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Restoring the Orthogonality of Two Polarizations in Radio Communication Systems, II

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The Poincaré sphere has been applied to the analysis of orthogonalizing two polarization ellipses by a differential phase shifter and a differential attenuator. The condition of minimum differential attenuation for removing a given amount of nonorthogonality is determined. A previously reported transformation via two nonorthogonal linear polarizations should be used for two slender ellipses.[1] *Another transformation via two oppositely rotating ellipses having parallel axes and equal axial ratios should be used for two fat ellipses. System applications of the transformations are discussed.*

I. INTRODUCTION

A method of recovering the orthogonality of two polarizations in a radio communication system was presented recently.[1] Two arbitrary polarization ellipses are first transformed simultaneously into two nonorthogonal linear polarizations by a differential phase shifter, and then the nonorthogonality is removed by a differential attenuator. It is of interest to ask whether that transformation is optimum for system applications.

Before proceeding further, we will define the optimum transformation. Clearly, it is desirable to minimize the differential attenuation that we must introduce to correct for nonorthogonality. However, in order to achieve maximum bandwidth, we also should minimize the differential phase shift even if we assume, as we do here, that the polarization characteristics of components in the system are not very frequency sensitive over the operating bandwidth. This assumption is, for example, expected to be valid for any depolarization in the main beam of reflector-type antennas provided there is no polarization distortion in the feed radiation. If necessary, we can apply polarization correction separately to each of the subbands within a wide band.

In a practical dual-polarization radio system, the two orthogonal polarizations feeding the transmitting antenna are either linear or circular. Signal generators and detectors are always linearly polarized; however, conversion to circular polarization for radio transmission is sometimes made to avoid effects such as Faraday rotation. If the transmission medium and the radiating systems introduce only moderate polarization distortion, the dual polarization signals appear as two slender or two fat polarization ellipses at the receiving terminal, depending on whether linear or circular polarizations are being used. This classification into two types of elliptical shapes suggests an optimum transformation for each type. However, the validity of the transformations presented in the next section is independent of the shapes of the ellipses. A practical design of adjustable differential phase-shifters and attenuators has been suggested by E. A. Ohm.[2]

The following analysis will be presented with the aid of the Poincaré sphere. This geometrical representation of the polarization of a plane electromagnetic wave was introduced to radio engineers by G. A. Deschamps.[3] For convenience of the reader, a summary of the Poincaré spherical representation is given in the Appendix.

II. ANALYSIS

2.1 *Minimum Differential Attenuation*

Let us first find the condition for minimum differential attenuation required for removing a given amount of nonorthogonality between two polarizations. Each polarization is represented by a point on the Poincaré sphere. If the great circle arc connecting two points on the sphere is a semicircle, then the two polarizations are orthogonal to each other. The degree of nonorthogonality between two polarizations can be measured by the deviation of the great circle arc from a semicircle. Let two nonorthogonal elliptically polarized waves be represented by points M_1 and M_2 on Poincaré sphere as shown in Fig. 1a. The great circle arc connecting M_1 and M_2 intersects the equator at C. The longitudes of M_1 and M_2 with respect to C are twice the orientation angles of the two polarization ellipses with respect to the X-axis of a set of X-Y coordinates. This relationship is sketched in Fig. 1b. Since M_1 and M_2 are nonorthogonal to each other, $\overset{\frown}{M_1C} + \overset{\frown}{M_2C} < \pi$. Orthogonalization is accomplished by stretching the great circle arc $\overset{\frown}{M_1M_2}$ to the value of π.

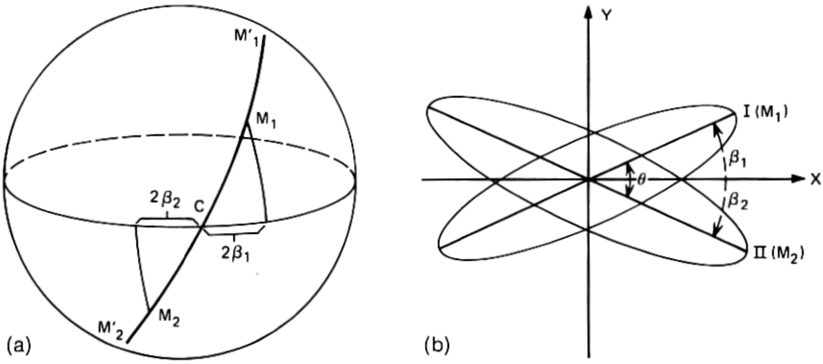


Fig. 1—Orthogonalization by differential attenuation.

