THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 53, No. 7, September 1974
Printed in U.S.A.

Transverse Coupling in Fiber Optics
Part III: Bending Losses

By J. A. ARNAUD

(Manuscript received February 20, 1974)

A general method is proposed to evaluate the radiation loss of bent open waveguides. This method consists in evaluating the coupling between the waveguide mode and the whispering-gallery modes that can be associated with the surrounding medium. The expression obtained for a reactive surface coincides with a previous result by Miller and Talanov.[1] *We investigate in detail the radiation loss of the fundamental* (HE_{11}) *mode of a dielectric rod coupled to a slab. This arrangement, described in Part II of this article series,*[2] *provides a useful mode-selection technique. The radiation loss is given by a simple closed-form expression. We find that for a wavelength of 1 μm and a rod radius of 5 μm in physical contact with the slab, the bending loss is less than 1 dB/km if the radius of curvature, in the plane of the slab, exceeds 22 mm.*

I. INTRODUCTION

Open waveguides support modes whose phase velocity is smaller than the velocity of plane waves in the surrounding medium. Thus, no radiation takes place under normal conditions. If the fiber is bent, however, the phase velocity increases in proportion to the distance from the curvature center. At some radius, it exceeds the velocity of plane waves in the medium and a radiation loss is suffered. This effect is of great practical importance in fiber communication because it sets a limit on how sharp bends can be made without resulting in untolerable loss. For most single-mode optical-glass fibers, a radius of curvature of the order of 1 cm can be tolerated. For gas lenses and weakly guiding millimeter-wave systems, the minimum radius is sometimes as large as 100 m. The relative insensitivity of glass fibers to bends results from the rather large change in refractive index in the cross section that they provide. It constitutes their main advantage compared to other guiding systems.

Different methods have been used to evaluate the radiation losses of curved waveguides. For simple geometries, it is possible to solve the boundary value problem using a cylindrical coordinate system, the loss being given by the imaginary part of the propagation constant. This method was used by Miller and Talanov[1] for a reactive surface, and by Marcatili[3] for dielectric slabs with various imbedding materials and for waveguides with rectangular cross section. Another method consists in evaluating the power radiated at the radius where the phase velocity of the guided mode becomes equal to the velocity in the surrounding medium. This approach was followed by Shevchenko[4] and Marcuse.[5] Ray pictures have also been used to describe the propagation in curved dielectric fibers.[6]

In most practical situations, the fiber curvature is not a constant but varies along the fiber axis (z). This results in an additional radiation loss suffered at the transition between straight and curved sections of the fiber. This transition effect has been discussed in detail by Shevchenko[7] for the case of a reactive surface. Because the radiation from the bend itself has the form $\exp(-\rho/\rho_o)$, where ρ denotes the radius of curvature and ρ_o a constant, and the radiations at the junctions have the form $1/\rho^2$, the latter becomes significant for large ρ, that is, for very low radiation losses. It should be noted, however, that the mechanical strength of a fiber usually prevents an abrupt curve. The transition radiation, therefore, is usually unimportant for single-mode communication fibers (we assume that the fiber is otherwise perfect). It is more important that, when a highly multimoded fiber is bent in some random manner, the ray slopes associated with the guided waves tend to increase in proportion to the square root of the length of the fiber. Eventually, the rays cease to be totally reflected. This is easily understood if we observe that randomly bent fibers are analogous to mechanical oscillators (e.g., harmonic oscillators for the case of square-law fibers) driven by random forces $f(t)$ proportional to the curvature of the fiber $C(z)$.[8–10] The equivalent mechanical oscillator gains energy as time goes on; that is, the amplitude and momentum increase until the limit is reached. Note that, even in nominally single-mode fibers, higher-order modes can be excited and can propagate over a certain length (see Part II of this article for numerical values[2]) and cause significant pulse spreading. This problem will not be discussed further here. In what follows, we assume that the fiber curvature is a constant. This is the case if the fiber is wound on a cylindrical drum. Our calculation, therefore, constitutes a first step in the evaluation of the losses of randomly bent fibers.

To evaluate the radiation loss of curved fibers, we shall use an approach akin to the one used in Part II of this article series.[2] For curved geometries, the guided mode is coupled to the whispering-gallery type of radiation modes rather than to sinusoidal standing waves. We first assume that the surrounding medium, which we will call a substrate, is bounded at some radius ρ_b and has finite dissipation losses. The loss suffered by the waveguide mode as a result of its coupling to the substrate is evaluated. Radius ρ_b is subsequently allowed to reach infinity and the dissipation loss to vanish.

These whispering-gallery modes are characterized by a circular caustic with radius ρ_s. The behavior of the field is oscillatory outside the caustic and exponential inside the caustic. At the caustic, the phase velocity is just equal to the phase velocity of plane waves. It is not difficult, therefore, to find for what values of ρ_s synchronism with the waveguide mode is achieved. Although ρ_s exceeds only slightly the rod curvature radius ρ, the distance $y_o \equiv \rho_s - \rho$ is the most critical parameter that influences the loss. Once the synchronism conditions have been obtained, we evaluate the coupling coefficients and the mode-number densities using the method described in Part II. We shall use for the waveguide field an expression applicable to scalar fields (described in Part I of this series[2]), which is slightly simpler than the expression applicable to the Maxwell field.

