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# Optimal Trade-Off of Mode-Mixing Optical Filtering and Index Difference in Digital Fiber Optic Communication Systems

By S. D. PERSONICK

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*In a digital fiber optical communication system, the optical power required at the receiver input to achieve a desired error rate depends upon the shape of the received pulses. In systems employing multimode fibers and/or broadband sources, we can experience pulse spreading in propagation because of the group velocity differences of different modes or because of dispersion. In an effort to control or compensate for pulse spreading, we can trade off coupling efficiency between the light source and the fiber (by varying the core-cladding index difference or bandlimiting the source), scattering loss in the fiber (by introducing mode coupling), and equalization in the receiver at baseband. This paper investigates the optimal trade-off for various fiber-source combinations.*

## I. INTRODUCTION AND REVIEW OF BACKGROUND MATERIAL

In digital fiber optic communication systems, as in other digital systems, the received power required at a repeater to achieve a desired error rate depends upon the shape of the received pulses. A previous paper[1] showed that the minimum average power requirement results from a pulse that is sufficiently narrow so that its energy spectrum is almost constant for all frequencies passed by the receiver (ideally, an impulse). For other received pulse shapes, we can define the additional power required, in decibels, as a "power penalty" for not having impulse-shaped pulses. Typical calculations of this power penalty for "on-off" signaling and a receiver employing avalanche gain with a high impedance front end[1] are shown for various families of received pulse shapes in Fig. 1. In that figure, the parameter $\sigma/T$ is defined as follows:

$$\frac{\sigma^2}{T^2} = \frac{1}{T^2}\left\{\frac{1}{A}\int h_p(t)t^2dt - \left[\frac{1}{A}\int h_p(t)tdt\right]^2\right\}, \tag{1}$$

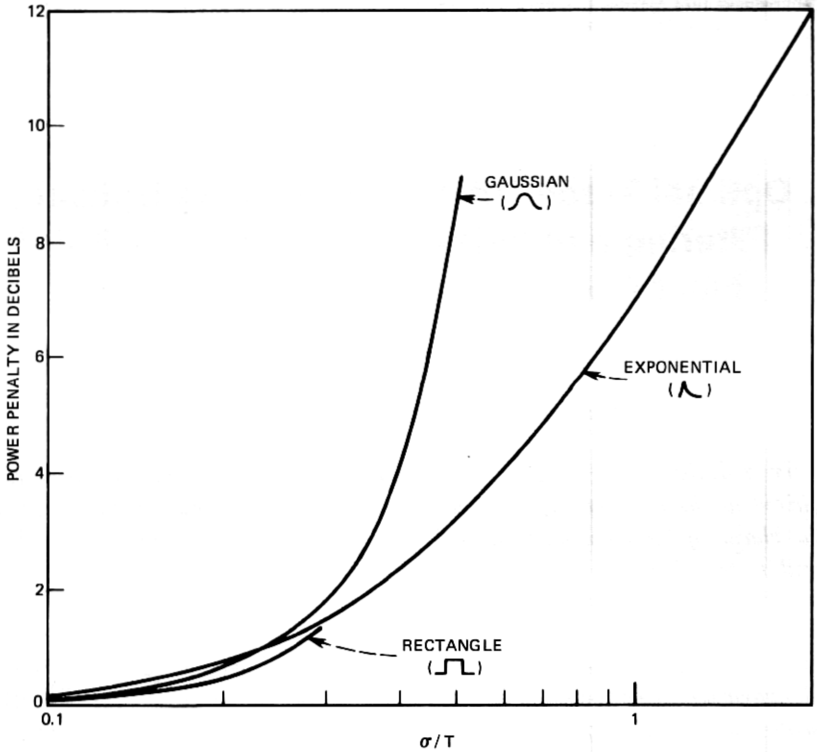


Fig. 1—Typical power penalty vs $\sigma/T$.

where

$$T = \text{spacing in time between binary digits}$$
$$h_p(t) = \text{received optical pulse shape}$$
$$A = \text{area under } h_p(t).$$

We shall refer to $\sigma$ as the rms pulse width.*

It has been shown[2,3] that, in long fibers, the "power impulse response" of the fiber approaches a Gaussian shape. In the rest of this paper, we assume that the received optical pulse is Gaussian in shape and that it has an rms width determined by the fiber delay distortion. That is, we assume that the rms width of the fiber input pulse is sufficiently small so that the power penalty associated with the rms

* In a Gaussian-shaped pulse, the rms width is about 0.425 times the full width between half-amplitude points. In a rectangular pulse, the rms width is $1/\sqrt{12}$ the full width.

sum of that width and the rms fiber impulse response width is the same as the power penalty associated with the rms fiber impulse response width alone.

From various heuristic analyses (see the appendix), we can conclude that the rms width of the received pulse is approximately the rms sum of the delay distortion in the fiber resulting from material dispersion (because of the variation of group velocity with wavelength associated with the use of a broadband source) and the delay distortion associated with the spread in the group delays of various fiber modes (when a multimode fiber is used). That is,

$$\sigma = (\sigma^2_{\text{dispersion}} + \sigma^2_{\text{mode}})^{\frac{1}{2}}, \tag{2}$$

where $\sigma_{\text{dispersion}} \sim$ optical bandwidth·fiber length $= B \cdot L$ and where $\sigma_{\text{mode}}$ is determined as follows[4]

*Case* 1. Conventional clad multimode fibers without mode coupling.

$$\sigma_{\text{mode}} = 0.289 \frac{\Delta n}{c} L,$$

where

$n$ = index of refraction of the core

$\Delta$ = (index of refraction of the core − index of refraction of the cladding)/$n$

$c$ = speed of light.

