THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 52, No. 1, January, 1973
Printed in U.S.A.

Crosstalk in Uniformly Coupled Lossy Transmission Lines

By J. C. ISAACS, JR., and N. A. STRAKHOV

(Manuscript received July 5, 1972)

The crosstalk between two identical, uniformly coupled, lossy transmission lines is examined. Equations are derived which can be solved to obtain formulas for the near-end crosstalk (NEXT) and far-end crosstalk (FEXT). An example is worked which illustrates the mutual influence of the two lines in terms of the modal voltages and currents. The mutual influence of the two lines is also studied by comparing the results of this example with the "classical" crosstalk formulas which assume weak coupling and neglect the influence of the disturbed line on the disturbing line. It is shown that the influence of the disturbed line on the disturbing line can be neglected for NEXT for most weak coupling situations. For sufficiently high frequencies and/or long line lengths, however, this influence cannot be neglected for FEXT.

I. INTRODUCTION

One of the earliest analyses of crosstalk in coupled transmission lines was made by Campbell;[1] later Shelkunoff and Odarenko[2] used a similar method to analyze the crosstalk in coaxial structures. These "classical" formulas were derived for two parallel transmission lines with weak coupling and matched terminations. One drawback of these analyses is that they do not take into account the effect of the disturbed line on the disturbing line. However, their crosstalk formulas are simple in form and easy to analyze. Also, they are applicable to any parallel, uniformly coupled transmission lines with weak coupling.

Somewhat later an analysis of coupled transmission lines was made by Rice.[3] His results apply under quite general conditions and are expressed in compact matrix notation. However, his results have apparently not influenced current analyses, possibly because the formulas are more complicated to analyze than those in Refs. 1 and 2. Coupling between two pairs under similarly general conditions is given by Kuznetsov and Stratonovich.[4] Although the emphasis is in obtaining results for the time domain, the basic approach is similar to the one we will follow. The specific time domain results do not apply to the transmission lines of interest here because the frequency dependence of the primary constants is not taken into account. More recent analyses[5,6] have relaxed the assumptions of weak coupling and matched lines and do take into account the effect of the disturbed line on the disturbing line. Unfortunately, these analyses focus attention on the lossless case in order to obtain crosstalk formulas which can be readily calculated. While the lossless case may be of interest for some line lengths and frequency ranges, it does not cover many applications which are of great practical interest.

In this paper, a fairly general analytical model is presented for two identical, parallel, uniformly coupled transmission lines with a common ground return. This model does not assume weak coupling, matched terminations, or lossless lines. The resultant crosstalk equations, although somewhat unwieldy, can be evaluated with the aid of a computer.

The motivation for this study was, in part, to assist in the analysis of special cables being utilized in the interconnection of equipment racks. These cables, referred to as flat flexible cables, have conductors that are not twisted and therefore can couple to each other strongly under certain conditions. The results of this study are also of interest to those studying longitudinal mode coupling effects in multipair cable.

II. DERIVATION OF CROSSTALK BETWEEN TRANSMISSION LINES WITH ARBITRARY CONSTANT COUPLING

The starting point for this analysis is the set of coupled differential equations which are assumed to govern the two transmission lines. They are

$$\frac{dE_1}{dx} = -(R + j\omega L)I_1 - j\omega L_c I_2 \tag{1a}$$

$$\frac{dI_1}{dx} = -(G + j\omega C)E_1 - j\omega C_c E_2 \tag{1b}$$

$$\frac{dE_2}{dx} = -(R + j\omega L)I_2 - j\omega L_c I_1 \tag{1c}$$

$$\frac{dI_2}{dx} = -(G + j\omega C)E_2 - j\omega C_c E_1 \tag{1d}$$

where

E_i is the voltage across transmission line i, $i = 1, 2$
I_i is the current flowing in transmission line i, $i = 1, 2$
R, L, G, C are standard distributed resistance, inductance, conductance, and capacitance, respectively
ω is frequency in radians/s
L_c, C_c are "coupling" inductance and capacitance; the relationship to physical quantities will be derived in a later section.

