}(\bar{a}). \tag{8}$$

Values of $\bar{\alpha}/\alpha_a$ are given on Fig. 11. At constant loads on the last trunk (shown by dashed lines for 100 call seconds per hour, or CCS) the ratios $\bar{\alpha}/\alpha_{\bar{a}}$ are relatively constant; hence a simple table, Table I,

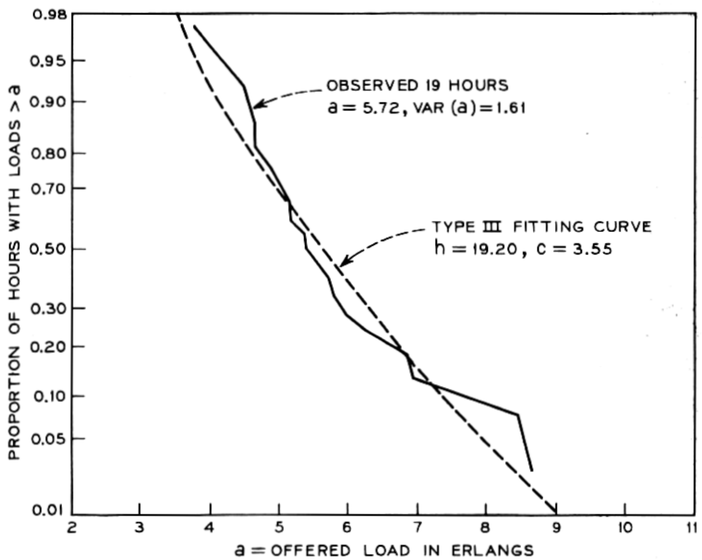

Fig. 9—Variations in day-to-day busy hour loads, Kildare group No. 67, 9 trunks, 20 days.

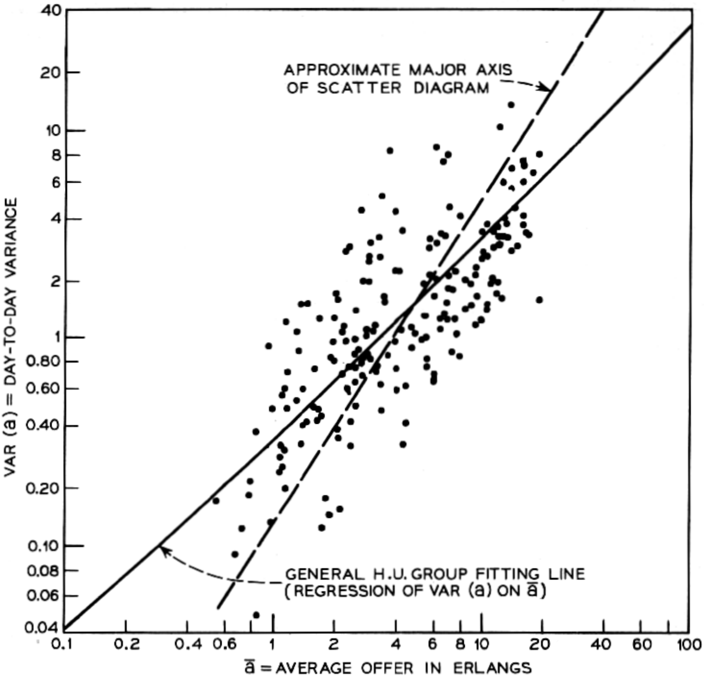

Fig. 10—Day-to-day variance vs average load in clock hours, 72 interlocal high-usage groups, 3 hours, Kildare, 1958.

using the last trunk load as the index is adequate for many working purposes. An example of the need for correcting by the factors of Table I is shown in Fig. 12. In the left diagram, uncorrected $\alpha_{\bar{a}}$ values are plotted against average overflow loads, $\bar{\alpha}_1$, calculated by the believed generally unbiased procedure

$$\bar{\alpha}_1 = \frac{1}{s} \sum_{i=1}^{s} \ell_i \frac{n_i/m_i}{1 - n_1/m_i} \tag{9}$$

where s is the number of hours summarized for each group. [Compare with equation (1).]

After correction the overflow estimates are shown in the right diagram to be in much better agreement with the $\bar{\alpha}_1$ values.

2.4 *Estimation of Average Offered Loads from Average Loads Carried*

To obviate the labor of calculating individual hourly estimates of the offered loads from observed hourly carried loads on high-usage

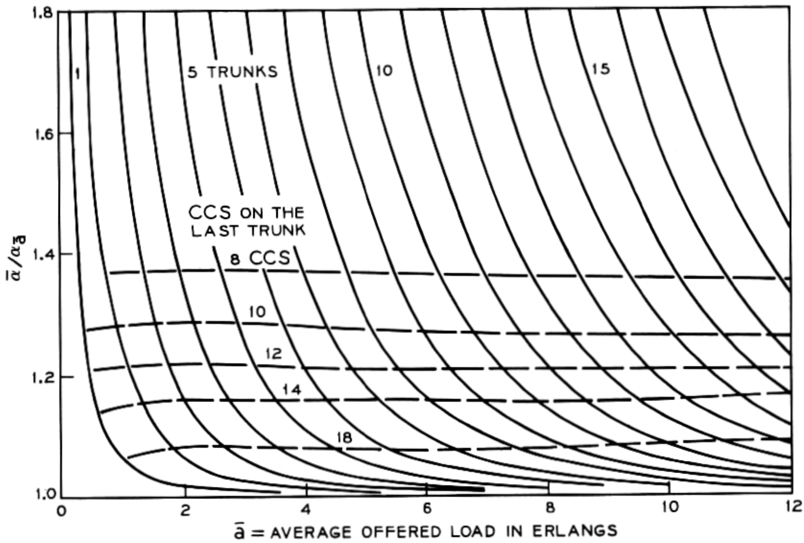

Fig. 11—Correction required in $\alpha_{\bar{a}}$ to estimate $\bar{\alpha}$.