Formulas applicable to arbitrary open waveguides are obtained in Section II. We investigate in Sections III and IV the case of round fibers whose dimensions are large compared with the wavelength, both in free space and coupled to slabs providing mode selection. The latter configuration is closely related to the single-material fiber proposed by Kaiser, Marcatili, and Miller,[11] and to slab-coupled guides.[12] Simple closed-form formulas are obtained for the bending losses in all cases.

II. GENERAL FORMULA

Let us consider an opened waveguide with propagation constant h and radius of curvature ρ as shown in Fig. 1. The general expression for the radiation loss given in Part II is

$$\mathcal{L} = \pi N C^2, \tag{1}$$

where N denotes the mode number density in the substrate (or surrounding medium), that is, the number of substrate modes whose

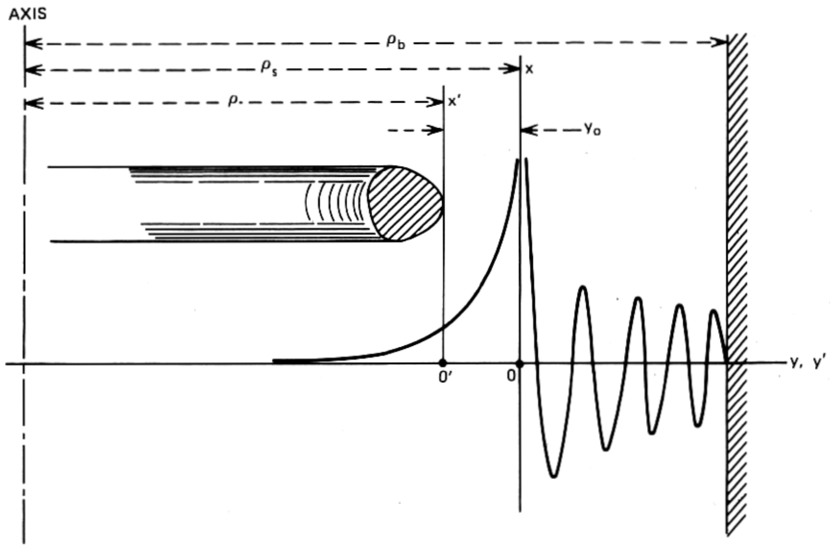

Fig. 1—Curved fiber. The radiation loss is obtained by evaluating the coupling between the straight fiber mode and the whispering-gallery mode shown with the caustic (or turning point) at ρ_s.

propagation constant lies between h and $h + dh$ at the waveguide radius ρ, divided by dh. The term C^2 is the coupling coefficient between the waveguide mode (fields E, H) and a synchronous substrate mode (fields E_s, H_s). For the Maxwell field (see Part I) we have

$$C = \frac{1}{2}\int_{-\infty}^{+\infty} (E_x H_{sz} + E_z H_{sx} - E_{sx} H_z - E_{sz} H_x) dx, \tag{2a}$$

if the powers are normalized to unity; that is, if

$$\int \mathbf{E} \times \mathbf{H} \cdot \mathbf{d}S = \int \mathbf{E}_s \times \mathbf{H}_s \cdot \mathbf{d}S = 1. \tag{2b}$$

The integral in eq. (2a) can be evaluated, for instance, along the x' axis shown in Fig. 1.

When the transverse variations of permittivity are small, the scalar parabolic (Fock) approximation is usually applicable. In that case (see Part I),

$$C = \frac{1}{2}\int_{-\infty}^{+\infty} (E\partial E_s/\partial y - E_s \partial E/\partial y) dx, \tag{3a}$$

with

$$\int kE^2 dS = \int k_s E_s^2 dS = 1. \tag{3b}$$

The terms k and k_s denote propagation constants in the media.* In eq. (1) the product NC^2 is to be evaluated in the limit where the boundaries imposed on the substrate extend to infinity. In that limit $N \to \infty$, and $C^2 \to 0$, but NC^2 remains finite.

The waveguide mode, with propagation constant h, is synchronous to a whispering-gallery mode with propagation constant h_s in the y,z plane, whose caustic radius ρ_s is given by

$$h\rho = h_s\rho_s. \tag{4a}$$

When the waveguide field is uniform along the x axis, h_s is simply equal to k, the bulk propagation constant, and, in free space, $h_s = k = \omega/c$.

More generally, if k_x denotes the x component of the wave vector, then

$$h_s = (k^2 - k_x^2)^{\frac{1}{2}}. \tag{5}$$

The distance y_o between the waveguide and the caustic is from eq. (4a):

$$y_o \approx \rho(1 - h_s/h). \tag{4b}$$

The field of whispering-gallery modes is easily obtained from ray-optics considerations, as illustrated in Fig. 2, in the paraxial JWKB approximation. For a detailed discussion the reader should see Ref. 13. We obtain

$$E_s(y) = \begin{cases} y^{-\frac{1}{4}} \sin\left[(2^{\frac{3}{2}}/3)h_s\rho_s^{-\frac{1}{2}}y^{\frac{3}{2}} + \pi/4\right], & y > 0 \\ \frac{1}{2}(-y)^{-\frac{1}{4}} \exp\left[-(2^{\frac{3}{2}}/3)h_s\rho_s^{-\frac{1}{2}}(-y)^{\frac{3}{2}}\right], & y < 0. \end{cases} \tag{6a}$$