A number of assumptions are tacitly implied in order for the equations to describe the physical situation. These will now be discussed.

The first and most basic assumption is that only two sets of voltages and currents are involved in the coupling mechanism. This assumption is readily met in the case of unbalanced transmission lines shown in Fig. 1a. However, for balanced transmission lines, depicted in Fig. 1b, other voltages and currents may play a role in the coupling mechanism. They will only be negligible if each transmission line is well balanced with respect to ground.

Another important assumption is that the power propagating down the transmission lines is essentially described by TEM modes. This assumption is required to assure that the telegrapher's equations [i.e., (1) with $L_c = C_c = 0$] are valid.

For the time being, it is not necessary to specify whether or not R, L, G, C, C_c, and L_c are frequency independent. However, if these results are translated from the frequency domain to the time domain, the frequency dependence of these parameters will have to be specified.

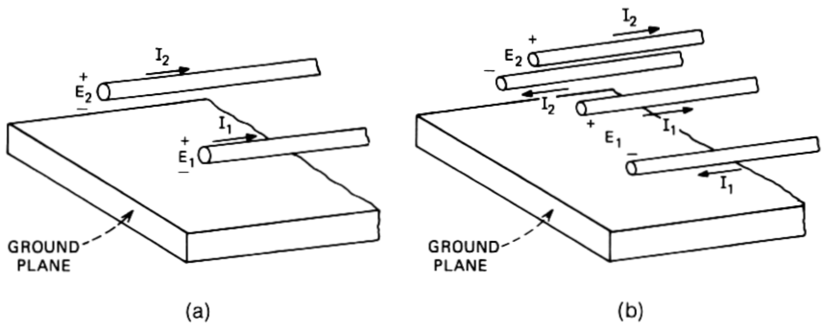

Fig. 1—(a) Unbalanced transmission lines. (b) Balanced transmission lines.

Of course, it is a fundamental assumption of this analysis that the six parameters are independent of x.

Differentiation of (1a) and (1c) with respect to x and substitution of (1b) and (1d) for the appropriate quantities result in

$$\frac{d^2E_1}{dx^2} = A_{11}E_1 + A_{12}E_2 \tag{2a}$$

$$\frac{d^2E_2}{dx^2} = A_{12}E_1 + A_{22}E_2 \tag{2b}$$

where

$$A_{11} = (R + j\omega L)(G + j\omega C) - \omega^2 L_c C_c \tag{3}$$

$$A_{22} = A_{11} \tag{4}$$

$$A_{12} = (R + j\omega L)j\omega C_c + (G + j\omega C)j\omega L_c. \tag{5}$$

Assuming a solution of the form $E_1 = A_1 e^{\gamma x}$ and $E_2 = A_2 e^{\gamma x}$ for (2) yields

$$\gamma = \pm\gamma^+ \quad \text{or} \quad \pm\gamma^-$$

where

$$\gamma^+ = \sqrt{A_{11} + A_{12}} = \{[R + j\omega(L + L_c)][G + j\omega(C + C_c)]\}^{\frac{1}{2}} \tag{6}$$

$$\gamma^- = \sqrt{A_{11} - A_{12}} = \{[R + j\omega(L - L_c)][G + j\omega(C - C_c)]\}^{\frac{1}{2}} \tag{7}$$

and

$$A_2 = \begin{cases} A_1 & \text{if} \quad \gamma = \pm\gamma^+ \\ -A_1 & \text{if} \quad \gamma = \pm\gamma^-. \end{cases}$$

