THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 51, No. 2, February, 1972
Printed in U.S.A.

Delay Distortion in Weakly Guiding Optical Fibers Due to Elliptic Deformation of the Boundary

By W. O. SCHLOSSER

(Manuscript received September 13, 1971)

The delay distortion in glass fiber optical waveguides due to small elliptical deformations of the cross section is calculated. Simple approximations are given for the case of small differences in index of refraction between core and cladding (weak guidance). Since the delay distortion is quadratically dependent on the index difference it is found that it is generally possible to keep it negligible by judiciously choosing the guide parameters.

I. INTRODUCTION

Recent results[1] indicate that glass fibers are a potential transmission medium for optical communication. For high-capacity systems (in excess of 100 MBaud) dispersion and the associated delay distortion is an important factor to be considered. For such an application the fiber cannot be used in a frequency range where more than one mode propagates, since the difference in group velocity between the various modes causes excessive delay distortion.[2] In the single-mode range the two best known sources of delay distortion are the material and waveguide dispersion.[2] In this paper we will treat a different source of dispersion. If the fiber is elliptically deformed two different polarizations with different group velocities are possible. We will calculate the delay distortion due to these elliptical deformations and establish simple relationships between the allowable delay distortion and the tolerances with which the fiber has to be manufactured.

II. DEFORMATION OF A SQUARE DIELECTRIC WAVEGUIDE

Let us consider the dielectric waveguide of square cross section first, since its properties are very similar to the round waveguide[3] and the mode structure is quite simple. This will allow us to explain the effects of deformation more easily than for the round waveguide, where the necessary formalism clouds the physics somewhat. For a rectangular dielectric waveguide the HE_{11} dominant mode can be thought of as being composed of the E mode on a slab of height $2a$ and the H mode on a slab of height $2b$ (Fig. 1). The propagation constant is approximated by

$$\beta^2 = n_c^2 k_0^2 - (\beta_H^2 + \beta_E^2) \tag{1}$$

where β_H and β_E can be determined from the characteristic equations for the slab modes:

$$[(n_c^2 - n^2)k_0^2 - \beta_H^2]^{1/2} = \beta_H \tan \beta_H a \qquad \text{H mode,} \tag{2a}$$

$$\frac{n_c^2}{n^2}[(n_c^2 - n^2)k_0^2 - \beta_E^2]^{1/2} = \beta_E \tan \beta_E b \qquad \text{E mode.} \tag{2b}$$

We postulate that the deformation of a quadratic cross section into a rectangular one corresponds to the elliptic deformation of a circular cross section. The height $2a$ of the square is increased by $2\Delta a$ and the width is decreased by $2\Delta a$. The change in propagation constant β due to this deformation can be calculated from

$$\Delta\beta = \left(\frac{\partial\beta}{\partial a} - \frac{\partial\beta}{\partial b}\right)\bigg|_{a=b=a_0} \cdot \Delta a = \frac{1}{\beta}\left(\beta_E \frac{\partial\beta_E}{\partial b} - \beta_H \frac{\partial\beta_H}{\partial a}\right)\bigg|_{a=b=a_0} \cdot \Delta a. \tag{3}$$

This shows that the effects from the widening of one dimension and narrowing of the other tend to cancel each other. From the well known properties of β we can make predictions about $\Delta\beta$. For small differences of refractive indices $n_c \approx n$ and β is given by $\beta = n \cdot k_0 + \Delta \cdot f(a, b, k_0)$ where $\Delta = (n_c - n)/n$. The derivative of β with respect to the dimensions a, b will therefore have a factor Δ and $\Delta\beta$ must hence possess this factor. We observe furthermore from the equations (2a) and (2b)

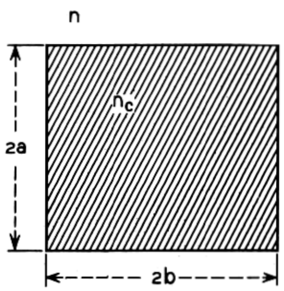

Fig. 1—Cross section of rectangular waveguide.

that β_E and β_H obey the same characteristic equation except for a factor $n_c^2/n^2 = 1 + 2\Delta$. The difference,

$$\beta_H \frac{\partial \beta_H}{\partial a} - \beta_E \frac{\partial \beta_E}{\partial b}\bigg|_{a=b=a_0},$$

will therefore be proportional to Δ^2. We can thus expect $\Delta\beta$ to be quadratic in Δ and the difference in group delay between the two polarizations will have the form

$$\Delta\tau = \Delta^2 \cdot \frac{\Delta a}{a} f(a, k_0, \Delta). \tag{4}$$

Recently, a letter by R. B. Dyott and J. R. Stern[4] has been published, approximating the phase difference between the two polarizations by the phase difference of two modes on circular guides of different diameters. This results in an approximation which overestimates the delay distortion considerably. We can see that easily by calculating the variation in propagation constant due to an increase of height and width by $2\Delta a$,

$$\Delta\beta = \frac{\partial \beta}{\partial a} \Delta a = -\left(\frac{1}{\beta_H}\frac{\partial \beta_H}{\partial a} + \frac{1}{\beta_E}\frac{\partial \beta_E}{\partial b}\right)\bigg|_{a=b=a_0} \cdot \Delta a.$$