These expressions are asymptotic forms of

$$E_s(y) = \pi^{\frac{1}{2}}(2h_s^2/\rho_s)^{1/2}\mathrm{Ai}\left[(2h_s^2/\rho_s)^{\frac{1}{3}}y\right], \tag{6b}$$

where (Ai) denotes an Airy function of the first kind, for large $|y|$. Alternatively, these expressions can be obtained from asymptotic forms of Bessel's functions with large arguments and orders [see, for example, the first eq. (42) in Ref. 3]. With sufficient accuracy we have

$$\int E_s^2(y)dy \approx \frac{1}{2}\int_0^{\rho_b-\rho_s} y^{-\frac{1}{2}}dy = (\rho_b - \rho)^{\frac{1}{2}}, \tag{7}$$

* Within the Fock approximation, the difference between k and k_s can be ignored in evaluating the powers in eq. (3b). However, for improved accuracy, we retain the distinction when n is not close to unity (e.g., $n = 1.41$).

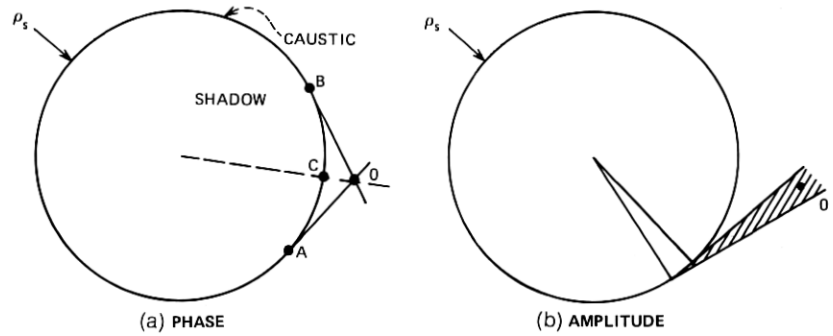

Fig. 2—Illustration of how the field of whispering-gallery modes given in eq. (6) is obtained in the JWKB approximation. (a) The argument of the sine function is h_s times the difference in length between the ray AO and the caustic arc AC. (b) The amplitude term is obtained from a power conservation argument applied to the ray pencil.

because only the oscillatory part of $E_s(y)$ contributes significantly to the total power, $\langle \cos^2 x \rangle \approx \frac{1}{2}$, and $\rho_b - \rho_s \approx \rho_b - \rho$ in the limit $\rho_b \to \infty$.

The reciprocal of the mode-number density N is the change in propagation constant at the waveguide radius ρ corresponding to a change in the argument of the cosine function in eq. (6) equal to π. We find by differentiation of this argument

$$N = (2^{\frac{1}{2}}/\pi)\rho^{\frac{1}{2}}(\rho_b - \rho)^{\frac{1}{2}}. \tag{8}$$

Thus, if we characterize the y dependence of the radiation field by a function

$$S(y) \equiv \pi N E_s^2(y) \Big/ \left[\int k_s E_s^2(y) dy \right], \tag{9a}$$

we obtain, with the help of eqs. (6), (7), and (8),

$$S(y) = (2^{\frac{1}{2}}/4)\rho^{\frac{1}{2}}(-y)^{-\frac{1}{2}}k^{-1} \exp\left[-(2^{\frac{5}{2}}/3)h\rho^{-\frac{1}{2}}(-y)^{\frac{3}{2}}\right], \quad y < 0. \tag{9b}$$

In eq. (9b) we have replaced ρ_s by ρ and h_s by h. Therefore, the value of $S(y)$ at $y = -y_o$, where y_o is given in eq. (4b), is

$$S(y_o) = (2^{\frac{1}{2}}/4)(1 - h_s/h)^{-\frac{1}{2}}k^{-1} \exp\left[-(2^{\frac{5}{2}}/3)h\rho(1 - h_s/h)^{\frac{3}{2}}\right]. \tag{10a}$$

This expression is the asymptotic form of

$$S(y_o) = 2^{\frac{1}{2}}\pi h^{-1}(2h^2\rho^2)^{1/6}\mathrm{Ai}^2\left[(2h^2\rho^2)^{\frac{1}{3}}(1 - h_s/h)\right]. \tag{10b}$$

Because we are limiting ourselves to small delays, $1 - h_s/h \ll 1$, we have

$$p \equiv (h^2 - h_s^2)^{\frac{1}{2}} \approx 2^{\frac{1}{2}}h(1 - h_s/h)^{\frac{1}{2}}. \tag{11}$$

Note, for later use, that at the waveguide radius ρ, $E_s^{-1}(\partial E_s/\partial y)$ is equal to p. Using eq. (11), eq. (10) can be written in the simpler form

$$S(y_o) = (\tfrac{1}{2}p) \exp\left[-\rho/(\tfrac{3}{2}p^{-3}h^2)\right]. \tag{12}$$

Let us now consider a waveguide structure, such as a reactive surface or a slab, uniform along the x axis, and fields that are independent of x. Because the field of the waveguide has an $\exp(-py)$ dependence on y, the general expression for the radiation loss, eqs. (1) and (3), is

$$\mathfrak{L} = p^2 \left[E^2 \Big/ \int_{-\infty}^{+\infty} kE^2 dy\right] S(y_o), \tag{13}$$

$S(y_o)$ being given in eq. (12).