Therefore, the general solutions for $E_1(x)$ and $E_2(x)$ are expressed as

$$E_1(x) = A^+e^{\gamma^+x} + A^-e^{\gamma^-x} + B^+e^{-\gamma^+x} + B^-e^{-\gamma^-x} \tag{8a}$$

$$E_2(x) = A^+e^{\gamma^+x} - A^-e^{\gamma^-x} + B^+e^{-\gamma^+x} - B^-e^{-\gamma^-x} \tag{8b}$$

where the four constants A^+, A^-, B^+, and B^- will be determined from boundary conditions. The corresponding expressions for the two currents can be obtained by solving (1a) and (1c). After the required algebraic manipulations, one obtains

$$I_1(x) = -\frac{1}{Z^+}A^+e^{\gamma^+x} - \frac{1}{Z^-}A^-e^{\gamma^-x} + \frac{1}{Z^+}B^+e^{-\gamma^+x} + \frac{1}{Z^-}B^-e^{-\gamma^-x} \tag{9a}$$

$$I_2(x) = -\frac{1}{Z^+}A^+e^{\gamma^+x} + \frac{1}{Z^-}A^-e^{\gamma^-x} + \frac{1}{Z^+}B^+e^{-\gamma^+x} - \frac{1}{Z^-}B^-e^{-\gamma^-x} \tag{9b}$$

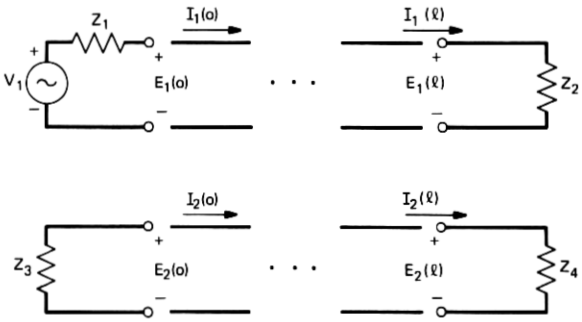

Fig. 2—Boundary conditions imposed on coupled transmission lines.

where

$$Z^+ = \left[\frac{R + j\omega(L + L_c)}{G + j\omega(C + C_c)}\right]^{\frac{1}{2}} \tag{10}$$

$$Z^- = \left[\frac{R + j\omega(L - L_c)}{G + j\omega(C - C_c)}\right]^{\frac{1}{2}}. \tag{11}$$

The boundary conditions that will be imposed are shown in Fig. 2. The corresponding boundary condition equations are:

$$V_1 = Z_1 I_1(o) + E_1(o) \tag{12a}$$

$$O = Z_3 I_2(o) + E_2(o) \tag{12b}$$

$$O = Z_2 I_1(l) - E_1(l) \tag{12c}$$

$$O = Z_4 I_2(l) - E_2(l). \tag{12d}$$

Substituting (8) and (9) into (12) results in four equations for the four unknowns A^+, A^-, B^+, and B^-. Solving for these quantities and substituting them into (8) yield a solution* of the form

$$\frac{E_1(x)}{V_1} = \tfrac{1}{2}R^+(x) + \tfrac{1}{2}R^-(x) \tag{13a}$$

$$\frac{E_2(x)}{V_1} = \tfrac{1}{2}R^+(x) - \tfrac{1}{2}R^-(x). \tag{13b}$$

The near-end crosstalk is given by $E_2(o)/E_1(o)$ while† the far-end crosstalk (equal level) is given by $E_2(l)/E_1(l)$.

* In principle, (13) could be derived from eqs. (1.25) and (1.30) of Ref. 3. However, applying the boundary conditions (12) to these equations leads to sufficient algebraic complication that it is easier to derive (13) directly.

† The conventional definition of near-end crosstalk is $E_2(o)/V_1$ which is equivalent to the above definition (except for a factor of 2) under the conditions of loose coupling and matched terminations.

Obtaining expressions for $R^+(x)$ and $R^-(x)$ involves very extensive algebra for arbitrary impedances and in general does not lend insight into the coupling process. For applications where switching circuits are involved, several special cases of interest partially simplify the algebra in obtaining expressions for the near-end and far-end crosstalk. Some of these cases are:

(*i*) $Z_1 = Z_3$ and $Z_2 = Z_4$,[7] (possible application to analog switching systems).
(*ii*) $Z_1 = Z_3 = Z$, $Z_2 = Z_4 = \infty$ (possible application to switching systems using "totem pole" logic).
(*iii*) $Z_1 = 0$, $Z_2 = Z_3 = Z_4 = \infty$ (possible application to switching systems using simple transistor logic).

