The Bell System Technical Journal
Vol. 53, No. 8, October 1974
Printed in U.S.A.

A Proposed Multiple-Beam Microwave Antenna for Earth Stations and Satellites

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(Manuscript received October 3, 1973)

An offset Cassegrainian antenna with essentially zero aperture blockage is expected to support closely spaced well-isolated beams suitable for earth stations and satellites. Each beam is fed with a separate small-flare-angle corrugated horn and has good area efficiency over a 1.75:1 bandwidth. Each beam also has good cross-polarization properties. The antenna is compact, and the design appears practical for a 4- and 6-GHz earth station, a 20- and 30-GHz earth station, and a 20- and 30-GHz satellite.

I. INTRODUCTION

Satellite communication systems with large capacities can be achieved if the satellites and earth stations are provided with multiple-narrow-beam antennas.[1] The capacity is proportional to the number of satellites, and thus it is important to use as many as practical in the limited orbital space. A moderate number of the resulting closely spaced satellites can be served by a single antenna at each earth-station site if the antenna is patterned after the offset Cassegrainian antenna shown in Fig. 1. This design allows an orderly expansion in communication capacity by the addition of feed horns. Since only one antenna is needed at each site, the design also permits a large saving in earth-station costs. Good multiple-beam performance can be achieved across all up/down pairs of satellite frequency bands, including those well below 10 GHz. At 20 and 30 GHz, a large earth-station antenna with acceptable thermal and wind distortion is hard to achieve. However, with the design outlined here, these problems can be largely overcome because the main reflector and subreflector can be fixed in position, thus allowing a stiffer structure. The steering of each beam is achieved by moving one of the feed horns, resulting in a steerable angle sufficient for tracking near-synchronous satellites.

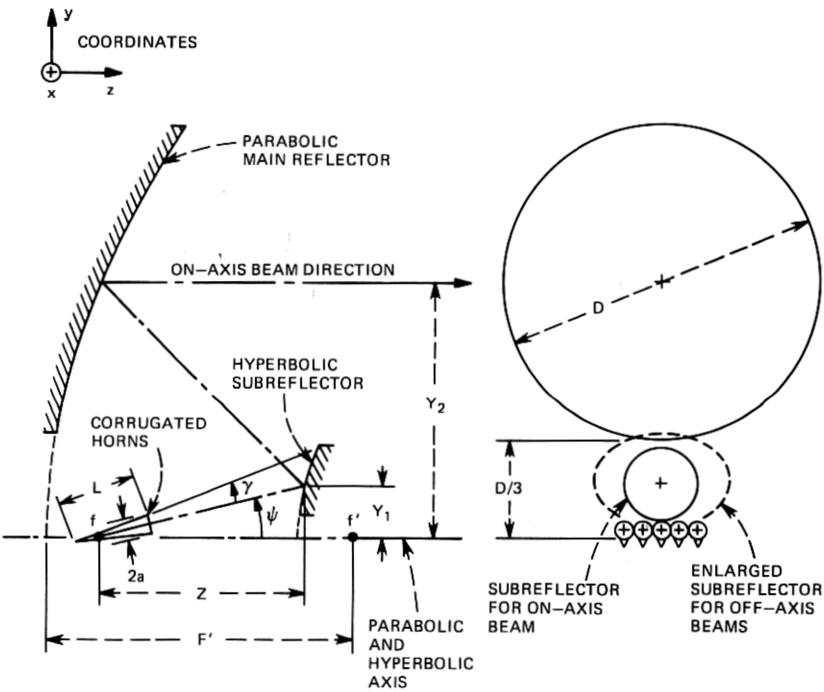

Fig. 1—Geometry of antenna and feed system. The feed horns are scaled for a 3m 20- and 30-GHz satellite antenna. For a 30m 4- and 6-GHz earth-station antenna, L and $2a$ are half as large as shown.