*Case* 2. Conventional clad fibers with complete mode coupling after a distance $L_C$.

$$\sigma_{\text{mode}} = 0.289 \frac{\Delta n}{c} \sqrt{LL_C} \qquad \text{for} \quad L > L_C,$$

$$= 0.289 \frac{\Delta n}{c} L \qquad \text{for} \quad L \leqq L_C.$$

*Case* 3. Ideal graded index fiber without mode coupling.

$$\sigma_{\text{mode}} = 0.037 \frac{\Delta^2 n}{c} L.$$

*Case* 4. Other fibers[2] can be treated once the techniques outlined below are understood.

From the definitions of $\sigma_{\text{dispersion}}$ and $\sigma_{\text{mode}}$ above, we see that, for a broadband (incoherent) source, the material dispersion contribution to $\sigma$ can be controlled by limiting the optical bandwidth $B$ being used. However, if the optical source bandwidth, $B_o$, must be reduced by filtering, then the average power into the guide will be reduced by the factor $B/B_o$. Similarly, we can control the mode delay spread by reducing the index difference $\Delta$. If a multimode (incoherent) source is being used, the average power into the guide is proportional to $\Delta$. Thus, we trade off input power against mode delay spread. Furthermore, even when we use a coherent source which in principle can be focused into any fiber, we must be careful when $\Delta$ becomes significantly smaller than 0.005, since the fiber loss at bends becomes large. (Exactly what value of $\Delta$ is too small to be practical is an open question.) For fibers with mode coupling, we can control $\sigma_{\text{mode}}$ by decreasing $L_C$ (increasing the mode coupling). However, this causes the coupling to radiating modes to increase, thus increasing the fiber loss.[5] Here again, there is a trade-off between the average power we receive at the fiber output and the received rms pulse width. For a *fixed shape* of the mechanical spectrum of fiber geometry perturbations, the radiation loss per unit length of the fiber resulting from mode coupling is inversely proportional to $L_C$, i.e.,

$$\text{radiation loss in nepers} = \alpha_o L/L_C, \tag{3}$$

where

$\alpha_o$ = constant depending upon the *shape* of the mechanical spectrum of the geometry perturbations causing coupling (and possibly upon the index difference $\Delta$). $L_C$ depends upon the *amplitude* of the mechanical perturbations.

In the following sections we derive the optimal trade-off between $\sigma$, $B$, $\Delta$, and $L_C$ for various combinations of sources and fibers to maximize the allowable fiber length $L$ between the optical source and the repeater.

## II. ANALYSIS

### 2.1 Incoherent source, conventional clad fiber, no mode coupling

Let the average power into the guide be $P_s$ when the index difference $\Delta$ is at some maximum practical value $\Delta_o$ and when the full source optical bandwidth $B_o$ is being used. Let the loss of the fiber be $\alpha$ nepers per kilometer. Let the power penalty from the nonzero value of the received rms pulse width, in nepers, be $f(\sigma/T)$. Let the required power at the receiver be $P_r$ when $\sigma/T = 0$. If we use a value of $\Delta \leqq \Delta_o$ and filter the source output to have an optical bandwidth $B \leqq B_o$, then we must have

$$P_s \frac{\Delta}{\Delta_o} \frac{B}{B_o} e^{-\alpha L} \geqq P_r e^{f(\sigma/T)}, \tag{4}$$

where, from (2),

$$\sigma = \left\{ \left( C_1 \frac{B}{B_o} L \right)^2 + \left( C_2 \frac{\Delta}{\Delta_o} L \right)^2 \right\}^{\frac{1}{2}} = \{ \sigma_d^2 + \sigma_m^2 \}^{\frac{1}{2}}$$

and $C_1$, $C_2$ are constants.

We rewrite (4) as follows (using equality to maximize $L$),

$$\frac{P_s}{P_r} e^{-\alpha L} = \left\{ \frac{\Delta}{\Delta_o} \frac{B}{B_o} e^{-f(\sigma/T)} \right\}^{-1}.$$

To maximize $L$, we must choose $\Delta/\Delta_o$ and $B/B_o$ to maximize the term in braces subject to the constraint that these ratios cannot exceed unity. We define $-10 \log$ (term in braces) as the "excess loss."