Since the two derivatives are added, $\Delta\beta$ will only be of order Δ and not Δ^2 as in equation (4). In practical cases Δ is in the order of 10^{-2} and therefore this approximation gives a considerably bigger value than (4).

III. DEFORMATION OF A CIRCULAR DIELECTRIC WAVEGUIDE

The function $f(a, k, \Delta)$ in equation (4) has to be determined for the circular fiber. This will now be done by first determining $\Delta\beta$ and then differentiating with respect to ω. We make use of the fact that the cross section of the guide is only slightly elliptic (Fig. 2). In this case the propagation constant β is approximated by the first two terms of a Taylor series

$$\beta = \beta_0 + \left(\frac{e}{d}\right)^2 \partial\beta/\partial\left(\frac{e}{d}\right)^2\bigg|_{e=0} \tag{5}$$

where β_0 is the propagation constant on the circular guide and e the small excentricity. The propagation constant β is generally determined from the zeros of the characteristic equation:

$$F(d, e^2, \beta, k_0, \Delta) = 0. \tag{6}$$

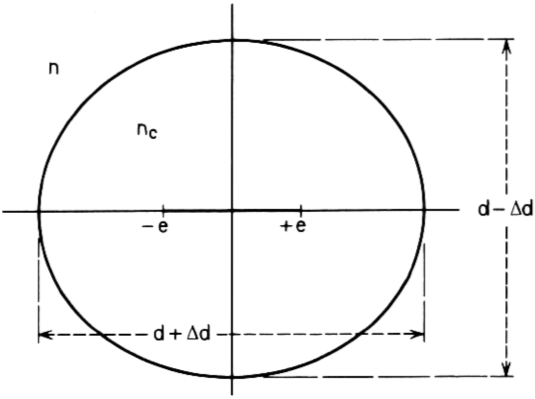

Fig. 2—Cross section of elliptically deformed fiber.

The derivative $\partial\beta/\partial(\frac{e}{d})^2$ can thus be expressed by the derivatives of F,

$$\partial\beta/\partial\left(\frac{e}{d}\right)^2\bigg|_{e=0} = \frac{\partial F/\partial\left(\frac{e}{d}\right)^2}{\partial F/\partial\beta}\bigg|_{e=0}. \tag{7}$$

The problem is now reduced to finding F. This has been done for small ellipticities in Ref. 5. The reduction of the results of Ref. 5 to the case of small difference in refractive indices finally yields $\Delta\beta$:

$$\Delta\beta = \Delta^2 2\,\frac{\Delta d}{d}\cdot n\cdot\left[\frac{\left(\frac{K_0(w)}{wK_1(w)} + \frac{1}{w^2}\right)u^4w^4}{J_1^2(u)v^6}\right], \tag{8}$$

where

$$u = \frac{d}{2}\,k_0\sqrt{n_c^2 - (\beta/k_0)^2}$$

$$w = \frac{d}{2}\,k_0\sqrt{(\beta/k_0)^2 - n^2}$$

$$v = \frac{d}{2}\,k_0\sqrt{n_c^2 - n^2}.$$

$\Delta\beta$ has the Δ dependence predicted from the quadratic case. It must be noted that the part in square brackets is only dependent on the parameter v, the normalized frequency. As shown in Ref. 6 the magnitude of v determines uniquely the properties of the mode independent

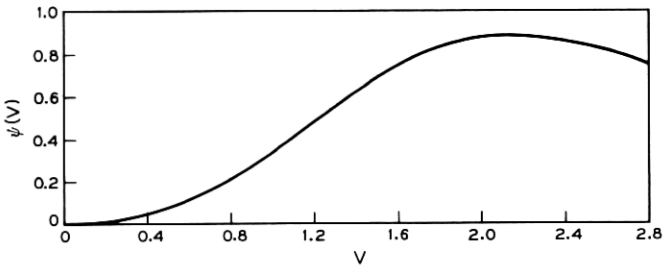

Fig. 3—The normalized perturbation $\psi(v)$ of the propagation constant.