These cases all involve somewhat bulky expressions, but they can be obtained with perseverance.

The case that will be studied in detail in the following section is $Z_1 = Z_2 = Z_3 = Z_4$. This case is of special interest for three reasons:

(*i*) The coupling capacitance and inductance can be related easily to physically measurable quantities.
(*ii*) The conditions under which the "classical" crosstalk formulas apply can be studied.
(*iii*) This case is of interest for many applications involving analog circuits.

III. RELATIONSHIP TO PHYSICAL QUANTITIES

The behavior of the coupling process is most easily illustrated by modifying the excitation assumed in (12). Instead of only exciting circuit 1, an excitation will also be applied to circuit 2 as shown in Fig. 3. The set of equations, (12), is modified by letting $Z_1 = Z_2 = Z_3 = Z_4$ and replacing (12b) by

$$\rho V_1 = Z_1 I_2(o) + E_2(o) \tag{14}$$

where ρ is a complex scalar. Obviously, the case $\rho = 0$ corresponds to the situation in Fig. 2 with equal terminating impedances. With this substitution, (13) becomes

$$\frac{E_1(x)}{V_1} = \frac{1+\rho}{2} R_o^+(x) + \frac{1-\rho}{2} R_o^-(x) \tag{15a}$$

$$\frac{E_2(x)}{V_1} = \frac{1+\rho}{2} R_o^+(x) - \frac{1-\rho}{2} R_o^-(x) \tag{15b}$$

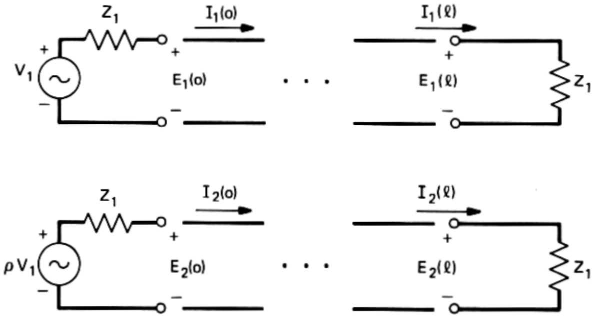

Fig. 3—An alternate means of exciting the coupled transmission lines.

where

$$R_o^+(x) = \frac{P_{00}e^{\gamma^+(l-x)} - P_{10}e^{-\gamma^+(l-x)}}{P_{00}^2 e^{\gamma^+ l} - P_{10}^2 e^{-\gamma^+ l}} \tag{16a}$$

$$R_o^-(x) = \frac{P_{01}e^{\gamma^-(l-x)} - P_{11}e^{-\gamma^-(l-x)}}{P_{01}^2 e^{\gamma^- l} - P_{11}^2 e^{-\gamma^- l}} \tag{16b}$$

and

$$P_{00} = 1 + \frac{Z_1}{Z^+} \tag{17a}$$

$$P_{10} = 1 - \frac{Z_1}{Z^+} \tag{17b}$$

$$P_{01} = 1 + \frac{Z_1}{Z^-} \tag{17c}$$

$$P_{11} = 1 - \frac{Z_1}{Z^-}. \tag{17d}$$

It is now apparent that any excitation of the two coupled transmission lines depicted in Fig. 3 will result in a response which will be a linear combination of the two functions $R_o^+(x)$ and $R_o^-(x)$. Therefore, these functions will be referred to as modes. They will now be examined in somewhat greater detail.

If $\rho = 1$, then (15) reduces to

$$\frac{E_1(x)}{V_1} = \frac{E_2(x)}{V_1} = R_o^+(x). \tag{18}$$

Therefore, if the two lines are energized with equal and in-phase sinusoids, the resulting voltage distributions are given by $R_o^+(x)$. Note that $R_o^+(x)$ contains terms with a + superscript and does not contain any terms with a − superscript. This, in turn, signifies that the propagation constant and the characteristic impedance associated with $R_o^+(x)$ are given by (6) and (10), respectively. This result will now be interpreted in terms of the distributed capacitance and inductance associated with the two transmission lines.