groups (which would in turn require a correction as discussed in Section 2.2), the carried loads ℓ are commonly averaged first, and this average $\bar{\ell}$ is then entered in Erlang loss theory curves or tables to obtain $a_{\bar{\ell}}$ from

$$\bar{\ell} = a_{\bar{\ell}}[1 - E_{1,x}(a_{\bar{\ell}})]. \tag{10}$$

One needs then to compare $a_{\bar{\ell}}$ with the true offer $\bar{a}$. This is done as follows. Choosing a value of $\bar{a}$ as the offer to x trunks, $\bar{\alpha}$ is obtained by correcting $\alpha_{\bar{a}}$ employing the appropriate factor from Fig. 11. One then obtains $\bar{\ell}$ from

$$\bar{\ell} = \bar{a} - \bar{\alpha} \tag{11}$$

TABLE I—CORRECTIONS IN $\alpha_{\bar{a}}$ TO ESTIMATE $\bar{\alpha}$

Load on the Last Trunk		Range of Corrections Seen in Fig. 11	Corrections to $\alpha_{\bar{a}}$ for Practical Use
CCS	erlangs		
8	0.22	1.32–1.37	1.33
10	0.28	1.22–1.28	1.25
12	0.33	1.16–1.21	1.20
14	0.39	1.11–1.16	1.15
18	0.50	1.05–1.09	1.08

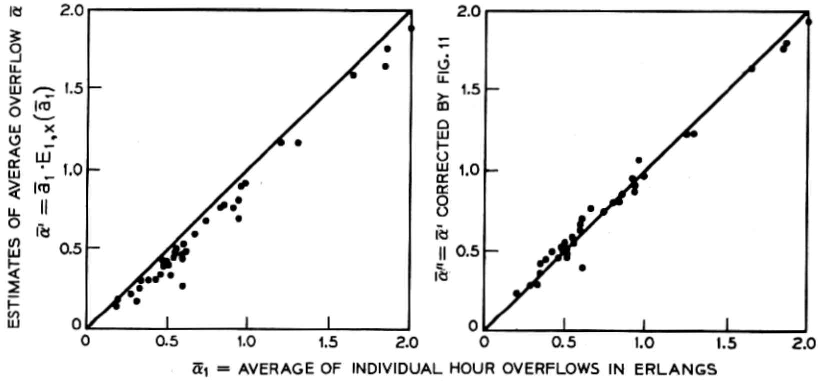

Fig. 12—Effect of corrections by Fig. 11 on estimates of overflows from high-usage groups of Kildare Tandem No. 1, 9–10 A.M.

since that load which does not overflow must naturally be carried. Relation (10) then gives $a_{\bar{l}}$. A field of values of $\bar{a}/a_{\bar{l}}$ have been calculated and form the curves of Fig. 13. We see that the ratio is sensitive both to proportion of overflow and to the number of trunks.

Corresponding data for Kildare groups are shown on Fig. 14. Similar forms of probability density distributions are observed.

It will be noted that there is a generally maximum ratio on Fig. 13 which tends to occur roughly at $E_{1,x}(a) = 0.2$, that is, at an expected overflow of 20 percent. It is interesting that this maximum correction required on $a_{\bar{l}}$ occurs squarely in the middle of the most common economic high-usage group operating levels.

For practical use we have constructed the traces of the 3-dimensional surface of Fig. 13 which correspond to several values of "economic CCS on the last trunk." (For a discussion of economic CCS, see Ref. 3, Section 7.6.) The theoretical $\bar{a}/a_{\bar{l}}$ ratios for 8, 14, 20, and 25 CCS on the last trunk are shown on Fig. 15. Comparison with data taken on a number of intertoll high-usage groups at Memphis, Tennessee, is shown in Fig. 16. Although the dispersion among individual groups is considerable, the grouped average values show reasonable agreement with theory. Table II shows the values of corrections to $a_{\bar{l}}$ required to best approximate the true offer $\bar{a}$ for 20 days of data, for the four selected last-trunk CCS. A brief study indicates that when the busy season includes fewer than 20 days, the required correction is somewhat smaller.