If a differential attenuation of $\dfrac{\tan\frac{1}{2}\overset{\frown}{M_1C}}{\tan\frac{1}{2}\overset{\frown}{M_1'C}} = \dfrac{\tan\frac{1}{2}\overset{\frown}{M_2C}}{\tan\frac{1}{2}\overset{\frown}{M_2'C}}$ is imposed along the X-axis such that $\overset{\frown}{M_1'C} + \overset{\frown}{M_2'C} = \pi$, then the two polarizations M_1 and M_2 will be orthogonalized. Since $\tan\frac{1}{2}\overset{\frown}{M_1'C}\tan\frac{1}{2}\overset{\frown}{M_2'C} = 1$, the required differential attenuation is minimized by maximizing $\sqrt{\tan\frac{1}{2}\overset{\frown}{M_1C}\tan\frac{1}{2}\overset{\frown}{M_2C}}$ with a proper differential phase shift, which implies the constraint that $\overset{\frown}{M_1C} + \overset{\frown}{M_2C} = \overset{\frown}{M_1M_2}$. In this way, the minimum differential attenuation needed is found to be $\tan\frac{1}{4}\overset{\frown}{M_1M_2}$ when $\overset{\frown}{M_1C} = \overset{\frown}{M_2C}$. For the given degree of nonorthogonality, $\pi - \overset{\frown}{M_1M_2}$, the orthogonalization can be performed by the minimum differential attenuation $\tan\frac{1}{4}\overset{\frown}{M_1M_2}$ only if the two elliptical polarizations have the same axial ratio. Thus the two arbitrary elliptical polarizations must first be transformed by a differential phase shifter to allow a minimum differential attenuation.

Furthermore, one wishes to obtain two orthogonal linear or circular polarizations immediately following orthogonalization. This requirement determines the prerequisite condition that two nonorthogonal elliptically polarized waves should first be transformed into two nonorthogonal linear polarizations or two oppositely rotating elliptical polarizations having parallel major and minor axes and equal axial ratios, both of which are limiting cases of two elliptical polarizations with the same axial ratio.

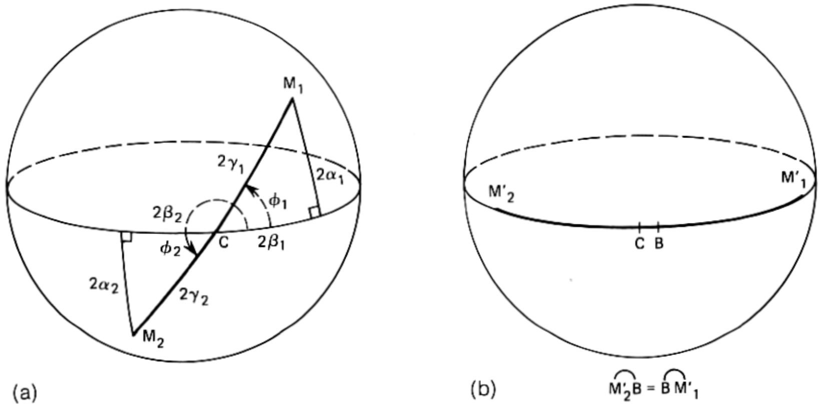


Fig. 2—Simultaneous transformation of two elliptical polarizations into two linear polarizations.

2.2 *Two Slender Ellipses*

If the transmitting antenna is fed by two orthogonal linear polarizations, moderate polarization distortion in a radio communication system will produce two slender polarization ellipses at the receiving terminal. At that point, the orthogonalization should begin by a simultaneous transformation of the two polarization ellipses into two linear polarizations. This transformation has been obtained previously;[1] however, it will be described here in terms of the Poincaré sphere.

