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Restoring the Orthogonality of Two Polarizations in Radio Communication Systems, I

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The transmission medium or the radiating system often fails to maintain the orthogonality between two polarizations. Using a differential phase shifter and a differential attenuator, the orthogonality can be recovered by the transformation of two nonorthogonal elliptically polarized waves into two orthogonal polarizations.

I. INTRODUCTION

Making use of two orthogonal polarizations simultaneously can double the capacity of a radio communication system.[1] The requirement of maintaining orthogonality is easily fulfilled when the transmitting and receiving antennas are aligned in the beam maximum of each other. Sometimes this alignment is not possible; for example, in some satellite communication systems the ground stations are distributed across the beamwidth of the satellite antenna. Then any polarization variation within the antenna beamwidth will give rise to cross-polarization coupling. Rain attenuation may also introduce significant depolarization at the higher microwave frequencies.[2]

In practice the two orthogonal polarizations leaving the transmitter are either two orthogonal linearly polarized waves or two opposite circularly polarized waves. Whatever be the cause of polarization distortion, the failure to maintain orthogonality will produce two nonorthogonal elliptically polarized waves at the receiving terminal. The purpose of this paper is to present a method of recovering orthogonality by transforming the two nonorthogonal elliptically polarized waves into two orthogonal linear polarizations. The transformation employs a differential phase shifter and a differential attenuator.

II. ANALYSIS

Two nonorthogonal elliptically polarized waves, I and II, can be represented by two polarization ellipses with their major axes oriented at an arbitrary angle θ with respect to each other as shown in Fig. 1. The axial ratios A_1 and A_2 can be of the same or opposite sign depending on whether the polarization ellipses are rotating in the same or opposite directions. Let us find the representations of these two elliptically polarized waves in the X−Y coordinates where the X_1- and X_2-axes are counterclockwise rotations of β_1 and β_2 respectively from the X-axis. (β_2 in the configuration of Fig. 1 is a negative value.) The ratio of clockwise to counterclockwise circularly polarized components for the polarization ellipse I (wave approaching) is

$$q_1 = \frac{1 + A_1}{1 - A_1} \tag{1}$$

where A_1 is positive if the elliptic polarization is rotating clockwise and negative if the elliptic polarization is rotating counterclockwise. When the X-axis is used as phase reference for the two circularly polarized components, this ratio becomes

$$q_1' = \frac{1 + A_1}{1 - A_1} e^{-j2\beta_1}. \tag{2}$$

The ratio of the Y component to X component for the polarization

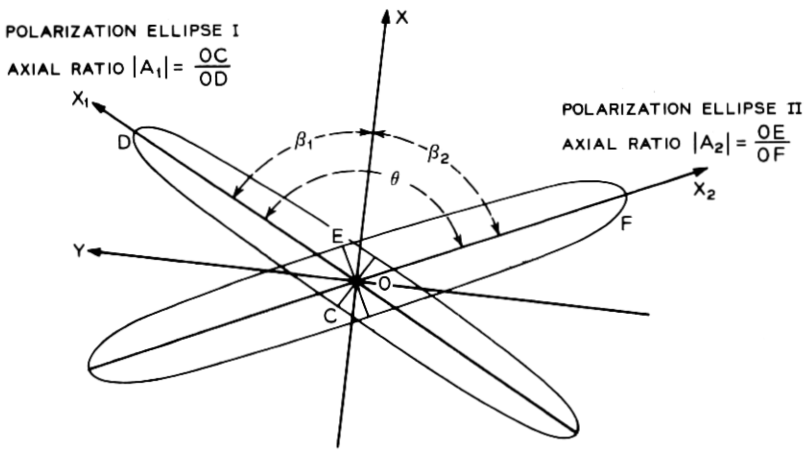

Fig. 1—Two nonorthogonal elliptically polarized waves.

ellipse I is[3]

$$P_1 = p_1 e^{-j(\pi/2)} = \frac{1 - q_1'}{1 + q_1'} e^{-j(\pi/2)}. \tag{3}$$

Substituting equation (2) into equation (3) yields an expression for P_1; similarly we can find P_2 for the polarization ellipse II.

$$P_i = \sqrt{\frac{(1 + A_i^2) - (1 - A_i^2)\cos 2\beta_i}{(1 + A_i^2) + (1 - A_i^2)\cos 2\beta_i}} \cdot \exp\left\{j \tan^{-1}\left[\frac{2A_i}{(1 - A_i^2)\sin 2\beta_i}\right]\right\}, \qquad i = 1, 2 \tag{4}$$

$0 < \llcorner P_i < \pi$ when $A_i > 0$; $-(\pi/2) < \llcorner P_i < \pi/2$ when $\sin 2\beta_i > 0$.