By equating appropriate partial derivatives of the excess loss to zero, we obtain the following equations for optimizing $\hat{\Delta}$ and $\hat{B}$ ($\Delta$ and $B$ which minimize excess loss).

$$f'\left(\frac{\sigma}{T}\right) \frac{\sigma_m^2}{\sigma T} = 1 \qquad \text{provided } \hat{\Delta}/\Delta_o \leqq 1, \qquad \text{otherwise } \hat{\Delta}/\Delta_o = 1, \tag{5a}$$

$$f'\left(\frac{\sigma}{T}\right) \frac{\sigma_d^2}{\sigma T} = 1 \qquad \text{provided } \hat{B}/B_o < 1, \qquad \text{otherwise } \hat{B}/B_o = 1, \tag{5b}$$

where $f'(z) = d/dx[f(x)]|_{x=z}$ and $\sigma_m$ and $\sigma_d$ are defined in (4). For sufficiently long lengths $L$, where both $\hat{\Delta}/\Delta_o$ and $\hat{B}/B_o$ are less than unity, we obtain [by adding (5a) to 5(b)]

$$\sigma_m = \sigma_d = \frac{xT}{\sqrt{2}}, \tag{6}$$

where $x$ is the solution $f'(x)x/2 = 1$. More specifically, we obtain the following

$$C_1 \frac{\hat{B}L}{B_o} = \frac{xT}{\sqrt{2}}, \qquad C_2 \frac{\hat{\Delta}L}{\Delta_o} = \frac{xT}{\sqrt{2}} \tag{7}$$

and therefore*

$$\text{excess loss} = -10 \log \left[ \frac{x^2 T^2}{2L^2 C_1 C_2} e^{-f(x)} \right]$$

for

$$\frac{L}{T} \geqq \text{maximum of} \left\{ \frac{x}{\sqrt{2}C_1}, \frac{x}{\sqrt{2}C_2} \right\}.$$

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* Throughout this paper we use the parameter $L/T$ (guide length/time slot width) frequently. The larger the fiber length or the smaller the time slot width, the more excess loss must be incurred to control or compensate for pulse spreading.

From the Gaussian power penalty curve of Fig. 1, we obtain the value of $x$ where $f'(x)x = 2$ to be 0.37. At that value of $x$,

$$-10 \log e^{-f(x)} = 3.3 \text{ dB}.$$

As $L/T$ decreases, either $\hat{\Delta}/\Delta_o$ or $\hat{B}/B_o$ will eventually reach unity. When that happens, $\sigma$ will approach $\sigma_d$ or $\sigma_m$, respectively, for shorter lengths $L$. Then, from (5), $\sigma/T$ will approach the solution of $f'(z)z = 1$. Furthermore, the excess loss will approach either

$$\text{excess loss} \rightarrow -10 \log \left[ \frac{zT}{LC_1} e^{-f(z)} \right] \tag{8}$$

if $\hat{\Delta}/\Delta_o$ reaches unity first and $L/T > z/C_1$ or

$$\text{excess loss} \rightarrow -10 \log \left[ \frac{zT}{LC_2} e^{-f(z)} \right] \tag{9}$$

if $\hat{B}/B_o$ reaches unity first and $L/T > z/C_2$.

From the Gaussian curve of Fig. 1, the solution of $f'(z)\, z = 1$ is $z = 0.3$. At that value of $z$, $-10 \log e^{-f(z)} = 1.8$ dB.

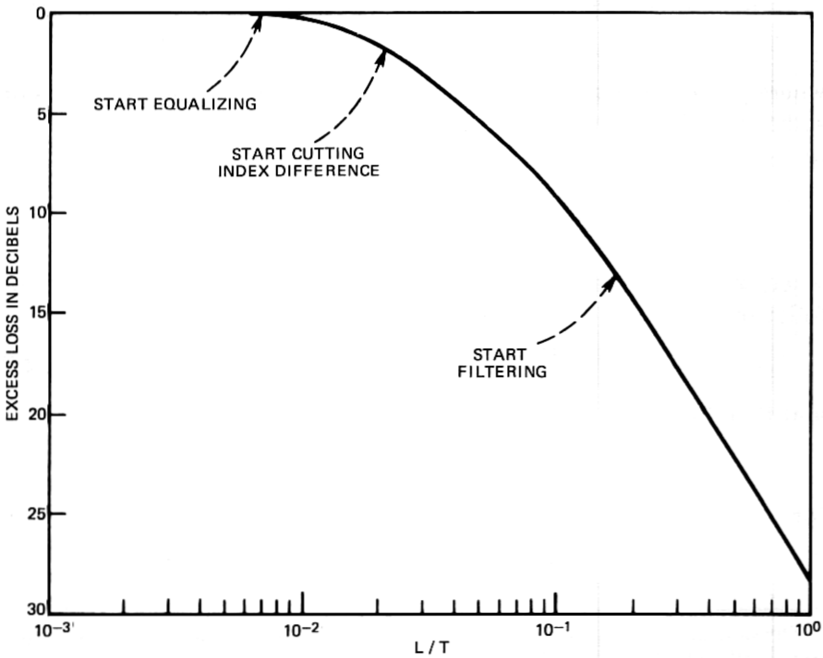


Fig. 2—Excess loss vs $L/T$ for conventional fiber.

For $L/T$ sufficiently small so that both $\hat{\Delta}/\Delta_o$ and $\hat{B}/B_o = 1$, the excess loss will approach zero as $L/T$ decreases.

*Example* 1

In a fused silica fiber, the material dispersion has been measured at 9 ps/km per angstrom of wavelength difference. With a typical GaAs LED* (light-emitting diode), this results in a value of $\sigma_d$ of 1.5 ns/km of pulse spreading at the full bandwidth $B_o$. Thus, $C_1$ can be set at 1.5 ns/km. For a fiber with a maximum index difference $\Delta$ of 0.01, $C_2$ would be given by 14.5 ns/km.