Let two nonorthogonal elliptic polarizations be represented by two points, M_1 and M_2 in Fig. 2a. The intersection C of the great circle arc with the equator designates a set of coordinate axes X-Y which is shown in Fig. 1b. The polarization ratios of the two polarization ellipses in terms of these X-Y coordinates will be related by $\tan \phi_1 = \tan \phi_2$. Replacing each side of this equation by eq. (11) yields:

$$\tan 2\alpha_1 \csc 2\beta_1 = \tan 2\alpha_2 \csc 2\beta_2. \tag{1}$$

Substituting $\beta_2 = \beta_1 - \theta$ into eq. (1), one obtains the solution for the orientation of the X-Y axes

$$\beta_1 = \frac{1}{2} \cot^{-1} \left[\frac{\cos 2\theta - \dfrac{\tan 2\alpha_2}{\tan 2\alpha_1}}{\sin 2\theta} \right], \quad 0 < \beta_1 < \frac{\pi}{2}. \tag{2}$$

The arc $\overset{\frown}{M_1 M_2}$ can be rotated onto the equator by applying the following differential phase delay to the components in the Y direction

$$\phi = \tan^{-1} [\tan 2\alpha_1 \csc 2\beta_1]. \tag{3}$$

Now the angle ψ between the two linear polarizations represented by M_1' and M_2' in Fig. 2b is half of the sum of the arcs $\overset{\frown}{CM_1}$ and $\overset{\frown}{CM_2}$

$$\psi = \gamma_1 + \gamma_2 \tag{4}$$

where $\gamma_i = \tan^{-1} \sqrt{\dfrac{(1 + \tan^2 \alpha_i) - (1 - \tan^2 \alpha_i) \cos 2\beta_i}{(1 + \tan^2 \alpha_i) + (1 - \tan^2 \alpha_i) \cos 2\beta_i}}$ is obtained using eq. (10). This angle ψ may be changed to $\pi/2$ if a differential attenuation of tan $(\psi/2)$ is imposed on the components in the direction B bisecting the two linear polarizations. This direction will be oriented at an angle

$$x = \tfrac{1}{2}(\gamma_1 - \gamma_2) \tag{5}$$

with respect to the X direction of the coordinates in Fig. 1b.

The above equations can be easily identified with those in Ref. 1. One notes that the transformations are valid for the two polarizations located on the same side as well as on opposite sides of the equator of the sphere.

2.3 *Two Fat Ellipses*

If the transmitting antenna is fed by two orthogonal circular polarizations, two fat polarization ellipses will appear at the receiving terminal if the radio communication system is moderately contaminated by polarization distortion. Here the orthogonalization should be started by the simultaneous transformation of two given ellipses into two oppositely rotating ellipses having parallel major and minor axes and equal axial ratios. One looks for an X′-Y′ cartesian coordinate system in terms of which the polarization ratios of the two ellipses become complex conjugates of each other after a proper differential phase delay is introduced along the axes. The X′-Y′ coordinates correspond to the point C′, and the proper differential delay is represented by Δ on the Poincaré sphere as shown in Fig. 3a. Let us write down the expression for the polarization ratio in terms of the X′-Y′ coordinates[1]

$$P_i' = \sqrt{\frac{(1 + \tan^2 \alpha_i) - (1 - \tan^2 \alpha_i) \cos 2\beta_i'}{(1 + \tan^2 \alpha_i) + (1 - \tan^2 \alpha_i) \cos 2\beta_i'}}\, e^{j \tan^{-1} \left[\frac{2 \tan \alpha_i}{(1 - \tan^2 \alpha_i) \sin 2\beta_i'} \right]} \tag{6}$$

$$0 < \underline{/P_i'} < \pi \text{ when } \tan \alpha_i > 0;$$

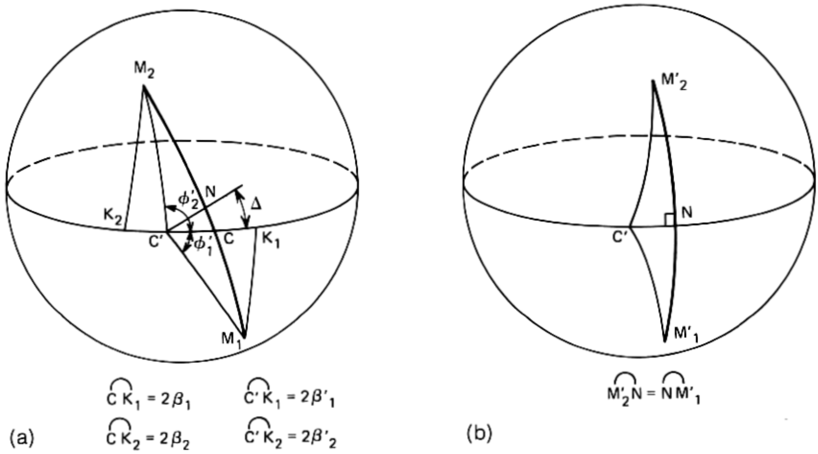


Fig. 3—Simultaneous transformation of two polarization ellipses into two ellipses with parallel axes and equal axial ratios.