In order to convert an elliptic polarization into a linear polarization, the phase difference between the X and Y components must be eliminated by a proper differential phase shifter. The condition for simultaneous transformations of two elliptic polarizations into two linear polarizations is

$$\llcorner P_1 = \llcorner P_2 \quad \text{or} \quad \llcorner P_1 + \pi = \llcorner P_2 . \tag{5}$$

The above equations are equivalent to

$$\tan \llcorner P_1 = \tan \llcorner P_2 . \tag{6}$$

Substituting equations (4) into equation (6), and using the relation $\beta_2 = \beta_1 - \theta$, the solution for β_1 is

$$\beta_1 = \tfrac{1}{2} \cot^{-1}\left[\frac{\cos 2\theta - \dfrac{A_2(1 - A_1^2)}{A_1(1 - A_2^2)}}{\sin 2\theta}\right], \qquad 0 < \beta_1 < \frac{\pi}{2}. \tag{7}$$

The above expression fixes the orientation of the X − Y coordinates. By applying a phase delay $\llcorner P_1$ to the components in the Y direction, the two elliptically polarized waves I and II are transformed into two linearly polarized waves L1 and L2 as illustrated in Fig. 2* for the case of $\llcorner P_1 + \pi = \llcorner P_2$.

The angle between the two linear polarizations is

$$\psi = \tan^{-1}|P_1| \pm \tan^{-1}|P_2| \quad \text{when} \quad \begin{cases} \llcorner P_1 + \pi = \llcorner P_2 \\ \llcorner P_1 = \llcorner P_2 \end{cases} \tag{8}$$

* When $\llcorner P_1 < 0$, the transformation into two linear polarizations can be also obtained by a phase delay $|\llcorner P_1|$ in the X direction.

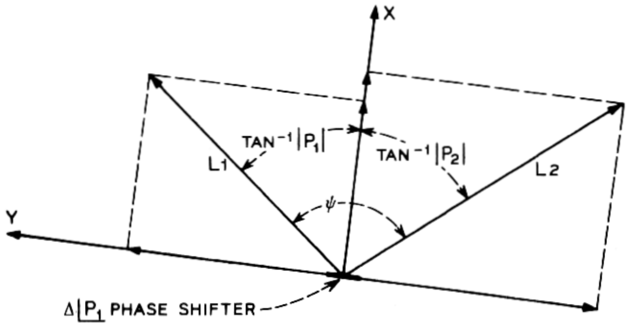

Fig. 2—Transformation into two linear polarizations.

where $|P_1|$ and $|P_2|$ are given by equations (4). Note that ψ is not necessarily a right angle. However, this angle can be changed to a right angle in the following manner. If $\psi < \pi/2$, an attenuation of $\tan(\psi/2)$ can be imposed on the components of L1 and L2 in the direction B (which bisects the angle ψ) as shown in Fig. 3a; i.e., at an angle

$$\chi = \tfrac{1}{2}[\tan^{-1}|P_1| \mp \tan^{-1}|P_2|] \quad \text{when} \quad \begin{cases} \angle P_1 + \pi = \angle P_2 \\ \angle P_1 = \angle P_2 \end{cases} \tag{9}$$

with respect to the X direction. If $\psi > \pi/2$, an attenuation of $\cot \psi/2$ can be imposed on the components in the direction perpendicular to B as shown in Fig. 3b. The above orthogonalization by differential attenuation is the inverse of depolarization by attenuation due to oblate raindrops.[2] Obviously differential gain may also provide orthogonalization or depolarization.

The two orthogonal linear polarizations obtained by the above scheme may not be aligned with the polarization axes of the receiver.

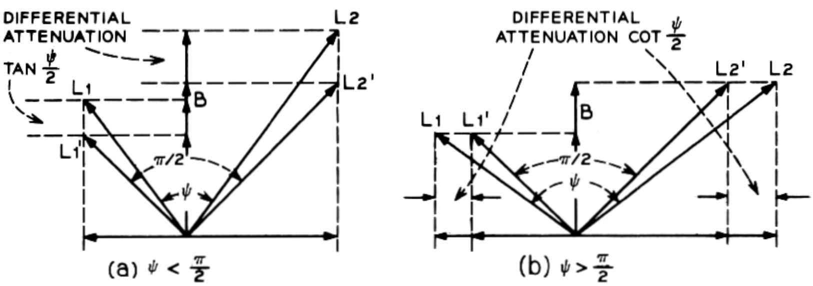

Fig. 3—Orthogonalization by differential attenuation.

In order to accomplish the alignment, one can use a $\Delta\pi$ phase shifter properly oriented as shown in Fig. 4 where L1′ and L2′ are rotated into L1″ and L2″.[4] This $\Delta\pi$ phase shifter can be also placed in front of or immediately after the $\Delta\underline{|P_1}$ phase shifter. Then the orientations of the differential phase shifters and the differential attenuator must be changed accordingly.