For a reactive surface with normalized susceptance p (see Fig. 3), we have

$$E^2 \Big/ \int_{-\infty}^{+\infty} kE^2 dy = 2p/k. \tag{14}$$

Therefore, the radiation loss (in nepers/unit length) is, substituting eqs. (12) and (14) in eq. (13),

$$\mathfrak{L} = (p^2/k) \exp(-\rho/\rho_o), \tag{15a}$$

where

$$\rho_o \equiv \tfrac{3}{2}k^2/p^3. \tag{15b}$$

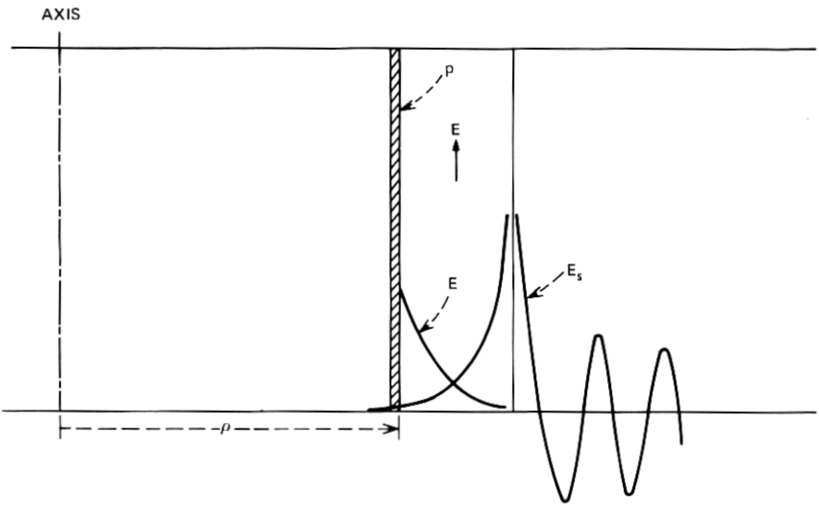

Fig. 3—Radiation loss of a curved reactive surface with normalized susceptance p.

This expression coincides with a previous result by Miller and Talanov.[1*]

For arbitrary waveguides with field $E(x)$, we need consider radiation modes for all values of k_x and not just $k_x = 0$ as before. A simplification comes from the fact that, as k_x increases, the caustic moves away from the waveguide and the coupling becomes negligible for some small value of k_x.

Let us introduce boundaries at $x = \pm L/2$ and assume symmetry for simplicity. The bending loss associated with some k_x is

$$\mathcal{L}(k_x) = S(y_o, k_x)R(k_x), \tag{16}$$

where $R(k_x)$ is essentially the spatial spectrum of the waveguide field

$$R(k_x) = \left[\int_{-\infty}^{+\infty} \tfrac{1}{2}(p_o E - \partial E/\partial y) \cdot \cos(k_x x)dx\right]^2 \Big/ \left(\frac{L}{2}\int knE^2 dxdy\right), \tag{17}$$

where

$$p_o \equiv (h^2 - k^2)^{\frac{1}{2}} \approx 2^{\frac{1}{2}} h(1 - k/h)^{\frac{1}{2}}. \tag{18}$$

The quantity $S(y_o, k_x)$ given in eq. (10) is a function of k_x through the dependence of h_s on k_x, and is given, according to eq. (5), by

$$h_s = (k^2 - k_x^2)^{\frac{1}{2}} \approx k - \tfrac{1}{2}k_x^2/k. \tag{19}$$

To first order in k_x we have

$$(2^{\frac{5}{2}}/3)h\rho(1 - h_s/h)^{\frac{3}{2}} \approx \rho/\rho_o + (\rho p_o/k^2)k_x^2. \tag{20}$$

This approximation supposes $k_x \ll p_o$ or $\rho \gg \rho_o$, where ρ_o is the critical radius given in eq. (15b). These assumptions are acceptable for long fibers, where only very small bending losses can be tolerated. The dependence of h_s on k_x is important only in the exponential term. Summing $\mathcal{L}(k_x)$ in eq. (16), with $S(y_o, k_x)$ given by eqs. (10) and (20), over k_x we obtain

$$\begin{aligned} \mathcal{L} &= (L/2\pi)\int_0^\infty \mathcal{L}(k_x)dk_x \\ &= (1/4\pi p_o)\exp(-\rho/\rho_o) \times \int_0^\infty LR(k_x)\exp(-\rho p_o k^{-2}k_x^2)dk_x. \end{aligned} \tag{21}$$

$\rho_o \equiv \frac{3}{2}k^2/p_o^3.$

---

* The results in Ref. 1 are applicable to anisotropic bent surfaces and arbitrary k_x. For our case, the expression for the loss $\mathcal{L}$ reads [as in Ref. 1, eq. (13.18)]

$$\rho\mathcal{L} = \gamma\alpha^2(1 + \alpha^2)^{-\frac{1}{2}}[\alpha + (1 + \alpha^2)^{\frac{1}{2}}]\uparrow[-2(1 + \alpha^3)^{\frac{1}{2}}\gamma]\exp(2\alpha\gamma),$$

where ↑ means "to the power," $\gamma \equiv k\rho$ and $\alpha \equiv p/k$. This expression goes to eq. (15) when $\alpha \to 0$, as one can (but not too easily) verify.