Figure 4 shows a cross section of the two coupled transmission lines, assuming symmetric excitation ($\rho = 1$). According to (18), the voltages on the two lines are equal at every point x; this fact is indicated on Fig. 4. The capacitance per unit length of each conductor to the ground plane is denoted by C_g, while the coupling between conductors is denoted by C_{12}.

Since there is no potential difference across C_{12}, the signals propagating along the two transmission lines are not affected by it. Therefore, each signal propagates along its respective transmission line as if the two lines were uncoupled and with distributed capacitance:

$$C + C_c = C_g. \tag{19}$$

The distributed inductance can be expressed in terms of C_g using the relationship:

$$(C + C_c)(L + L_c) = \mu\epsilon, \tag{20}$$

(See, for example, Chapter I, Sec. 4, eq. (31) of Ref 8.) This formula is applicable to the case where the frequency of excitation is sufficiently high that the magnetic field penetrating the metal conductors contributes a negligible amount to the coupling inductance. Thus

$$L + L_c = \frac{\mu\epsilon}{C_g}. \tag{21}$$

Turning now to the case $\rho = -1$, eq. (15) yields

$$\frac{E_1(x)}{V_1} = -\frac{E_2(x)}{V_1} = R_o^-(x). \tag{22}$$

The propagation constant and characteristic impedance associated with this mode are expressed by (7) and (11), respectively. As with the previous mode, this mode behaves as if the two lines were uncoupled but with primary constants R, G, $C - C_c$, and $L - L_c$. To see how these are related to the physical capacitance, it is convenient to depict the voltages and capacitances as shown in Fig. 5.

As indicated by (22), the voltages on each transmission line are equal but opposite in sign. A vertical line between the two conductors

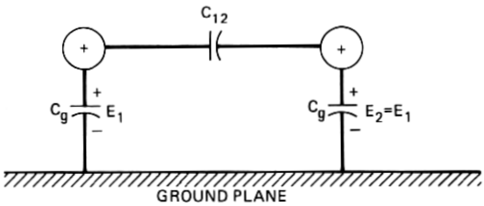

Fig. 4—Cross section of two coupled transmission lines—symmetric excitation.

must therefore constitute a surface of ground potential. Thus, the total capacitance to ground influencing a signal propagating along either line is given by

$$C - C_c = C_g + 2C_{12}. \tag{23}$$

As in the previous case, one may take $(C - C_c)(L - L_c) = \mu\epsilon$ if the frequency is sufficiently high. Thus

$$L - L_c = \frac{\mu\epsilon}{C_g + 2C_{12}}. \tag{24}$$

Combining (19) and (23) to solve for C and C_c yields

$$C = C_g + C_{12} \tag{25}$$

and

$$C_c = -C_{12}, \tag{26}$$

while combining (21) and (24) to solve for L and L_c results in

$$L = \mu\epsilon \frac{C_g + C_{12}}{C_g(C_g + 2C_{12})} \tag{27}$$

and

$$L_c = \mu\epsilon \frac{C_{12}}{C_g(C_g + 2C_{12})}. \tag{28}$$

One final observation is that, in the higher frequency bands of in-

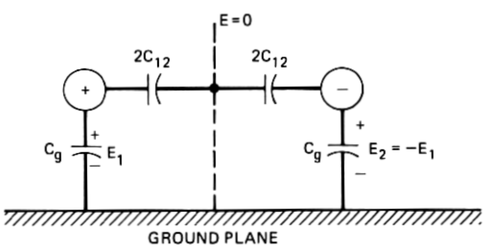

Fig. 5—Cross section of two coupled transmission lines—asymmetrical case.

terest, the following approximations can be made with little error:

$$\gamma^+ \cong \frac{R}{2}\frac{1}{Z^+} + j\omega\sqrt{\mu\epsilon} \tag{29a}$$

$$\gamma^- \cong \frac{R}{2}\frac{1}{Z^-} + j\omega\sqrt{\mu\epsilon} \tag{29b}$$

$$Z^+ \cong \left(\frac{L + L_c}{C + C_c}\right)^{\frac{1}{2}} \tag{29c}$$

$$Z^- \cong \left(\frac{L - L_c}{C - C_c}\right)^{\frac{1}{2}}. \tag{29d}$$

(See Chapter II, Sec. 13, eqs. (18), (8), and (6) of Ref. 8.)