From (7) we obtain

$$\text{excess loss} = -10 \log \frac{(0.37T)^2}{2L^2(1.5)(14.5)} e^{-f(0.37)}$$

$$= -20 \log \frac{T}{L} + 28.3 \text{ dB}$$

for $L/T > 0.174$, where the length $L$ is in kilometers and the time slot width $T$ is in nanoseconds.

From (8) we find that, for $0.0206 < L/T < 0.174$, the excess loss asymptotically approaches

$$\text{excess loss} \rightarrow -10 \log \left[ \frac{zT}{LC_2} e^{-f(z)} \right] = -10 \log \frac{T}{L} + 18.6 \text{ dB}$$

for $0.0206 < L/T < 0.174$.

For $L/T < 0.0206$, the excess loss asymptotically approaches zero. Figure 2 is a plot of excess loss vs $L/T$ in this example.

### 2.2 Incoherent source, ideal graded index fiber, no mode coupling

Following the same procedures as in 2.1, we can replace $\sigma_m$ for a self-focusing fiber by $C_3(\Delta/\Delta_o)^2L$ (where $\Delta_o$ is the maximum allowable value of $\Delta$). We then obtain the following set of equations which determine the values of $\Delta$ and $B$ that minimize the excess loss

$$f'\left(\frac{\sigma}{T}\right)\frac{2\sigma_m^2}{\sigma T} = 1 \quad \text{provided } \hat{\Delta}/\Delta_o \leqq 1, \text{ otherwise } \hat{\Delta}/\Delta_o = 1, \tag{9a}$$

$$f'\left(\frac{\sigma}{T}\right)\frac{\sigma_d^2}{\sigma T} = 1 \quad \text{provided } \hat{B}/B_c \leqq 1, \text{ otherwise } \hat{B}/B_o = 1. \tag{9b}$$

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* Assuming a Gaussian-shaped optical spectrum with bandwidth between the half-power points of about 400 Å.

For sufficiently long lengths $L$, where both $\hat{\Delta}/\Delta_o$ and $\hat{B}/B_o$ are less than unity, we obtain [by adding (9a) to twice (9b)]

$$\sigma_m = C_3 \left(\frac{\hat{\Delta}}{\Delta_o}\right)^2 L = \frac{x'T}{\sqrt{3}} \tag{10}$$

$$\sigma_d = C_1 \frac{\hat{B}}{B_o} L = x'T \sqrt{\frac{2}{3}}$$

$$\text{excess loss} = -10 \log \left[ \frac{(x')^{\frac{3}{2}} T^{\frac{3}{2}} \left(\frac{4}{27}\right)^{\frac{1}{4}}}{L^{\frac{3}{2}} C_1 \sqrt{C_3}} e^{-f(x')} \right],$$

where $x'$ is the solution of $f'(x')x' = 1.5$, and where we must have

$$\frac{L}{T} > \text{maximum of} \left\{ \frac{x'}{C_1}\sqrt{\frac{2}{3}} \quad \text{and} \quad \frac{x'}{C_3}\sqrt{\frac{1}{3}} \right\}.$$

For the Gaussian power penalty curve shown in Fig. 1, we have $x' = 0.34$ and $-10 \log e^{-f(x')} = 2.6$ dB.

As before, as $L/T$ decreases, either $\hat{B}/B_o$ or $\hat{\Delta}/\Delta_o$ will reach unity. Thereafter, for smaller values of $L/T$ we have the following: either

$$\sigma \to \sigma_d; \text{ excess loss} \to -10 \log \frac{zT}{LC_1} e^{-f(z)}, \tag{11}$$

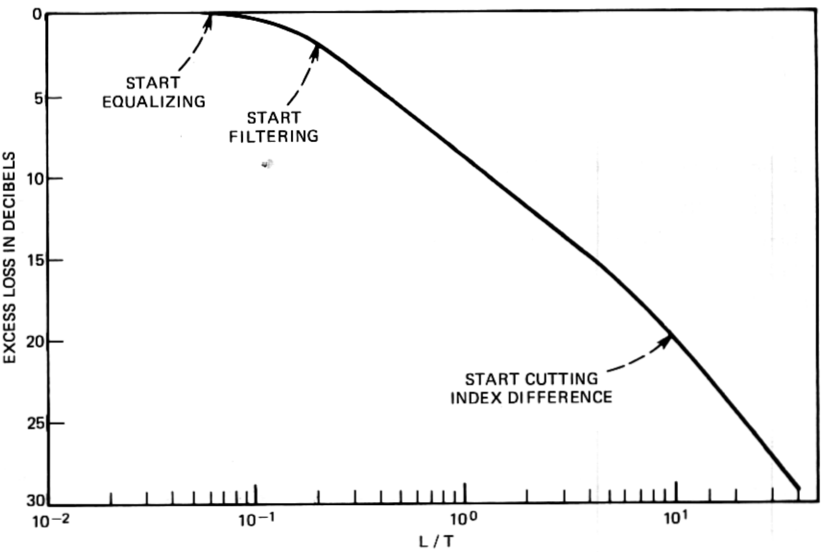


Fig. 3—Excess loss vs $L/T$ for graded index fiber.

where $f'(z)z = 1$, provided $\hat{\Delta}/\Delta_o$ reaches unity first and $L/T > z/C_1$, or

$$\sigma \longrightarrow \sigma_m; \text{ excess loss} \longrightarrow -10 \log \left[ \left( \frac{z'T}{LC_3} \right)^{\frac{1}{2}} e^{-f(z')} \right],$$

where $f'(z')2z' = 1$, provided $\hat{B}/B_o$ reaches unity first and $L/T > z'/C_3$. For the Gaussian power penalty curve of Fig. 1, we obtain $z = 0.3$, $-10 \log e^{-f(z)} = 1.8$ dB, $z' = 0.24$, $-10 \log e^{-f(z')} = 0.98$ dB.