This is a general expression for the bending loss of open waveguides. Note that some of the assumptions could be relaxed with little additional complication. The integration over k_x, for instance, could be performed without using the approximation in eq. (20), and we could use the Maxwell field instead of the scalar field.

III. BENDING LOSS OF A DIELECTRIC ROD

We consider in this section the fundamental HE_{11} mode of a dielectric rod with refractive index n and radius a, with $a \gg \lambda$ (see Fig. 4). The asymptotic form of the field, given in Part II, is

$$E(r) = J_0(gr), \quad r < a, \tag{21a}$$

with

$$\begin{aligned} J_0(ga) &= 0, \\ ga &\equiv (k^2n^2 - h^2)^{\frac{1}{2}}a = 2.4\cdots \\ h &\approx kn - 2.87/kna^2. \end{aligned} \tag{21b}$$

Therefore,

$$\int E^2 2\pi r dr \approx 2\pi \int_0^a J_0^2(gr) r dr = 0.83a^2. \tag{22}$$

By specifying the continuity of the first derivative of the field, we obtain the field near the boundary (which is small but not strictly zero):

$$E(x, y) \approx (1.25/p_o a) \exp(-p_o x'^2/2a) \exp(-p_o y'). \tag{23}$$

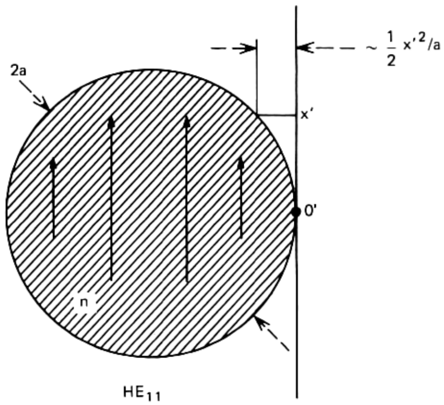

Fig. 4—Field of the fundamental HE_{11} mode of a dielectric rod. The important quantity is the field averaged over the x' axis. Only the close neighborhood of point 0 contributes significantly to the radiation loss.

The parameter $R(k_x)$ defined in eq. (17) is, in the present case,

$$LR(k_x) = (3.75/kna^4)\left[\int_{-\infty}^{+\infty} \exp\,(-p_o x^2/2a)\cos\,(k_x x)dx\right]^2. \quad (24)$$

Using the equality

$$\left[\int_{-\infty}^{+\infty} \exp\,(-\alpha x^2)\cos\,(k_x x)dx\right]^2 = (\pi/\alpha)\exp\,(-k_x^2/2\alpha),$$

we obtain

$$LR(k_x) = (7.5\pi/p_o kna^3)\exp\,(-ak_x^2/p_o). \quad (25)$$

The total loss is obtained by substituting this result in eq. (21) and integrating

$$\mathfrak{L} = (1.6/p_0^2 kna^3)(\rho p_o k^{-2} + a/p_o)^{-\frac{1}{2}}\exp\,(-\rho/\rho_o). \quad (26a)$$

The term ρ_o is the same as in eq. (15b). The exponential term coincides with the exponential term in Ref. 3 and is applicable to fibers with rectangular cross section. For $n = 1.01$, $\lambda = 1\ \mu$m, and $a = 10\ \mu$m, we obtain

$$\mathfrak{L} \approx 18 \times 10^6(\rho + 480)^{-\frac{1}{2}}\exp\,(-\rho/80)\ \text{dB/km}, \quad (26b)$$

where ρ is expressed in micrometers.

The loss in dB/km is plotted in Fig. 5 as a function of ρ for $\lambda = 1$ μm, $n = 1.01$, and $a = 10\ \mu$m. The bending loss is considerably

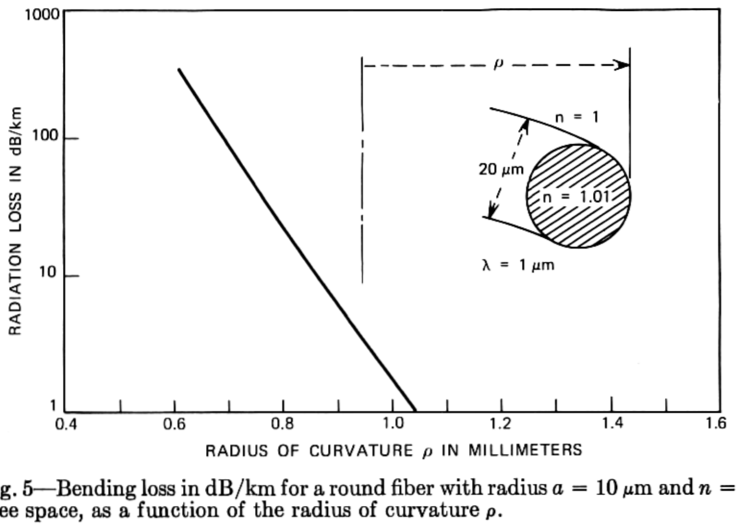

Fig. 5—Bending loss in dB/km for a round fiber with radius $a = 10\ \mu$m and $n = 1.01$ in free space, as a function of the radius of curvature ρ.