Substituting eqs. (25) to (28) into the above equations yields

$$\gamma^+ \cong \frac{1}{2}\frac{RC_g}{\sqrt{\mu\epsilon}} + j\omega\sqrt{\mu\epsilon} \tag{30a}$$

$$\gamma^- \cong \frac{1}{2}\frac{R(C_g + 2C_{12})}{\sqrt{\mu\epsilon}} + j\omega\sqrt{\mu\epsilon} \tag{30b}$$

$$Z^+ \cong \frac{\sqrt{\mu\epsilon}}{C_g} \tag{30c}$$

$$Z^- \cong \frac{\sqrt{\mu\epsilon}}{C_g + 2C_{12}}. \tag{30d}$$

Thus, the $R_o^-(x)$ mode has a higher loss and smaller characteristic impedance than the $R_o^+(x)$ mode.

It is now possible to outline a measurement procedure that will yield all quantities required to evaluate (13). Since the effective dielectric constant surrounding most physical transmission lines is determined by the detailed geometry of the insulation and shields surrounding each conductor, the quantity $\sqrt{\mu\epsilon}$ will be assumed unknown for the following procedure, even though μ and ϵ may be known for each constituent material in the transmission line.

Step 1. Measure C_g and C_{12}.

Step 2. Terminate the coupled pairs in four equal impedances Z_1 and energize the two lines from the same voltage generator. The generator frequency should be in the range for which the approximations leading to (29) and (30) are valid. In other words, terminate and excite the lines as shown in Fig. 3 with $\rho = 1$.

Step 3. Adjust all four impedances, Z_1, until $R_o^+(l)$ is a maximum. It is easy to show that in this case $Z_1 = Z^+$. Using (30c) and the value of C_g from Step 1 gives $\sqrt{\mu\epsilon}$.

Step 4. With $Z_1 = Z^+$, measure $R_o^+(l)$ which equals $(\frac{1}{2}) \exp(-\gamma^+ l)$. With γ^+ given by (30a), R can be solved for directly, given the length of the coupled lines, l.

Step 5. The remaining quantities, γ^- and Z^-, can now be evaluated using (30b) and (30d). Note that there is an additional check on the value of $\sqrt{\mu\epsilon}$ through the imaginary part of γ^+.

IV. COMPARISON OF RESULTS WITH CLASSICAL CROSSTALK FORMULAS

We now use the results of the previous section to analyze the "classical" crosstalk formulas as derived by Shelkunoff and Odarenko.[2] Their analysis assumed uniform weak coupling between two parallel transmission lines terminated in their characteristic impedances. Assuming the two transmission lines had identical primary and secondary constants, they derived the following formulas for near-end crosstalk (NEXT) and far-end crosstalk (FEXT):

$$N(\omega) = \frac{Z_{12}}{4Z_0\gamma_0(\omega)}(1 - e^{-2\gamma_0(\omega)l}), \tag{31}$$

$$F(\omega) = \frac{Z_{12}'}{2Z_0} l, \tag{32}$$

where Z_{12}, Z_{12}' are the mutual impedances between the two lines for NEXT and FEXT, respectively, Z_0 and γ_0 the secondary quantities of an isolated line [i.e., (10) and (6) with $L_c = C_c = 0$], and l the length of the lines. The above formulas were derived neglecting the effects of the disturbed line on the disturbing line. In the discussion that follows, we shall examine the validity of this assumption.