where $i = 1, 2$;

$$-\frac{\pi}{2} < \underline{|P_i'} < \frac{\pi}{2} \text{ when } \sin 2\beta_i' > 0 .$$

Combining the condition $|P_1'| = |P_2'|$ and the relation $\beta_2' = \beta_1' - \theta$, one obtains

$$\beta_1' = \frac{1}{2} \tan^{-1} \left[\frac{\dfrac{(1 + \tan^2 \alpha_2)(1 - \tan^2 \alpha_1)}{(1 + \tan^2 \alpha_1)(1 - \tan^2 \alpha_2)} - \cos 2\theta}{\sin 2\theta} \right], \quad 0 < \beta_1' < \frac{\pi}{2} \tag{7}$$

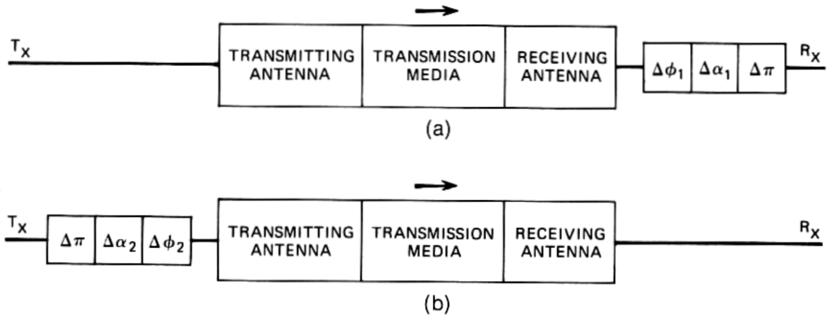
which fixes the X′-Y′ axes. The required differential phase delay Δ along the Y′ axis is determined by

$$\underline{|P_1'} - \Delta = -\left(\underline{|P_2'} - \Delta\right)$$

which gives

$$\Delta = \tfrac{1}{2}\left(\underline{|P_1'} + \underline{|P_2'}\right). \tag{8}$$

The above differential phase shift corresponds to the rotation of the great circle arc $\overset{\frown}{M_1M_2}$ about the point C′. The transformed ellipses are represented by M_1' and M_2' where $\overset{\frown}{M_1'N} = \overset{\frown}{M_2'N}$ and $\overset{\frown}{M_1'M_2'}$ is perpendicular to the equator as shown in Fig. 3b. The parallel major axes of the transformed ellipses are oriented with respect to the X′ axis at



$\Delta\phi_1, \Delta\phi_2$ DIFFERENTIAL PHASE – SHIFTER

$\Delta\alpha_1, \Delta\alpha_2$ DIFFERENTIAL ATTENUATOR

$\Delta\pi$ PHASE – SHIFTER WILL PERFORM POLARIZATION TRACKING FOR ORTHOGONAL LINEAR POLARIZATIONS ($\Delta\frac{\pi}{2}$ PHASE – SHIFT AT BOTH TRANSMITTING AND RECEIVING ENDS WILL BE NEEDED FOR ORTHOGONAL CIRCULAR POLARIZATIONS)

Fig. 4—Locations of the orthogonalization device.

an angle equal to $\frac{1}{2}\,\widehat{NC'}$. Using eq. (13), we have

$$\tfrac{1}{2}\,\widehat{NC'} = \tfrac{1}{2}\tan^{-1}\left\{\frac{2|P_1'|}{1-|P_1'|^2}\cos\tfrac{1}{2}\left[\angle P_1' - \angle P_2'\right]\right\}. \tag{9}$$

Now two oppositely rotating circular polarizations can be obtained by imposing a differential attenuation along the major axis of the transformed ellipses. The value of differential attenuation is also given by $\tan(\psi/2)$ where ψ is determined by eq. (4) of the preceding section.

In addition to the differential phase shift for orthogonalization as discussed above, circular polarization systems require $\Delta(\pi/2)$ phase shift* at both transmitting and receiving ends to convert to the final linearly polarized ports. The same amount of additional differential phase shift overall will also be needed in a system using orthogonal linear polarizations, if a $\Delta\pi$ phase shifter is used for polarization tracking.