The offset Cassegrain is also appropriate for use aboard a satellite because all beams, including those moderately far off-axis, have high area efficiencies and low side-lobe levels. However, good results on a satellite are restricted to bands well above 10 GHz because the antenna size is limited by the launch vehicle.

It has been previously shown that a multiple-beam antenna can be achieved in a variety of ways,[2–5] where each approach has emphasized one feature desired in a practical antenna. By combining several of these with a corrugated feed horn[6] and an enlarged subreflector, it is possible to achieve a compact antenna with exceptionally good multiple-beam characteristics. In particular, in the offset Cassegrainian antenna shown in Fig. 1:

(*i*) An offset design essentially eliminates beam blockage, thus allowing a significant reduction in side-lobe level.[7] This, in turn, results in higher isolation between beams and a lower antenna noise temperature.

(*ii*) The Cassegrainian feed system is compact and has a large focal-length-to-diameter (F/D) ratio.[8] The large F/D ratio reduces aberrations to an acceptable level, even when a beam is moderately far off-axis.

(*iii*) A corrugated feed horn is essentially a Gaussian-beam launcher[9] and, as such, it can be used to achieve beams with low side-lobe levels. The corresponding feed-horn aperture[6] is small enough to allow the beams to be closely spaced.

(*iv*) An enlarged subreflector, as indicated by the dashed line in Fig. 1, allows the main reflector to be properly illuminated, even when a beam is moderately far off-axis.

These features can be achieved over a wide range of antenna parameters. Using the results developed here, the sample calculations summarized in Table I show that (*i*) the off-axis beam angles are practical, (*ii*) the coma aberration is small, (*iii*) the feed-horn dimensions are reasonable, and (*iv*) the isolations between beams are large.

II. OFF-AXIS DESIGN CRITERIA

Consider a parabolic reflector that is circularly symmetric and illuminated with a feed at its prime focus. If the aperture is large in wavelengths and the prime focal-length-to-diameter, F'/D, ratio is 2 or more, it is well-known that a beam can be scanned over tens of beamwidths by lateral displacement of the feed.[2] A Cassegrainian antenna normally has a secondary focal length F larger than F', and thus a larger F/D ratio.[8] Consequently, a scanned beam can also be obtained by displacing a feed at the secondary focus.[5] For the small off-axis angle reported in Ref. 5 (4 beamwidths $\equiv 0.9°$), the on-axis and off-axis beam characteristics are nearly identical, and the residual differences can be readily explained in terms of an equivalent parabola.[5,8] The equivalent parabola, in turn, has characteristics identical to those of a prime-focus parabola. Consequently, the prime-focus theory[2] can be used to predict the off-axis equivalent-parabola results, and thus the Cassegrainian results. This chain of reasoning assumes that the equivalent-parabola concept is valid for the antenna parameters (F'/D and F/D ratios and off-axis angles) considered here. In support of this assumption, it is of interest to note that the chief off-axis beam parameter of a prime-focus parabola, namely,[2]

$$X' = \frac{N\left(\frac{D}{F'}\right)^2}{1 + 0.02\left(\frac{D}{F'}\right)^2}, \tag{1}$$

where N is the off-axis angle in half-power beamwidths, has a value in Ref. 5 of about 30. Thus, the equivalent-parabola concept is valid for X' values at least through 30. Furthermore, the known results indicate that the region of validity can be extrapolated to X' values well beyond 30. In particular, Ref. 5 shows that the coma lobe, which is the first side lobe aimed toward the on-axis direction, increases very slowly as a function of off-axis beam angle. From Ref. 2, it is also known that an increase in coma-lobe level is a sensitive leading indicator of serious aberration problems, and that X' increases rapidly with coma-lobe level. It follows that X' in Ref. 5 can be much larger than 30 before a larger increase in coma-lobe level signals the onset of serious aberrations. The upper limit of X' should and can be calculated but, in the meantime, some of the results in Table I include an engineering judgment that the equivalent-parabola concept is valid for X' values through 45. Even if the upper limit turns out to be somewhat less, the offset Cassegrain can still support a respectable number of multiple beams, i.e., for $X' = 30$, the number of 1°-spaced beams from the earth-station antenna of Table I is 7 rather than 11.