Referring to (6), (7), (10), and (11), and assuming $L_c \ll L$, $C_c \ll C$, and $Z_1 = Z_0$, it can be shown that

$$Z^{\pm} \cong Z_0 \pm \delta \tag{33a}$$

$$\delta = \frac{j\omega}{2\gamma_0}(L_c - C_c Z_0^2) \tag{33b}$$

$$\left|\frac{\delta}{Z_0}\right| \ll 1 \tag{33c}$$

and

$$\gamma^{\pm} = \gamma_0 \pm \epsilon \tag{34a}$$

$$\epsilon = j\omega \frac{(L_c + C_c Z_0^2)}{2Z_0} \tag{34b}$$

$$\left|\frac{\epsilon}{\gamma_o}\right| \ll 1. \tag{34c}$$

Now using (33), (34), (13), (16), and (17) one can show that

$$N(\omega) = \frac{E_2(o)}{E_1(o)} \cong \frac{8\delta}{Z_0} \frac{\left(1 - e^{-2\gamma_0 l}\left[\cosh(2\epsilon l) - j\frac{\delta}{Z_0}\sinh(2\epsilon l)\right]\right)}{16} \tag{35}$$

where the approximation is obtained by only assuming weak coupling. Now for sufficiently small $|\epsilon l|$, the term in brackets is approximately unity so that (35) becomes

$$N(\omega) \cong \frac{\delta}{Z_0}(1 - e^{-2\gamma_0 l})$$

$$= \frac{j\omega}{4\gamma_0 Z_0}(L_c - C_c Z_0^2)(1 - e^{-2\gamma_0 l}). \tag{36}$$

This agrees with the Shelkunoff and Odarenko result, (31), with $Z_{12} = j\omega(L_c - C_c Z_0^2)$.

For larger values of l, the exponential term in (35) can usually be neglected for lossy lines. In the lossless case, the term in brackets will cause a departure from the classical formula for sufficiently large $|\epsilon l|$; however, in weak coupling situations, the length and/or frequency required to invalidate the approximation $\cosh(2\epsilon l) \cong 1$ are usually large enough to invalidate the lossless assumption. Thus, for most practical situations involving weak coupling, eq. (31) is adequate.

We now consider far-end crosstalk. Again referring to (13), (16), and (17), letting $x = l$, and assuming the conditions for weak coupling exist, it can be shown that

$$F(\omega) \cong \frac{e^{\gamma^- l} - e^{\gamma^+ l}}{e^{\gamma^- l} + e^{\gamma^+ l}}. \tag{37}$$

Substituting (34a) into (37) results in

$$F(\omega) \cong \frac{e^{-\epsilon l} - e^{\epsilon l}}{e^{-\epsilon l} + e^{\epsilon l}}$$

$$= -\tanh(\epsilon l). \tag{38}$$

Referring to (34b), for sufficiently large ω, Z_0 is a real constant and ϵ is an imaginary number; thus, (38) can be written as

$$F(\omega) \cong -j \tan(-j\epsilon l) \tag{39a}$$

$$\cong -\epsilon l \qquad \text{for} \qquad |\epsilon l| \leqq \frac{\pi}{6}$$

$$= -j\omega \frac{(L_c + Z_0^2 C_c)l}{2Z_0}. \tag{39b}$$

Therefore, for $|\epsilon l| \leqq (\pi/6)$, eq. (39b) agrees with (32) with $Z'_{12} = -j\omega(L_c + Z_0^2 C_c)$. Shelkunoff and Odarenko[2] point out that (32) must not be carried to an absurd conclusion: namely, that most of the far-end power will reside in the disturbed circuit for sufficiently long transmission lines. They conjecture that, in the limiting case, the far-end power will divide equally between the two lines. Equation (39a) indicates that the far-end power oscillates back and forth between the two lines as a function of l (or frequency, since ϵ is a function of ω). Equation (39a) is valid over a larger range of l than (32) (or 39b), although it is not valid for all l, since it is based on an approximation, (34), which is multiplied by l. To be more specific, (6) and (7) can be written as

$$\gamma^\pm = [(\gamma_0^2 - \omega^2 L_c C_c) \pm j\omega\gamma_0(Z_0 C + L_c/Z_0))]^{\frac{1}{2}}$$

$$= \gamma_0\left[1 \pm \frac{j\omega}{\gamma_0 Z_0}(L_c + C_c Z_0^2) - \frac{\omega^2}{\gamma_0}L_c C_c\right]^{\frac{1}{2}}. \tag{40}$$