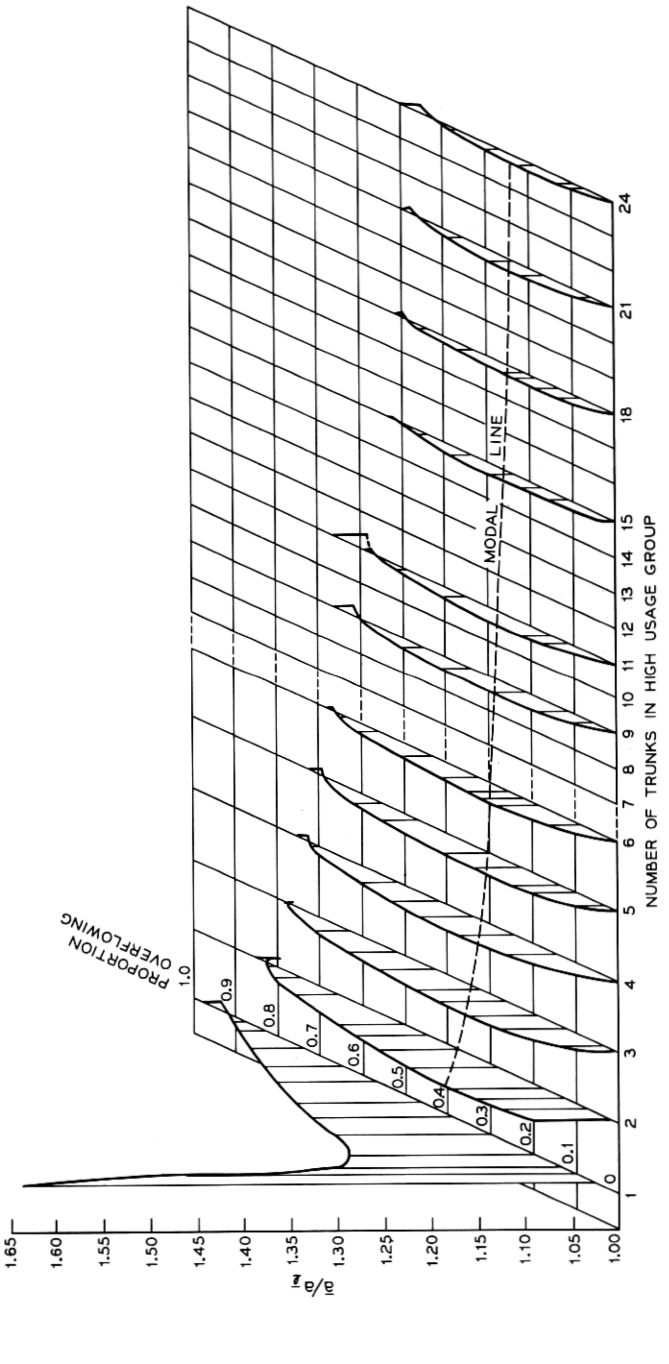

Fig. 13—Theoretical corrections required in $a_{\bar{l}}$ to obtain improved estimates of offered load $\bar{a}$.

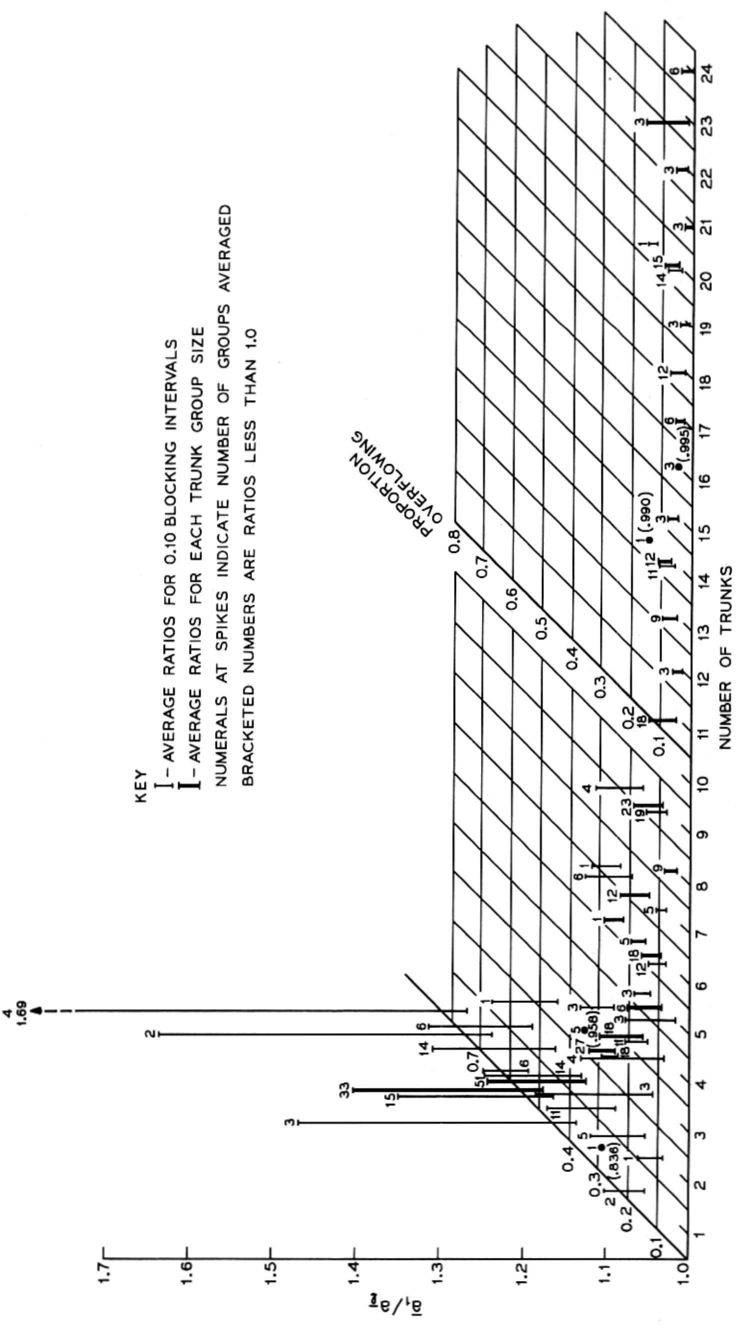

Fig. 14—Observed corrections required in $a_{\bar{t}}$ to obtain improved estimates of offered load $\bar{a}$, 100 high-usage groups in 3 tandems, Kildare, 1958, 3 hours for 20 days.