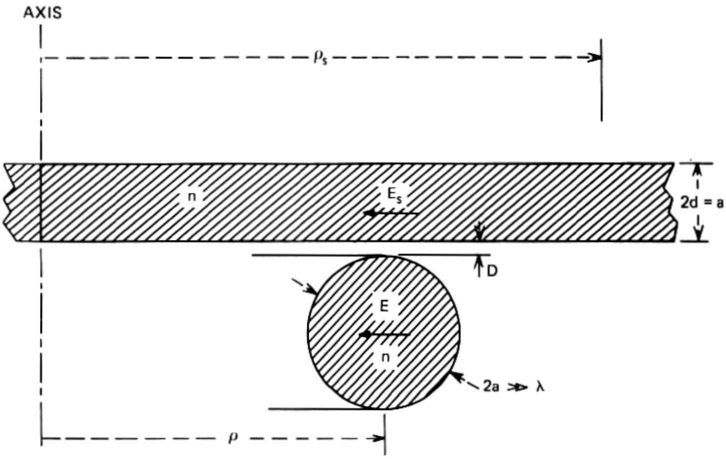

Fig. 6—Slab-loaded rod. For selection of a single mode, the slab thickness must be about half the rod diameter if the refractive indices are equal.

smaller for an oversized dielectric rod than for a reactive surface having the same curvature and the same propagation constant.

IV. RADIATION OF A BENT ROD LOADED BY A SLAB

We now investigate the radiation loss of a dielectric rod coupled to a slab having the same refractive index n shown in Figs. 6 and 7.

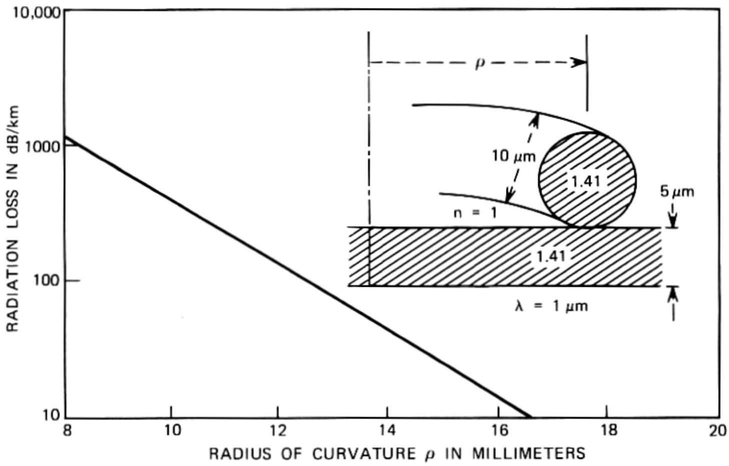

Fig. 7—Bending loss in dB/km as a function of the radius of curvature ρ for a slab-loaded rod with $n = 1.41$, $\lambda = 1\ \mu$m, and $a = 5\ \mu$m.

The mode selection mechanism provided by this arrangement was discussed in Part II. The thickness $2d$ of the slab is chosen equal to half the rod diameter $2a$ so that, in the absence of curvature, only the fundamental (HE_{11}) mode of the rod propagates without radiation loss.

The propagation constant h of the rod is given in eq. (21b). For the fundamental mode (H_1) of the slab, we have $k_x \approx \pi/2d = \pi/a$. Thus, from eq. (5), with k changed to kn, we obtain

$$h_s = kn - 4.9/kna^2. \tag{27}$$

Substituting these results in eq. (4b), we obtain the rod-caustic spacing y_o:

$$y_o = 2\rho/k^2n^2a^2. \tag{28}$$

It can be shown that the higher-order modes of the slab H_2, H_3, $\cdots$, corresponding to $k_x = 2\pi/a$, $3\pi/a$, $\cdots$, respectively, do not couple significantly to the rod and can be ignored.

The normalized field at the surface of the slab (as we have seen in Part II) is, with $d = a/2$,

$$E_s^2 \Big/ \int_{-\infty}^{+\infty} E_s^2(x)dx \approx 2\pi^2/p^2a^3. \tag{29}$$

Substituting the value of y_o given in eq. (28) in eq. (9b) and multiplying by the expression in eq. (29) to take into account the distribution of the radiation field in the x direction, we obtain

$$S(y_o) = (\pi^2/2)/p^2a^2. \tag{30}$$

Here it is legitimate to assume that only the average waveguide field is important. Thus, we can set $k_x = 0$ in eq. (25). Dividing by 2, because the field E_s is a constant along x instead of a cosine function, setting $L = 1$, and taking into account the spacing D between the rod and the slab, we obtain

$$R = (3.75\pi/pkna^3) \exp(-2pD). \tag{31}$$

Finally, we obtain the bending loss

$$\mathcal{L} = SR = 58(p^3kna^5)^{-1} \exp(-5.3\rho/k^2n^2a^3) \exp(-2pD). \tag{32}$$