III. DISCUSSION

The above analysis assumed that the device for orthogonalization would be located at the receiving terminal as shown in Fig. 4a. Since there always exist two elliptic polarizations which would become orthogonal after going through a linear transmission system with a certain polarization distortion, one can also put the differential ele-

* The notation, $\Delta(\pi/2)$, implies a differential phase shift of $\pi/2$.

ments at the transmitting terminal as shown in Fig. 4b. Another obvious corollary states that the dual-polarization radiation of an antenna always can be orthogonalized in any particular direction. The differential attenuator should be located as illustrated in Figs. 4a and b at the receiver or the transmitter in order to satisfy the condition for minimum differential attenuation. Sometimes it is desirable to use differential elements at both the transmitting and receiving terminals. For example, one may wish to eliminate the polarization distortions of the transmitting and receiving antennas separately.

It is often claimed that the use of circular polarization in a satellite communication system eliminates the need for polarization tracking. In order to realize this advantage, the depolarization of the satellite antenna radiation must be kept small over the entire coverage of ground stations. Furthermore, the matching requirement at each discontinuity of the waveguide feeding network and the radiating system is more stringent for circular polarization, because multiple reflections among the discontinuities often corrupt circular but not linear polarization.

IV. ACKNOWLEDGMENTS

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APPENDIX

Poincaré Spherical Representation

A polarization ellipse shown in Fig. 5a is completely characterized by the axial ratio, A = minor axis/major axis, and the orientation of the ellipse. The sense of rotation can be taken into account by giving $+$ or $-$ sign to the axial ratio. Now we define $\tan^{-1} A$ as the ellipticity angle α with $-45° \leqq \alpha \leqq 45°$, and take the angle between OX and the major axis as the orientation angle β with $-90° < \beta < 90°$. Then a point M on a sphere with longitude 2β and latitude 2α as shown in Fig. 5b completely specifies a state of polarization.

The points on the equator represent linear polarizations. The arc between two points along the equator is twice the angle between two linear polarizations. The points on the upper and lower hemispheres correspond to clockwise and counterclockwise (wave approaching) elliptical polarizations respectively, while the poles designate circular polarizations. For each point which represents an elliptic polarization, the projection K onto the equator defines the orientation of its major

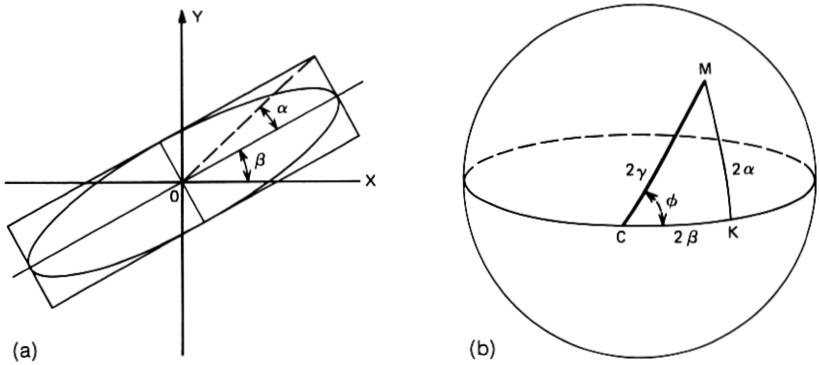

Fig. 5—Poincaré spherical representation of a polarization ellipse.

axis. Any set of X-Y coordinates can be specified by its X direction which in turn corresponds to a point C on the equator. Then the longitude of K can be measured with respect to C.

The polarization ratio P is a complex number defined as the ratio between the Y and X components of the electric vector. Using the expression for the polarization ratio in eq. (6) and the following formulas for the spherical triangle shown in Fig. 5b,

$$\cos 2\gamma = \cos 2\alpha \cos 2\beta \tag{10}$$

$$\tan \phi = \tan 2\alpha \csc 2\beta, \tag{11}$$

one can identify $P = \tan \gamma e^{j\phi}$. The orientation and the ellipticity can be expressed in terms of P as follows:

$$\sin 2\alpha = \sin 2\gamma \sin \phi \tag{12}$$

$$\tan 2\beta = \tan 2\gamma \cos \phi. \tag{13}$$

A differential phase delay of Δ of the Y component with respect to the X component will correspond to a clockwise rotation Δ of the arc $\overset{\frown}{CM}$ around the point "C" on the Poincaré sphere.

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