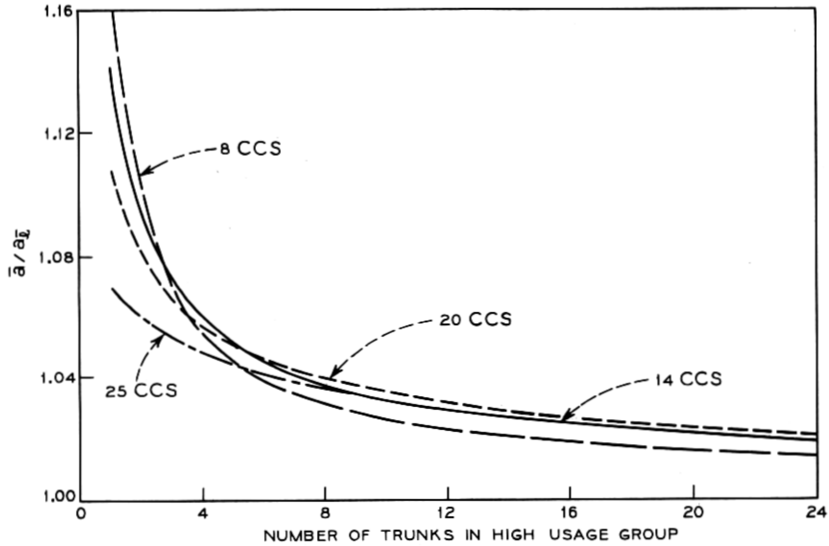

Fig. 15—Practical correction of $a_{\bar{l}}$ to estimate $\bar{a}$, with dependence on engineering level of load on "last trunk."

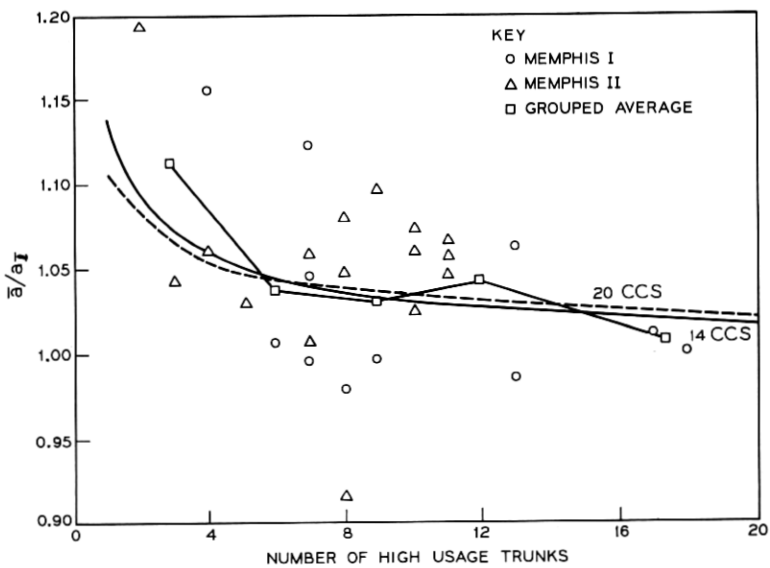

Fig. 16—Comparison of theory and data in corrections of $a_{\bar{l}}$ to estimate $\bar{a}$, intertoll data at Memphis, 1957–1958.

TABLE II—AVERAGE CORRECTIONS TO BE APPLIED TO $a_{\bar{\ell}}$ TO OBTAIN IMPROVED ESTIMATES OF $\bar{a}$
($\bar{\ell}$ determined as 20-day average)

No. Trunks	Correction Factors for y CCS on the Last Trunk			
	$y = 8$	$y = 14$	$y = 20$	$y = 25$
1	1.16	1.14	1.11	1.07
2	1.10	1.09	1.08	1.06
3	1.07	1.07	1.07	1.05
4	1.05	1.06	1.06	1.05
5	1.04	1.05	1.05	1.04
6	1.04	1.04	1.05	1.04
7	1.03	1.04	1.04	1.04
8	1.03	1.04	1.04	1.04
10	1.03	1.03	1.03	1.03
12	1.02	1.03	1.03	1.03
15	1.02	1.03	1.03	1.03
20	1.02	1.02	1.02	1.02
25 and up	1.01	1.02	1.02	1.02

2.5 *Estimation of Average Offered Loads from Average Overflow Ratios*

To monitor the level of traffic flow in some systems, a pair of registers which count numbers of calls offered m, and numbers of calls blocked n, are provided instead of the more common carried load meters. The value n/m is called the overflow ratio. Descloux[1] has shown that, under the condition that the observation period is long, say 20 holding times, little bias exists in estimates made of individual hourly offered loads to high-usage groups when a is determined from the Erlang loss relation

$$E_{1,x}(a) = n/m. \tag{12}$$

To increase the reliability of such estimates, the overflow ratios for a number of hours are commonly averaged and entered as for a single hour in equation (12). We inquire whether the estimate of average offer, $a_{\overline{n/m}}$, so obtained is unbiased, or requires correction. The expected loss probability when a load a, varying from hour to hour according to $\theta(a)$, is submitted to x trunks is

$$\bar{E}_{1,x}(a) = \int_0^\infty E_{1,x}(a)\theta(a)\,da. \tag{13}$$

When a constant load equal to the average load $\bar{a}$ is offered to x trunks, a loss probability E' will result,

$$E' = E_{1,x}(\bar{a}). \tag{14}$$

In general $\bar{E}$ will be different from E'.

Suppose that actual busy hour overflow ratios are observed over a period of days, and averaged giving $\overline{n/m}$. If $\bar{a}$ is estimated as $\bar{a}_1$ through equating $\overline{n/m}$ to $\bar{E}$ in equation (13) we should expect $\bar{a}_1$ to be unbiased since both $\overline{n/m}$ and $\bar{E}$ contemplate the presence of day-to-day variations, the first in reality, and the second by the inclusion of $\theta(a)$ in equation (13). However estimating $\bar{a}$ through substituting $\overline{n/m}$ for E' in equation (14) will yield $a_{\overline{n/m}}$, containing a certain amount of error since it merely determines the single hour load corresponding to the average loss.