An important parameter of an off-axis beam is the third-order phase error across the beam at the antenna aperture. This error, $\Delta\phi$, increases the level of the coma lobe.[2] For a symmetrical parabola illuminated with a feed displaced laterally from the prime focus, the peak value of $\Delta\phi$ at the edge of the aperture can be calculated from eq. (12) of Ref. 2. Similarly, when an offset parabola (as in Fig. 1) is illuminated with a feed displaced laterally from the prime focus (in the x direction in Fig. 1), the maximum third-order phase error, $\Delta\phi'$, which occurs at the side edge of the aperture, can be calculated from[10]

$$\Delta\phi' = \frac{2\pi}{32}\frac{F'}{\lambda}\frac{\sin\theta}{(F'/D)^3}\frac{1}{1 + (Y_2/2F')^2}, \tag{2}$$

where F' is the prime focal length, θ is the off-axis angle of the beam, D is the diameter of the offset aperture, and Y_2 (see Fig. 1) is the offset height of the aperture. Equation (2) assumes that the feed is also displaced slightly in the longitudinal direction (the $-z$ direction in Fig. 1) to cancel field curvature.

Comparison of eqs. (12) and (13) of Ref. 2 shows that $\Delta\phi'$ is proportional to X'. Noting that $\Delta\phi'$ in (2) is defined in terms of the aperture diameter, D, independently of whether the aperture is centered or offset, it follows that D in eq. (1) should be interpreted in the same way, i.e., it is the diameter of the offset aperture, D, and not the diameter of the aperture of the full parabola (8/3 D in Fig. 1).

If the prime-focus feed illuminating the offset parabola is replaced with a Cassegrainian feed system, as in Fig. 1, and the equivalent-parabola concept is valid, F' in (1) and (2) can be replaced with the Cassegrainian focal length F. In Fig. 1, F is the distance Z times the ratio of centerline-ray heights where they intercept the main and subreflector heights, i.e., $F = Z(Y_2/Y_1)$. For the antenna parameters listed in Table I, the values of $\Delta\phi$ calculated from (2) are substantially less than 90°. For these values, the first side lobe, or coma lobe, is increased in amplitude, but the side lobes which are positioned further out, i.e., those that determine the minimum spacing of well-isolated beams, are virtually unchanged. Accordingly, in the remainder of this paper, it is assumed that $\Delta\phi$ is zero. The corresponding values of X, which are calculated from (1) after replacing F' by F, are found to be 4.5 or less. From the plots given in Ref. 2, the off-axis and on-axis beam characteristics are essentially identical for these values of X.

III. BEAM SPACING

Suppose the amplitude distribution across an unblocked aperture is that of a dominant-mode Gaussian beam, that the amplitude at the edge is truncated at the −15-dB point, and that the phase front is uniform. The envelope of the resulting radiation pattern is shown in Fig. 2. For the offset Cassegrain shown in Fig. 1, the above amplitude and phase distribution can be achieved by placing a corrugated feed horn[6] at the secondary focal point, f. Comparison of Dragone's results[9] with the standard Gaussian-beam equations[11] shows that the radius of the beam, ω, at the −8.686-dB (or $1/e$ amplitude) point, is related to the feed-aperture radius, a, by

$$\omega = 0.647\, a. \tag{3}$$

The comparison also shows that the phase-front radius is equal to the slant length of the feed-horn, L. Using Gaussian-beam equations,[11] the beam parameters in any other region in the feed system can be calculated. One result is that the required feed-horn length, L, can be found from the half-angle, γ, subtended at the focus f by the subreflector, and the illumination taper, T, in dB, at the edge of the subreflector.