When $L/T$ is sufficiently small so that $\hat{\Delta}/\Delta_o$ and $\hat{B}/B_o$ are both equal to unity, then the excess loss asymptotically approaches zero for smaller $L/T$.

*Example* 2

From Example 1 we have $C_1$ typically 1.5 ns/km. From (2) we have $C_3$ typically 0.019 ns/km for an ideal graded index fiber with a maximum $\Delta = \Delta_o$ of 0.01.

From (10) we obtain

$$\text{excess loss} = -15 \log \left( \frac{T}{L} \right) + 4.85 \text{ dB}$$

for $L/T > 10.3$, where $L$ is in kilometers and $T$ is in nanoseconds.

From (11) we obtain

$$\text{excess loss} \rightarrow -10 \log \left( \frac{T}{L} \right) + 8.78 \text{ dB}$$

for $0.2 < L/T < 10.3$.

For $L/T < 0.2$, the excess loss asymptotically approaches zero as $L/T$ approaches zero.

Figure 3 shows a plot of excess loss vs $L/T$ for this example.

### 2.3 Incoherent source, conventional fiber with adjustable mode coupling

Now consider a conventional fiber with adjustable mode coupling. Using the same notation as in 2.1, we have the following condition from (2), (3), and (4)*

$$P_s \frac{\Delta}{\Delta_o} \frac{B}{B_o} e^{-\alpha L} e^{-\alpha_o L/Lc} \geqq P_r e^{f(\sigma/T)}, \tag{12}$$

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* As mentioned in Section I, the parameter $\alpha_o$ depends upon the shape of the mechanical coupling spectrum and possibly upon the index difference $\Delta$. Since this dependence upon $\Delta$ is not known analytically, we assume $\alpha_o$ = constant independent of $\Delta$. One could also assume $\alpha_o \propto \Delta^N$ for some $N$ (probably negative) and still obtain simple results similar to those that follow using analogous techniques.

where

$$\sigma = \left\{\left(C_1 \frac{B}{B_o} L\right)^2 + \left(C_2 \frac{\Delta}{\Delta_o} \sqrt{LL_C}\right)^2\right\}^{\frac{1}{2}} = \{\sigma_d^2 + \sigma_m^2\}^{\frac{1}{2}},$$

provided $L > L_C$.

By setting appropriate partial derivatives to zero, we can minimize the excess loss given by

$$\text{excess loss} = -10 \log \left\{\frac{\Delta}{\Delta_o} \frac{B}{B_o} e^{-\alpha_o L/L_C} e^{-f(\sigma/T)}\right\}. \tag{13}$$

The optimizing values of $\hat{\Delta}$, $\hat{B}$, and $\hat{L}_C$ satisfy the following equations (we are assuming $\alpha_o$ fixed by the shape of mechanical coupling spectrum).

$$f'\left(\frac{\sigma}{T}\right)\frac{\sigma_m^2}{\sigma T} = 1, \qquad \text{provided } \hat{\Delta}/\Delta_o \leqq 1, \quad \text{otherwise } \hat{\Delta}/\Delta_o = 1, \tag{14a}$$

$$f'\left(\frac{\sigma}{T}\right)\frac{\sigma_d^2}{\sigma T} = 1, \qquad \text{provided } \hat{B}/B_o \leqq 1, \quad \text{otherwise } \hat{B}/B_o = 1, \tag{14b}$$

$$\frac{\alpha_o L}{\hat{L}_C} = f'\left(\frac{\sigma}{T}\right)\frac{\sigma_m^2}{2\sigma T}, \qquad \text{provided } L > \hat{L}_C, \tag{14c}$$

$$\frac{\alpha_o L}{\hat{L}_C} = 0.5, \qquad \text{provided } \hat{\Delta}/\Delta_o \leqq 1 \text{ and } L > \hat{L}_C. \tag{14d}$$

For sufficiently long fibers and if $\alpha_o \leqq 0.5$, we will have $L > \hat{L}_C$, $\hat{\Delta}/\Delta_o < 1$, $\hat{B}/B_o < 1$, and therefore the following will hold:*

$$\text{excess loss} = -10 \log \left[\frac{x^2 T^2 e^{-0.5}}{2\sqrt{2}L^2 C_1 C_2 \sqrt{\alpha_o}} e^{-f(x)}\right], \tag{15}$$

where $x$ is the solution of $f'(x)x/2 = 1$

$$\frac{\alpha_o L}{\hat{L}_C} = 0.5, \qquad \frac{\hat{\Delta}}{\Delta_o} = \frac{xT}{2C_2 L\sqrt{\alpha_o}}, \qquad \frac{\hat{B}}{B_o} = \frac{xT}{\sqrt{2}LC_1},$$

provided

$$\frac{L}{T} \geqq \frac{x}{2C_2\sqrt{\alpha_o}}; \qquad \frac{L}{T} \geqq \frac{x}{\sqrt{2}C_1}; \qquad \alpha_o \leqq 0.5.$$

It is convenient to consider $L/T$ and $L/\hat{L}_C$ as separate parameters.