The term $\exp(-2pD)$ in eq. (32) is the same as in eq. (76) in Part II, and is applicable to the spurious H_{01} mode of the straight fiber. Thus, moving the fiber away from the slab reduces the mode discrimination and the bending loss of the fundamental mode in the same proportion. A trade-off is necessary, therefore, that depends on the application, e.g., integrated optics or long-distance communication. The prefactor in eq. (32) also has the same form as for the discrimination loss of the straight fiber, with a very fast a^{-5} dependence on the fiber radius a. The numerical factor, however, is almost 20 times smaller. This, of course, is a welcome result. The first exponential term in eq. (32) has the form

$$\exp(-\rho/\rho_o), \tag{33}$$

where the critical radius is now

$$\rho_o = k^2n^2a^3/5.3 = 15a^3/\lambda^2, \quad n = 1.41. \tag{34}$$

For example, if $\lambda = 1\ \mu\text{m}$ and $a = 5\ \mu\text{m}$, the critical radius is $\rho_0 = 1.9$ mm. If $n = 1.01$ and $a = 5\ \mu\text{m}$, the critical radius is $\rho_0 = 0.95$ mm.

The complete expression in eq. (32) is explicitly, for $n = 1.41$,

$$\mathfrak{L} = 2.3 \times 10^8 \lambda_{\mu m}^{-1}(a/\lambda)^{-5} \exp(-12.5D/\lambda) \cdot \exp[-\rho/(15a^3/\lambda^2)]\ \text{dB/km}. \tag{35}$$

This expression shows that the radiation loss does not exceed 1 dB/km if $\rho > 24$ mm, when $\lambda = 1\ \mu\text{m}$ and $a = 5\ \mu\text{m}$. If $n = 1.01$ or, almost equivalently, if the space between the fiber and the slab is filled up with a material whose refractive index is 99 percent of the rod and slab index, we have

$$\mathfrak{L} = 12 \times 10^{10} \lambda_{\mu m}^{-1}(a/\lambda)^{-5} \exp(-1.75D/\lambda) \cdot \exp[-\rho/(7.5a^3/\lambda^2)]\ \text{dB/km}. \tag{36}$$

For instance, for $\lambda = 1\ \mu\text{m}$, $a = 5\ \mu\text{m}$, and $D = 0$, a 1-dB/km bending loss is obtained for $\rho = 9$ mm.

The radiation losses of the higher-order modes (e.g., H_{01}) of the rod can be obtained along the same lines. Because h_{spurious} is smaller than h_s, the caustic radius ρ_s is, for spurious modes, smaller than ρ. Thus, the spurious modes see the oscillatory part of the whispering-gallery modes. The radiation losses of these spurious modes, already quite high, are not significantly affected by bending, except near the "sound barrier" $h \approx h_s$. Near this point, the JWKB approximation that we used to describe the radiation field is not applicable, and one needs the description in terms of Airy's functions. For the fundamental mode (HE_{11}) and first spurious mode (H_{01}), only the H_1 mode of the slab need be considered. For high-order modes, however, a sum needs to be performed over the various slab modes. We obtain from eq. (10b)

$$hS = 2^{\frac{1}{3}}\pi \sum_{m=1}^{\infty} m^2 (2h^2\rho^2)^{1/6}$$

$$\mathrm{Ai}^2\{(2h^2\rho^2)^{\frac{1}{3}}[1 - kn/h + \tfrac{1}{2}m^2(\pi/2hd)^2]\}. \quad (37)$$

The first factor m^2 in the sum expresses the fact that the slab field at the boundary increases in proportion to the slab mode number m. The term $\frac{1}{2}m^2(\pi/2hd)^2$ results from the change in the synchronism condition with the slab mode number m. This function is plotted in Fig. 8 as a function of: $1 - (kn/h)$ for $\lambda = 1$ μm, $n = 1.41$, a slab thickness $2d$ of 10 μm, and for a radius of curvature $\rho = 16$ mm. The propagation constants of HE_{11} and H_{01} modes for a rod with a radius of 10 μm are shown on the same figure. We observe that the radiation loss of spurious modes fluctuates by a factor as large as 100 (in dB/km) for small changes of parameters. Of course, if the radius of curvature is not a constant, these fluctuations tend to smooth out. Fig. 9 gives in more detail the region $h \approx kn$ for the same wavelength, slab thickness, and refractive index as in Fig. 8, and for $\rho = 16$ mm and 48 mm. The propagation constant of the rod modes are shown by arrows for a rod radius of 11.5 μm. This radius is to be preferred to the radius of 10 μm originally chosen because it provides a better discrimination ratio, almost 10^4 in dB/km, for $\rho = 48$ mm. The next 16 higher-order modes (each arrow, except the first, corresponds to a group of four modes) have somewhat less radiation loss than the H_{01} because they are

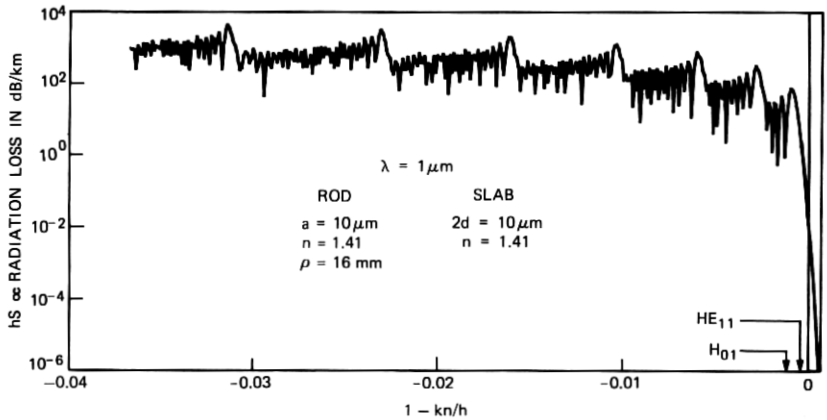

Fig. 8—Radiation loss (proportional to hS) resulting from the coupling of the waveguide to the various slab modes (H_1, H_2 $\cdots$), as a function of h (the waveguide propagation constant), for $2d = 10$ μm, $\lambda = 1$ μm, $n = 1.41$, and $\rho = 16$ mm.