A range of offered loads to trunk groups of size 1–24, covering the cases of 8 to 18 "CCS on the last trunk," has been studied and $\bar{a}_1/a_{\overline{n/m}}$ plotted against the numbers of trunks. This is shown in Fig. 17. For

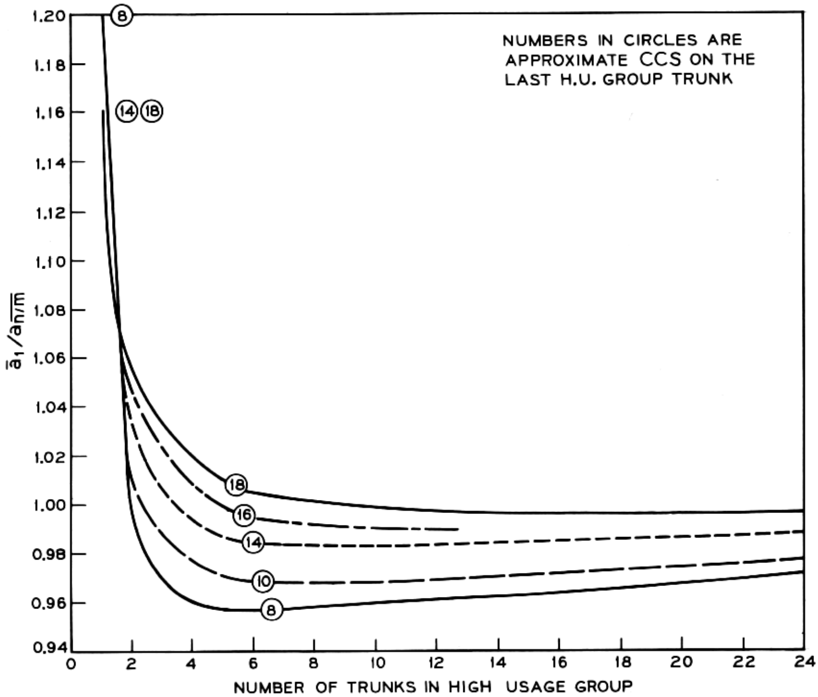

Fig. 17—Correction required in offered load estimated from averaged n/m readings.

one and two trunks the estimation procedure based on equation (14) considerably underestimates the average offered load; but beyond four trunks, overestimates of the true average offer are generally made. The cause of the inversion in the correction required lies in the shapes of the Erlang load versus loss curves in the ranges of interest. The one- and two-trunk curves are predominantly concave downwards (see for example Fig. 1), while for the larger trunk numbers the curves are concave upwards. Only for the one-trunk case are the corrections sufficient in most practical situations to warrant their making. Here a median correction is about 1.18.

Available data for groups having call and overflow counters, while showing considerable dispersion, confirm the shape and location of the correction curves of Fig. 17.

III. GROUPS WITHOUT ALTERNATE ROUTES

There has long been interest in the different theoretical formulas used by American and European administrations for engineering inter-local trunk groups. The former have relied on the cumulative Poisson while the latter have favored Erlang's loss formula. What do data taken on such groups indicate?

In 1959 data were taken on some 30 direct trunk groups terminating in the Arlington, Massachusetts, No. 5 crossbar office. Hourly observations comprised numbers of calls offered to each group, the number blocked, and, by switch counting, the average load carried. Illustrative of these is group No. 26 with 32 trunks observed for 4 hours a day for 29 business days. Offered loads were estimated for each hour by equation (1). The load versus blocking relationship observed for 116 hours is shown in Fig. 18. Superimposed is the Erlang loss relation. The agreement is seen to be very good.

On Fig. 19 are shown only the twenty-nine 10–11 A.M. busy hour loads of Fig. 18. The agreement with Erlang's theory is again understandably excellent. Other group data comparisons were nearly as satisfactory. We are led to conclude that Erlang's loss formula describes quite well the hourly blocking for conversation traffic on direct groups which do not have alternate routes. Apparently the return of blocked calls here was of a nature that caused little disturbance of the Poisson character of the offer—or perhaps they contributed to it!

In the lower section of Fig. 19 is shown a distribution of the busy hour loads offered to group No. 26 as they varied from day to day over the 6 weeks busy season. For the average offer of $\bar{a} = 21.37$

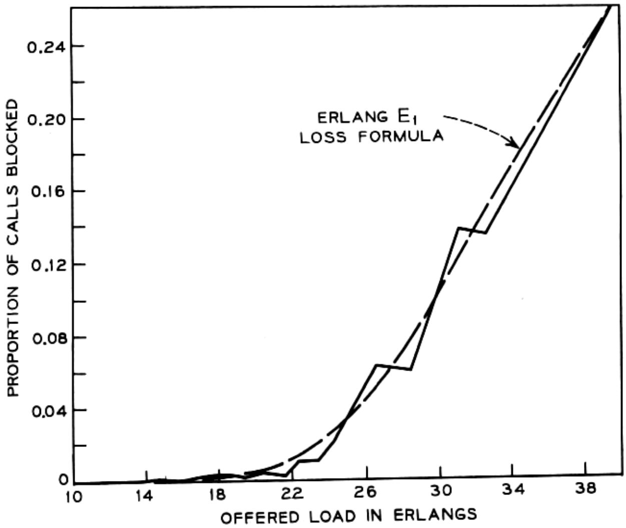

Fig. 18—Comparison of load vs loss data with Erlang's theory: full group No. 26 of 32 trunks, Arlington, 4 busiest hours of each day for 6 weeks, 1959.

erlangs, a variance $\mathrm{Var}(a) = 20.3$ was observed. Since the probability of loss curve in this range is concave upwards, the average loss for all the hours tends to exceed the loss at the average load. Thus the theoretical single hour loss at 21.37 erlangs offered is 0.0074. However the observed average loss was found to be 0.0197 when hourly losses were given equal weight (as in American practice). (If the losses had been weighted by the corresponding offered loads, the average loss would be still greater at 0.0267.)