* It is interesting to note that, with optimal mode coupling, in the region where $\hat{\Delta} < \Delta_o$, the optimal value of $\Delta$ is increased by the factor $\sqrt{0.5/\alpha_o}$ relative to the no-mode coupling case [see formulas for $\hat{\Delta}/\Delta_o$ in (7) and (15) and also (8) and (16)]. Further in this region ($\hat{\Delta} < \Delta_o$), the excess radiation loss from mode coupling ($\alpha_o L/L_c$) is always 0.5 neper.

As $L/T$ decreases, either $\hat{\Delta}/\Delta_o$ or $\hat{B}/B_o$ will reach unity.

If $\hat{B}/B_o$ reaches unity first, then the excess loss will asymptotically approach the following for smaller values of $L/T$.

$$\text{excess loss} \rightarrow -10 \log \left\{ \frac{zT}{C_2 L \sqrt{2\alpha_o}} e^{-f(z)} e^{-0.5} \right\} \tag{16}$$

$$\frac{\alpha_o L}{L_C} = 0.5, \qquad \frac{\hat{\Delta}}{\Delta_o} = \frac{zT}{C_2 L \sqrt{2\alpha_o}},$$

where $f'(z)z = 1$, provided $\hat{B}/B_o$ reaches unity first and

$$L/T > z/(C_2 \sqrt{2\alpha_0}).$$

If $\hat{\Delta}/\Delta_o$ reaches unity first, then the excess loss will asymptotically approach the following for smaller values of $L/T$

$$\text{excess loss} \rightarrow -10 \log \left\{ \frac{zT}{LC_1} e^{-f(z)} \right\} \tag{17}$$

$$\frac{\hat{B}}{B_o} = \frac{zT}{LC_1},$$

provided $\hat{\Delta}/\Delta_o$ reaches unity first and $L/T \geqq z/C_1$.

For values $L/T$ below that at which $\hat{\Delta}/\Delta_o$ and $\hat{B}/B_o$ both equal unity, the excess loss asymptotically approaches zero.

*Example* 3

Using the same parameter values as in Example 1 and assuming* $\alpha_o = 0.1$, we obtain the following

$$\text{excess loss} = -20 \log \frac{T}{L} + 27 \text{ dB}$$

for $L/T > 0.174$ km/ns,

$$\text{excess loss} \Rightarrow -10 \log \frac{T}{L} + 17.3 \text{ dB}$$

for $0.0462 < L/T < 0.174$, and

$$\text{excess loss} \Rightarrow 0 \text{ for } \frac{L}{T} < 0.0462.$$

Figure 4 shows a plot of excess loss vs $L/T$ for this example.

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* At this point, the achievable value of $\alpha_o$ in practical fibers is a subject of speculation. We choose $\alpha_o = 0.1$ arbitrarily.

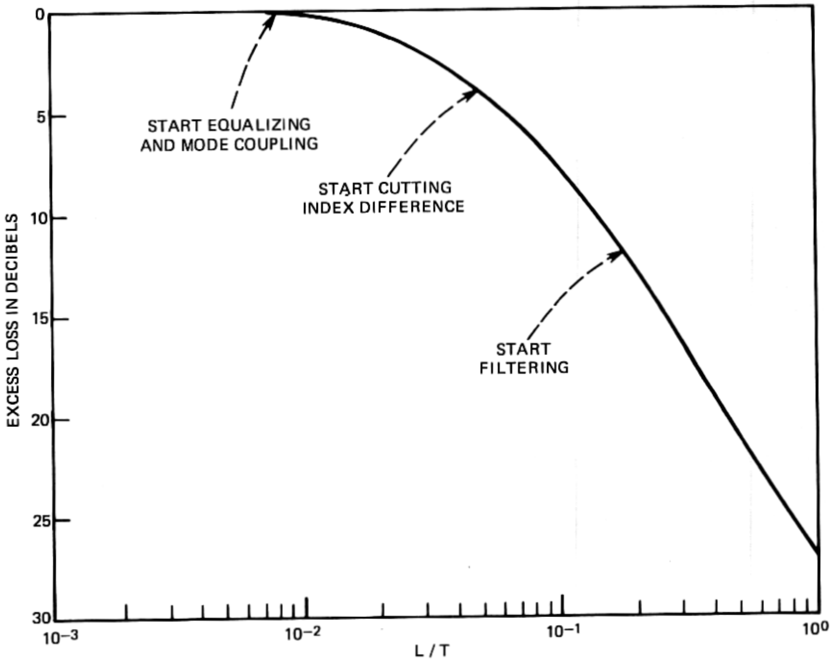


Fig. 4—Excess loss vs $L/T$ for conventional fiber with coupling ($\alpha_o = 0.1$).