of the guide parameters (only for small Δ of course). We can therefore express $\Delta\beta$ as the product of the guide parameters and a function $\psi(v)$ depending only on the binding properties of the mode:

$$\Delta\beta = \Delta^2 \cdot 2 \frac{\Delta d}{d} n \psi(v). \tag{9}$$

$\psi(v)$ is plotted in Fig. 3. The group delay difference between the two polarizations is given by

$$\Delta\tau = \frac{L \cdot n}{c} \Delta^2 2 \frac{\Delta d}{d} \frac{d}{dv} (v \cdot \psi(v)). \tag{10}$$

The v dependent part is plotted in Fig. 4. Since it never exceeds 1.6, we can use the upper limit

$$\boxed{\Delta\tau_{\max} = 3.2 \frac{L \cdot n}{c} \frac{\Delta d}{d} \Delta^2} \tag{11}$$

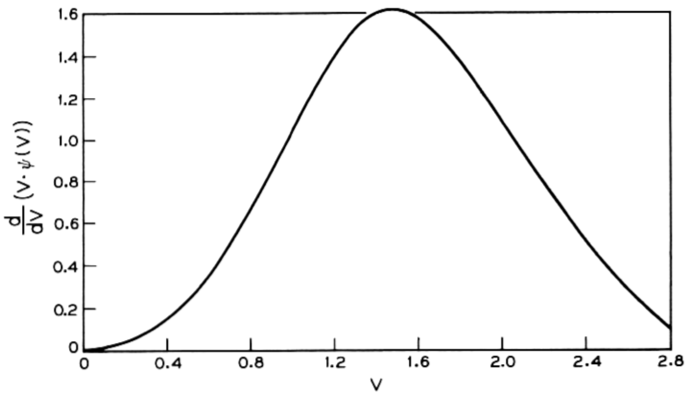

Fig. 4—The normalized delay distortion $d\ (v \cdot \psi(v))/dv$.

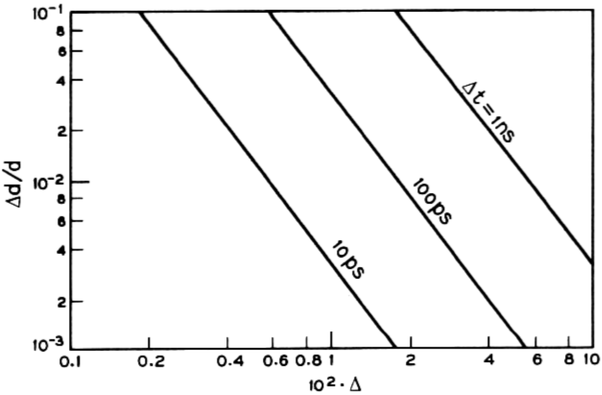

Fig. 5—The maximum diameter variation as a function of Δ.

to establish tolerances for $(\Delta d)/d$ as a function of Δ, assuming that the core diameter is chosen to get appropriate binding properties. Figure 5 shows the upper limit of $(\Delta d)/d$ as a function of Δ corresponding to various delay distortions. The length L of the guide is 3 km and $n = 1.5$. For 1 ns delay distortion and $\Delta = 1.8 \times 10^{-2}$ the diameter variation must be smaller than 10 percent which is not difficult to achieve. The requirements for a 10-ps delay distortion however ($\Delta < 1.8 \times 10^{-3}$ for $(\Delta d)/d < 10$ percent) are not so trivial anymore. We can conclude from these data that for a system in the several 100-Mb/s range the delay distortion due to elliptical deformation does not cause any difficult tolerance problems. At much higher speeds, however, the effect must be seriously considered.

REFERENCES

1. Kapron, F. P., Keck, D. B., and Maurer, R. D., "Radiation Loss in Glass Optical Waveguides," Appl. Phys. Letters, *17*, (1970), pp. 423–425.
2. Gloge, D., "Dispersion in Weakly Guiding Fibers," Appl. Opt., *10*, (1971), pp. 2442–2445.
3. Schlosser, W. O., and Unger, H. G., "Partially Filled Waveguides and Surface Waveguides of Rectangular Cross Section," Advances in Microwaves, *1*, (1966), pp. 319–387.
4. Dyott, R. B., and Stern, J. R., "Group Delay in Glass Fiber Waveguide," Elec. Letters, *7*, (1971), pp. 82–84.
5. Schlosser, W. O., "Die Stoerung der Eigenwerte des runden dielektrischen Drahtes bei schwacher elliptscher Deformation der Randkontur," AEU, *19*, (1965), pp. 1–8.
6. Gloge, D., "Weakly Guiding Fibers," Appl. Opt., *10*, (1971), pp. 2252–2258.