Clearly, if Erlang's formula describes well the losses for individual hours, it will not usually give an adequate estimate of the average loss over a series of busy hours. However, when the more conservative Poisson summation values,

$$P(x, \bar{a}) = \sum_{r=x}^{\infty} \frac{\bar{a}^r e^{-\bar{a}}}{r!},$$

are laid on Fig. 19, they are seen to pass directly through the (unweighted) average loss point, indicating that at least for the amount of day-to-day variation present in this sample of hours, the Poisson provides just the right amount of "improvement" to Erlang's loss formula.

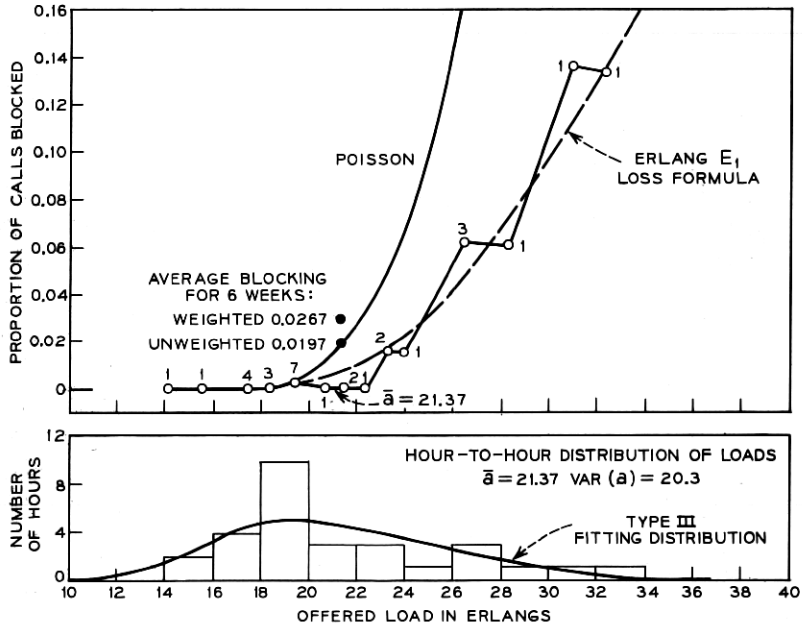

Fig. 19—Comparison of load vs loss data with Erlang's theory: full group No. 26 of 32 trunks, Arlington, 10–11 A.M. busy hour each day for 6 weeks, 1959. (Numbers at points are hours averaged.)

There is then a particular variance of day-to-day busy hour loads which when applied to Erlang's loss formula will just produce the average loss of the Poisson formula. Figure 20 shows a field of curves indicating the Var (a) required for average loads, $\bar{a}$, such that at the loss levels given, the Poisson summation will closely relate the average busy season busy hour loss to the average offer. On the same field are shown the actual variances versus averages observed for the trunk groups in the Arlington study mentioned above. It is seen that the variances found in practice are such that at commonly used interlocal average busy season busy hour grades of service, 0.005 to 0.03, the Poisson summation provides an excellent means of specifying group average capacity. It may be noted that in American practice, the objective grades of service are set to be met by the average (unweighted) blocking in the busy season busy hours.

The dashed line on Fig. 20 is a centrally located curve which describes the general relation seen between day-to-day variance and average load offered. It is interesting – and reasonable – that its equation is

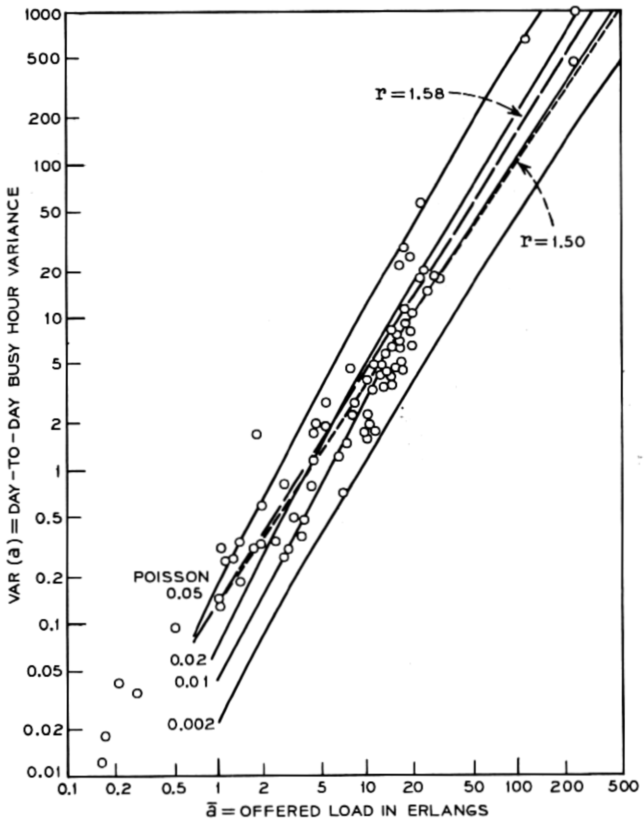

Fig. 20—Variations in time consistent hourly loads on 32 direct groups of trunks: Arlington, 4 busiest hours of each day for 6 weeks, 1959, compared with "Poisson assumption."