### *2.4 Laser source, conventional fiber with adjustable mode coupling*

Here we assume that, to avoid excessive loss at bends, the index difference in the fiber, $\Delta$, is fixed at some minimum allowable value $\Delta_{\min}$. To control pulse spreading we can trade off mode-coupling radiation loss against equalization penalty. The condition we must satisfy is

$$P_s e^{-\alpha L} e^{-\alpha_0 L/L_C} \geqq P_r e^{f(\sigma/T)}.$$

We wish to choose $\hat{L}_C$ to minimize the excess loss given by

$$\text{excess loss} = -10 \log \{e^{-\alpha_0 L/L_C} e^{-f(\sigma/T)}\}, \tag{18}$$

where*

$$\sigma = C_4 \sqrt{L L_C}, \qquad C_4 = 0.289 \Delta_{\min} n/c. \tag{19}$$

We obtain the optimizing equation:

$$\alpha_o L/\hat{L}_C = \frac{1}{2} f'\left(\frac{\sigma}{T}\right)\frac{\sigma}{T}. \tag{20}$$

* Material dispersion is assumed negligible for a coherent source. That is, we assume a single mode and a short-term bandwidth less than 1 Å.

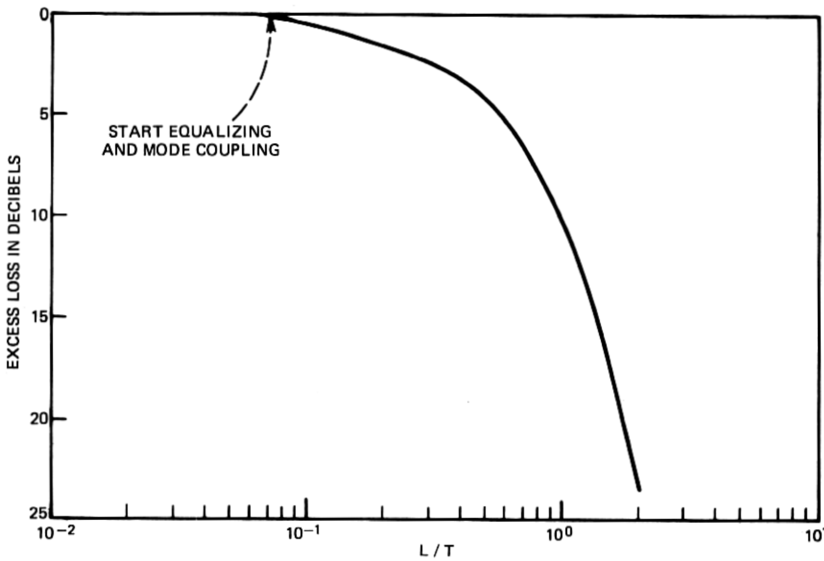


Fig. 5—Excess loss vs $L/T$—laser source, conventional fiber $\Delta_{min} = 0.001$, $\alpha_o = 0.1$.

To solve (20) we can pick a value of $\sigma/T$ and solve for $f'(\sigma/T)$ and $f(\sigma/T)$ graphically from Fig. 1. We then use those results in (20) to solve for $L/\hat{L}_C$. Then we substitute into (19) to find $L/T$ and into (18) to find the total excess loss.

*Example* 4

Using $\Delta_{min} = 0.001$ and $\alpha_o = 0.1$, Fig. 5 shows a plot of excess loss vs $L/T$ for this example.

## III. APPLICATIONS

If the optical power required at the receiver when the received pulses are very narrow is $P_r$ and the transmitted power (at maximum bandwidth and index difference) is $P_s$ and if the fiber loss in the absence of mode coupling loss is $\alpha L$, then we must have

$$10 \log (P_s e^{-\alpha L}) - \text{excess loss } (L) = 10 \log (P_r)$$

or, equivalently,*

$$10 \log \frac{P_s}{P_r} e^{-\alpha L} = \text{“excess gain } (L)\text{”} \geqq \text{excess loss}(L).$$

* We define excess gain as number of decibels by which $P_s e^{-\alpha L}$ exceeds $P_r$. In effect, it is equal to the "allowable excess loss."

At a given bit rate, $1/T$, and given $\alpha$, $P_s$, and $P_r$, we can plot excess loss ($L$) and "excess gain ($L$)" simultaneously. The intersection of the two curves gives the maximum allowable distance $L$ between the transmitter and the receiver.

*Example* 5

Assume that, at a bit rate of 25 Mb/s ($T$ = 40 ns), the required received power $P_r$ is approximately $-58$ dBm. Assume that a conventional fiber with mode coupling ($\alpha_o = 0.1$) and loss $\alpha = 5$ dB/km is used. Assume that an incoherent source is being used with $P_s = -13$ dBm for $\Delta_{max} = 0.01$. Assume that $C_1$ and $C_2$ are 1.5 and 14.5 ns/km so that Fig. 4 applies. Figure 6 shows a plot of excess loss and excess gain vs $L$. It is apparent that the maximum length $L$ between the transmitter and the receiver is 6.8 km. At that distance, the excess loss = 11.5 dB. Further, we have $\hat{\Delta} \cong 0.0024$, $\hat{B}/B_o = 1$ and $\hat{L}_C = 1.36$ km.*

## IV. COMMENTS AND CONCLUSIONS

The purpose of this paper has been to show how we can combine analytical results on fibers and repeaters to determine maximum repeater spacings by optimization of available parameters. Since the fiber art is still young, many assumptions above are subject to question. We can summarize a few possible criticisms here.

It is not known whether the assumption of the Gaussian pulse shape leads to overly conservative estimates of the equalization penalty. With time and experiments, the Gaussian pulse shape approximation will probably be improved upon.