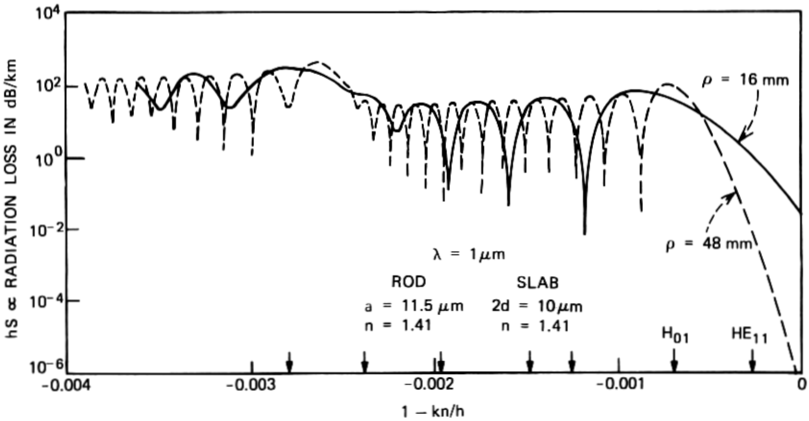

Fig. 9—Detail of Fig. 8 for $h \approx kn$, for $\rho = 16$ mm and 48 mm. The arrows denote the rod modes (HE_{11}, H_{01}, $\cdots$) for a rod radius of 11.5 μm.

beyond the "sound barrier" (we are ignoring the dips that would be smoothed out anyway in a real system). It is desirable, therefore, that a second slab, with smaller thickness, be coupled to the rod to damp strongly the modes just following the H_{01} group, and to improve further the discrimination against the other modes.

The bending loss that we have calculated is applicable to the fiber bent in the plane of the slab. When the fiber is bent in the perpendicular plane and with the same radius, the radiation loss is practically negligible. Slab-loaded fibers should be manufactured, therefore, in the shape of ribbons so that the mechanical rigidity of the fiber favors bending in the plane perpendicular rather than parallel to the slab. Physical considerations also suggest that a slight curvature of the slab, which is unimportant when the fiber is straight, may help to reduce the radiation loss.

V. ACKNOWLEDGMENT

The author expresses his thanks to E. A. J. Marcatili for many stimulating discussions and useful comments.

REFERENCES

1. M. A. Miller and V. I. Talanov, "Electromagnetic Surface Waves Guided by a Boundary with Small Curvature," Zh. T.F., *26*, No. 12 (1956), p. 2755.
2. J. Arnaud, "Transverse Coupling in Fiber Optics—Part I: Coupling Between Trapped Modes," B.S.T.J., *53*, No. 2 (February 1974), pp. 217–224; "Part II: Coupling to Mode Sinks," No. 4 (April 1974), pp. 675–696.
3. E. A. J. Marcatili, "Bends in Optical Dielectric Guides," B.S.T.J., *48*, No. 7 (September 1969), pp. 2103–2132.

4. V. V. Shevchenko, "Surface Wave Radiation Losses in Bent Waveguides," Izv. Vuz, Radiofiz, *14*, No. 5 (1971), p. 768.
5. D. Marcuse, *Light Transmission Optics*, New York: Van Nostrand Reinhold, 1972, p. 398.
6. A. W. Snyder and D. J. Mitchell, "Bending Losses of Multimode Optical Fibers," Elec. Letters, *10*, No. 1 (January 10, 1974).
7. V. V. Shevchenko, *Continuous Transitions in Open Waveguides; Introduction to the Theory*, Boulder, Colorado: The Golem Press, 1971, p. 65.
8. D. W. Berreman, "Growth of Oscillations of a Ray about the Irregularly Wavy Axis of a Lens Light Guide," B.S.T.J., *44*, No. 9 (November 1965), pp. 2117–2132.
9. J. A. Arnaud, "Graded Index Fibers Versus Cladded Fibers: a Mechanical Analogy," February 1972, unpublished work.
10. J. A. Arnaud, "Hamiltonian Theory of Beam Mode Propagation," in E. Wolf, ed., *Progress in Optics, XI*, Amsterdam: North Holland, 1973.
11. P. Kaiser, E. A. J. Marcatili, and S. E. Miller, "A New Optical Fiber," B.S.T.J., *52*, No. 2 (February 1973), pp. 265–269.
12. E. A. J. Marcatili, "Slab-Coupled Waveguides," B.S.T.J., *53*, No. 4 (April 1974), pp. 645–674.
13. J. B. Keller and S. I. Rubinow, "Asymptotic Solution of Eigenvalue Problems," Ann. Phys., *9* (1960), pp. 24–75.