Var $(a) = 0.13\ \bar{a}^r$ where $r = 1.58$, identical with that of the central axis line drawn through the high-usage group observations on Fig. 10. The dotted line on Fig. 20 results from choosing $r = 1.50$; it corresponds nicely with Poisson blocking of 0.01 for loads of 10 to 100 erlangs.*

* It may be noted that Bell System traffic capacity tables for nongraded groups carrying random and nonrandom offers have been generated at several blocking levels, and contemplate the following hour-to-hour load variations: Single hour blocking; "low" hour-to-hour variation ($r = 1.50$); "medium" hour-to-hour variation ($r = 1.70$); "high" hour-to-hour variation ($r = 1.84$). At loads of one erlang, these several traffic tables contain the same allowance for day-to-day variations. Below one erlang the theoretical day-to-day variations, while inverted from their relative positions at loads above one erlang, are small compared with the momentary variations inherent in random traffic.

IV. FINAL GROUPS

Much has been written in the past on estimation of the effect of the nonrandomness (peakedness) of overflow traffic from high-usage groups upon the trunking requirements in final groups.[3] Studies have also been made (as in the previous section) showing the added trunks required to accommodate the increased demand on groups whose offered loads show significant day-to-day variations.

The two effects will usually appear simultaneously in the engineering of final groups. Moreover, loads offered to final groups may be expected to show generally larger day-to-day busy hour variations than do loads to high usage and direct groups not having alternate routes. Figure 21 shows a field of variance versus average loads offered to 28 interlocal final groups in which negligible first-routed traffic was present. A central fitting line, having the equation

$$\log_{10} \text{Var}\,(a) = 1.8404 \log_{10} \bar{a} - 0.8861, \tag{15}$$

has been drawn through the points, or equivalently,

$$\text{Var}\,(a) = 0.13\bar{a}^{1.84}. \tag{16}$$

When this variation is assumed, the carrying capacities of 6 to 15

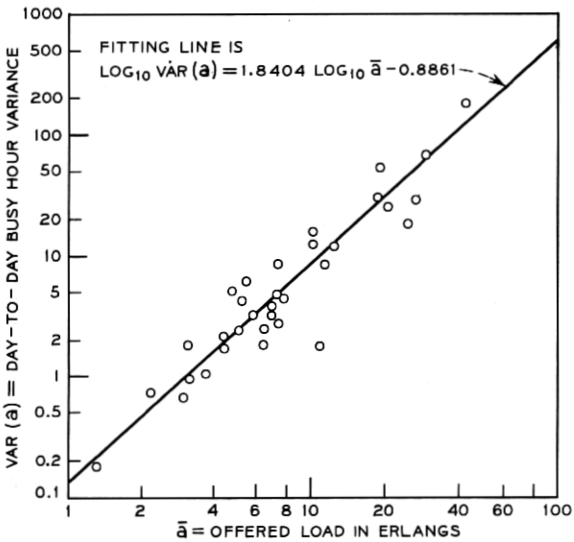

Fig. 21—Variations in busy hour loads offered to final groups in 28 interlocal alternate route systems.

trunks at an 0.02 average blocking are shown illustratively on Fig. 22. For 10 trunks, for example, the erlang capacity for random traffic is reduced from 5.08 to 4.47 erlangs, a drop of 12 percent resulting from the day-to-day variations.

Similarly, if day-to-day variations are *not* introduced, but instead a nonrandomness, characterized by a peakedness factor P.F. = (variance)/(mean) = 1.5, is assumed, the capacity of 10 trunks is reduced from 5.08 to 3.96 erlangs, a reduction of 22 percent.

When day-to-day variations and nonrandomness are jointly introduced, the capacity of a group is further reduced; thus the 10 trunks will now accommodate an offer of only 3.57 erlangs, a reduction of 30 percent from the basic Erlang loss formula value. It is clear that each cause can produce substantial reductions in trunking capacity.

When the offer to a final group contains a significant fraction of first-routed traffic, both the peakedness factor and the day-to-day variations will commonly be lowered; the trunking requirements will then be correspondingly reduced on both accounts.

An illustration of the importance of using theory which considers both day-to-day variations and the peakedness of the traffic is given in Fig. 23. The results are shown of observations on a Kildare final group of 59 trunks, with 62 subtending high-usage groups plus 4 first-routed traffic items. From high-usage group load and trunk configurations the estimated peakedness factor of the final group offered load is 1.93. The load-loss characteristics observed for the 20 days are:

Hour of Day	Average Offer (erlangs)	Day-to-Day Variance of the Offer	Average Blocking Observed
9–10 A.M.	26.5	28.5	0.001197
10–11 A.M.	44.0	180.3	0.04570

As seen on the figure, the simple Erlang loss values (dotted lines) are an order of magnitude below the observed losses. Although allowance for nonrandomness (dashed lines) makes a marked improvement, it is still far from describing the actual losses. Nor is the Poisson model (dots on Fig. 23), which might be expected to compensate partially for day-to-day variations in the 0.01 to 0.03 loss range, a nearly sufficient improvement. But employing the typical day-to-day variance magnitudes of Fig. 21 in conjunction with the estimated peakedness of the offer, the solid line load-loss curves for 58 and 60 trunks exactly bracket the average loss values seen for 20 days for the 9–10