It is not known yet how much control the designer will have over the mode coupling and the index difference. Future analyses will have to take into account the practical constraints on these parameters.

It is not known yet whether optical filters of the type assumed above can be built. Further, the above analysis neglects in-band insertion loss.

It is hoped that, although the above analysis is somewhat simplistic, it can serve as a guide to the fiber system designer by pointing out the concepts and trade-offs involved.

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* It is interesting to note that, if mode coupling were not allowed, the excess loss curve would be about the same (calculated from Fig. 2) and therefore the maximum length $L$ would be the same. However, at the maximum length $\hat{\Delta}$ would be 0.0011.

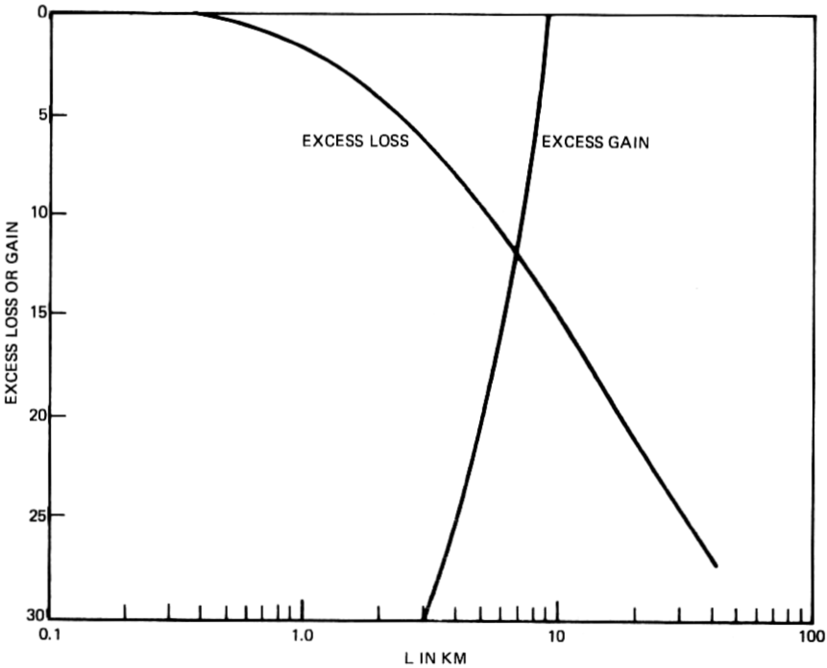


Fig. 6—Excess loss and gain vs $L$.

## APPENDIX

We wish to show heuristically that the total rms width of the fiber impulse response is the rms sum of the contribution from dispersion and the contribution resulting from mode delay spread.

Suppose that, if the fiber is excited by a narrow-band source at wavelength $\lambda$, the resultant output response is $h_\lambda(t)$. Let the mean arrival time and rms width of $h_\lambda(t)$ be defined as

$$\tau_\lambda = \frac{1}{A_\lambda}\int t h_\lambda(t)dt \tag{21}$$

$$\sigma_\lambda = \left\{\frac{1}{A_\lambda}\int t^2 h_\lambda(t)dt - \tau_\lambda^2\right\}^{\frac{1}{2}},$$

where

$$A_\lambda = \int h_\lambda(t)dt = \text{area of } h_\lambda(t).$$

Now suppose the fiber is excited by a narrow pulse from a broadband source having its output distributed in wavelength according to the spectrum $S(\lambda)$. Intuitively,* we can write the fiber output response as follows:

$$h(t) = \int S(\lambda)h_\lambda(t)d\lambda = \int [S(\lambda)A_\lambda]\left(\frac{h_\lambda(t)}{A_\lambda}\right)d\lambda. \tag{22}$$

Define $\tilde{S}(\lambda)$ as $S(\lambda)A_\lambda$ and let

$$\tilde{S} = \int S(\lambda)A_\lambda d\lambda = \int h(t)dt.$$

Let $\hat{S}(\lambda) = [S(\lambda)A_\lambda/\tilde{S}]$. It follows from (22) that the rms width of $h(t)$ is given by

$$\begin{aligned}\sigma &= \left\{\frac{1}{\tilde{S}}\int t^2h(t) - \left(\frac{1}{\tilde{S}}\int th(t)\right)^2\right\}^{\frac{1}{2}} \\ &= \left\{\left[\int \sigma_\lambda^2\hat{S}(\lambda)d\lambda\right]\right. \\ &\qquad \left. + \left[\int \tau^2(\lambda)\hat{S}(\lambda)d\lambda - \left(\int \tau(\lambda)\hat{S}(\lambda)d\lambda\right)^2\right]\right\}^{\frac{1}{2}}.\end{aligned} \tag{23}$$

The first term in square brackets in (23) is a weighted average of the mean square width of the narrow-band pulse at different wavelengths. The second term is the mean square deviation of the narrow-band-mean-arrival time, i.e., the dispersion $\sigma_d^2$. If we next assume that $\sigma_\lambda \approx$ constant $= \sigma_m$ (i.e., that the rms width of the narrow-band impulse response is not dependent upon wavelength within the band of interest), then we obtain

$$\sigma = \{\sigma_m^2 + \sigma_d^2\}^{\frac{1}{2}},$$

which is the desired result.

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* But not rigorously. This is where the argument becomes heuristic.