Assuming the conditions for weak coupling ($L_c \ll L$, $C_c \ll C$), the third term in the brackets is much smaller than the second term, and the second term in the brackets has a magnitude much less than unity, so (34) is a good first-order approximation to (40). Now for Z_0 a real constant, ϵ is an imaginary quantity. However, for any given frequency the higher order terms from (40) contain real parts which will dominate the behavior of the exponential terms in (37) for sufficiently large l. Thus, referring to (37), in the limit as $l \to \infty$,

$$\lim_{l\to\infty} |F(\omega)| = \lim_{l\to\infty} \left|\frac{1 - e^{(\gamma^+ - \gamma^-)l}}{1 + e^{(\gamma^+ - \gamma^-)l}}\right| = 1. \tag{41}$$

The same result is reached by fixing l and letting $\omega \to \infty$.

In summary, when weak coupling conditions exist, the effect of the disturbed line on the disturbing line can be neglected for most NEXT calculations; however, for a sufficiently large l and/or ω, the effect of the disturbed line on the disturbing line cannot be neglected for FEXT calculations. This is because, for certain values of l and/or ω, the far-end power in the disturbed line will be comparable to the far-end power in the disturbing line.

V. CONCLUSION

An analytical model for analyzing crosstalk between two identical, parallel, uniformly coupled transmission lines with ground return has been presented. Using this model, formulas were developed for the two sets of modal voltages and currents on the transmission lines. It was found that each mode has associated with it a propagation factor and characteristic impedance which, in general, are different for each mode.

By applying different sets of excitation voltages to the two lines (changing boundary conditions), the effect of each line on the other can be analyzed in terms of the modal quantities. Using this technique, formulas were derived for the coupling capacitance and inductance in terms of the distributed capacitance and distributed inductance of an isolated line, the distributed capacitance to ground for the nonisolated lines, and the permeability and permittivity of the medium surrounding the transmission lines.

The mutual influence of the two lines was also studied by assuming weak coupling between them and then deriving NEXT and FEXT formulas using this model. These formulas were compared with the classical formulas which do not take into account the influence of the disturbed line on the disturbing line. In the case of NEXT, the effect of the disturbed line on the disturbing line was found to be negligible for most practical cases. In the case of equal level FEXT, however, the effect of the disturbed line on the disturbing line can be quite significant for sufficiently large line length and/or frequency.

REFERENCES

1. Campbell, G. A., "Dr. Campbell's Memoranda of 1907 and 1912," B.S.T.J., *14*, No. 4 (October 1935), pp. 558–572.
2. Shelkunoff, S. A., and Odarenko, T. M., "Crosstalk Between Coaxial Transmission Lines," B.S.T.J., *16*, No. 2 (April 1937), pp. 144–164.
3. Rice, S. O., "Steady State Solutions of Transmission Line Equations," B.S.T.J., *20*, No. 2 (April 1941), pp. 131–178.

4. Kuznetsov, P. I., and Stratonovich, R. L., *The Propagation of Electromagnetic Waves in Multiconductor Transmission Lines*, 1964, London: Pergamon Press, Ltd., distributed by the MacMillian Company, New York. See Chapter 6 which is entitled: "The Propagation of Electromagnetic Waves Along Two Parallel Single-Wire Lines."
5. Amemiya, H., "Time-Domain Analysis of Multiple Parallel Transmission Lines," RCA Review, *28*, 1967, pp. 241–276.
6. Dvorak, V., "Numerical Solution of the Transient Response of a Distributed Parameter Transformer," IEEE Trans. on Circuit Theory (May 1970), pp. 270–273.
7. Remec, M. J., private communication.
8. King, R. W. P., *Transmission Line Theory*, New York: Dover Publications Inc., 1965.