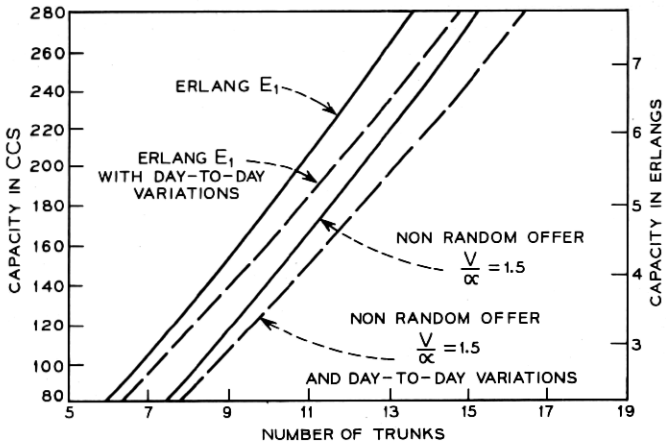

Fig. 22—Final trunk group engineering with various assumptions as to the character of offered loads. Average blocking = 0.02.

A.M. and the 10–11 A.M. hours on the 59-trunk final group. Construction of such load-loss relations is described in Ref. 4.

V. SUMMARY

Examples have been given comparing observation, simulation, and theory in various areas of traffic flow on trunks comprising direct and alternate routed plans. Particular attention has been drawn to the need for developing adequate relationships between offered, carried, and overflow loads, both single hour and average busy season, suitable to each operating condition. Where possible, a physical understanding of the principal factors is followed by statistical theory which may require approximate numerical calculations. Further insight may be gained from controlled simulations. In order for the relationships developed to be useful, they must finally be found to agree with the flow of traffic in real situations.

The following particular results have been determined:

(*i*) Single hour load-loss relationships on high-usage groups are found to be well described by Erlang's $E_{1,x}$ (a) loss formula. Comparisons are made with data and simulations (Figs. 1, 2, 3, 4).

(*ii*) Estimating single hour loads offered to high-usage groups from observed carried loads by use of Erlang's loss relationship yields too-large values at the heavier occupancy levels, and slightly too-small values at low occupancies. This is explained and illustrated by recourse to regression theory (Fig. 5). The magnitude of the corrections needed are indicated in Figs. 6, 7, 8.

(*iii*) If busy season busy hour carried loads are first averaged and this value entered in Erlang's E_1-relation the average offer will be underestimated. Corrections required for a 20-day busy season are given in Table II and Figs. 13, 14, 15, 16.

(*iv*) Estimates of the average offer obtained from averaged hourly blocking proportions show that both positive and negative corrections may be required (Fig. 17). Except for the 1-trunk case, however, the corrections indicated are small.

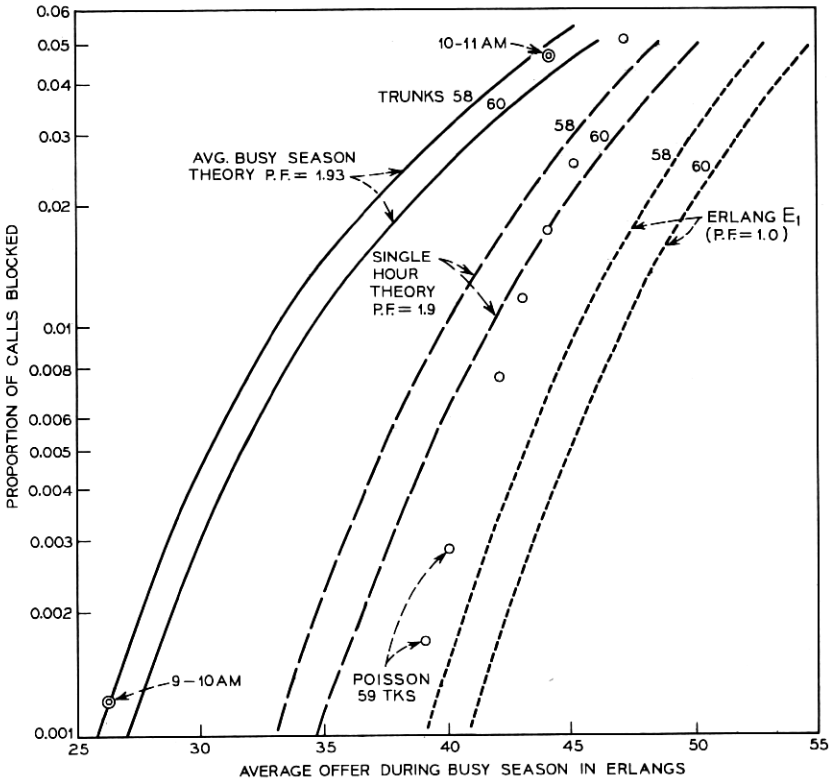

Fig. 23—Comparison of blocking theories with performance of Kildare final group No. 1, 62 high-usage groups plus 4 first-routed items, 59 trunks, P.F. = 1.93, 20 days, October–November 1958.

(*v*) When a series of busy hour loads is submitted to a high-usage group, the true average overflow load will exceed that estimated by entering Erlang's relation with the average offer. Corrections are given in Table I and Fig. 11.

(*vi*) For full groups (i.e., those without alternate routes) it is found that Erlang's formula describes well the load-loss relation for single hours (Figs. 18, 19).

(*vii*) When busy hour loads offered to full groups are averaged for the busy season, the corresponding average loss will usually be better estimated by the Poisson summation formula, if the loss values are in the common range from 0.005 to 0.03 blocking.

(*viii*) Loads offered to final groups will normally be nonrandom (peaked) and hence require special procedures for engineering. Examples are given in which both peakedness and day-to-day busy hour variations are present. Unless both influences are allowed for, the average load-average loss estimate may be far from that actually observed (Fig. 23).

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