III. DISCUSSION

The above analysis suggests a method for restoring the orthogonality of two polarizations at the receiving end of the system. This method can be used when the two transmitted polarizations are either linear or circular. The differential phase shifter and the differential attenuator can be installed in a circular waveguide feeding the antenna. It follows from the reciprocity principle that the orthogonality of two polarizations will be also preserved if the direction of transmission is reversed.

If the polarization distortions are time-varying, the practical implementation of the above scheme appears to require two beacon signals, each of which would occupy a separate narrow frequency band and would be transmitted on one of the two orthogonal polarizations. The components of each signal in the two orthogonal polarizations of the receiver can be measured. The amplitude ratio and phase difference of these two components completely specify an elliptically polarized wave for each signal.

Now let us obtain some feeling for the required differential attenuation. In the absence of differential attenuation, each of the two linear polarizations separated by an angle ψ will have a cross polarization of $|\tan \frac{1}{2}(\psi - \pi/2)|$ if their misalignment with respect to the two orthogonal linear polarizations of the receiver has been minimized. The

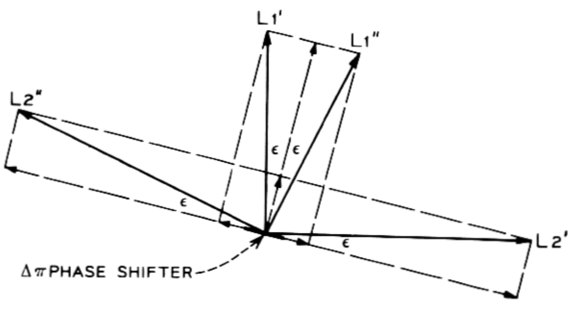

Fig. 4—Rotation by $\Delta\pi$ section.

complete elimination of cross-polarization coupling will need a differential attenuator of tan $(\psi/2)$. However, it is often only necessary to reduce the cross polarization to a certain system specification, such as -30 dB. Then the required differential attenuation will be tan $(\psi/2)/\tan(\psi_o/2)$ or its reciprocal where $|\tan \frac{1}{2}(\psi_o - \pi/2)|$ is the tolerable cross polarization. For various system specifications of tolerable cross polarization, Fig. 5 shows the required differential attenuation versus the original cross polarization. Furthermore, one notes that the differential attenuation only affects one component of each signal. The attenuation of each total signal is $\sin(\psi/2)/\sin(\psi_o/2)$ or $\cos(\psi/2)/\cos(\psi_o/2)$ when ψ is less or greater than $\pi/2$. In the case of weak nonorthogonality the attenuation of the total signal is approximately half of the differential attenuation.

A reviewer has called the author's attention to the practice of using adjustable squeeze sections[5] for the reduction of crosstalk in the horn-reflector-antenna system. A differential phase shifter alone may reduce the cross-polarization coupling; however, the complete elimination of cross polarization in general requires the proper combination of a differential phase shifter and a differential attenuator as described in this paper.

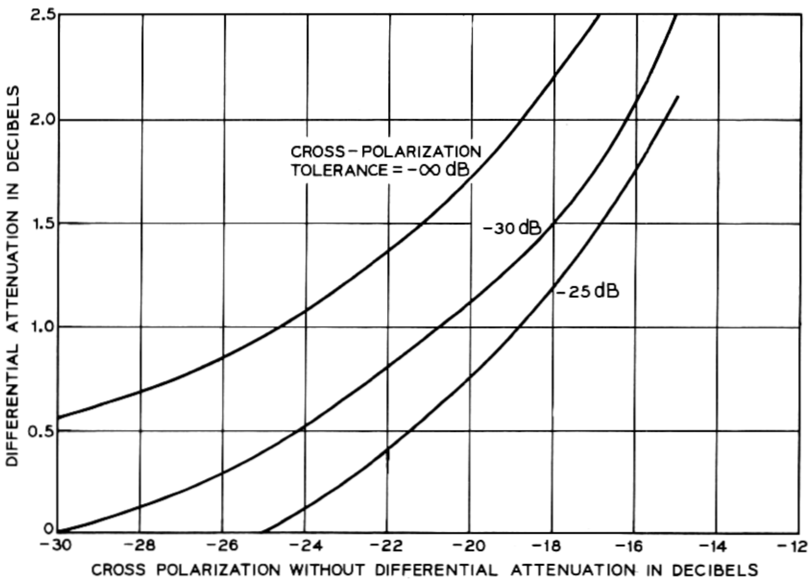

Fig. 5—Required differential attenuation for attaining a system specification of tolerable cross-polarization coupling.

IV. ACKNOWLEDGMENT

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