$$L = 0.076\, \frac{\lambda}{\gamma^2}\, T_{\mathrm{dB}}. \tag{4}$$

Equation (4) includes the feed-horn design criterion[6]

$$a^2/\lambda L = 1, \tag{5}$$

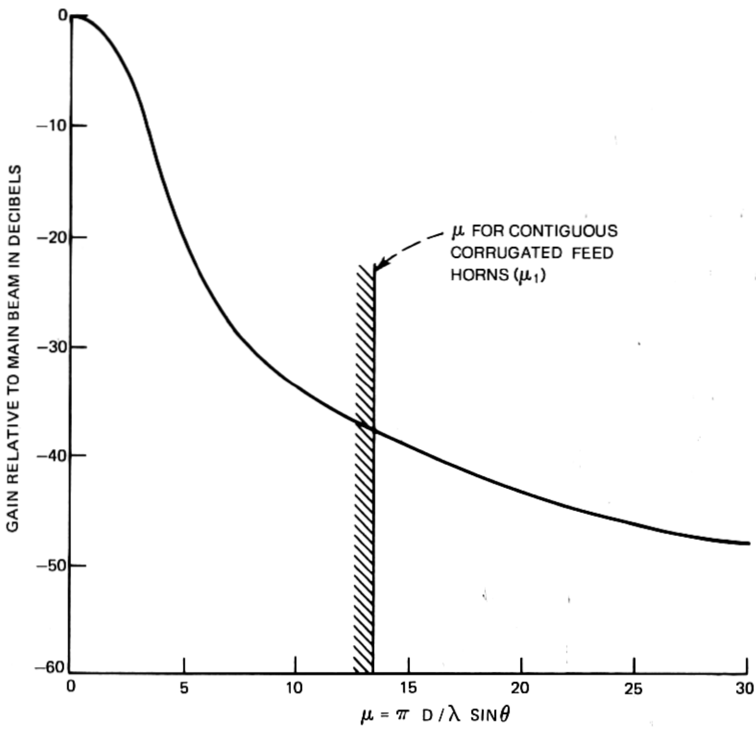

Fig. 2—Estimate of the side-lobe envelope resulting from a Gaussian illumination taper truncated at the −15 dB point, courtesy of T. S. Chu.

where, for a 1.75:1 bandwidth, λ is specified at the low end of the frequency range. Equation (4) is strictly valid only when $\gamma \gg \lambda/D_{sub}$, where D_{sub} is the diameter of the subreflector. For an equivalent parabola with focal length F,[8] it can be shown that the γ criterion is automatically satisfied when the F/D ratio is less than 5.

The corresponding feed-horn aperture radius, a, is found by solving $a^2/\lambda L = 1$ for a, and substituting L from (4):

$$a = 0.275 \frac{\lambda}{\gamma} \sqrt{T_{dB}}. \tag{6}$$

Suppose the antenna shown in Fig. 1 has a diameter-to-wavelength ratio, D/λ, in the hundreds, an equivalent focal length, F, and an F/D ratio larger than 2. Then if a second feed-horn is placed adjacent to the on-axis feed, the second beam will be aimed in an off-axis direction,

$\theta_1 = 2a/F$. Inserting a from Eq. (6) and noting[8] that $\gamma = D/2F$,

$$\theta_1 = 1.1 \frac{\lambda}{D} \sqrt{T_{dB}}. \tag{7}$$

Inserting (7) into the parameter on the abscissa of Fig. 2, the value of u for contiguous corrugated feed horns is

$$u_1 = 3.46\sqrt{T_{dB}}. \tag{8}$$

For $T = 15$ dB, $u_1 = 13.4$. From Fig. 2, the -3-dB beamwidth is 3.62; thus, u_1 corresponds to $13.4/3.62 = 3.7$ beamwidths. For $u_1 = 13.4$, Fig. 2 shows that the side-lobe envelope level is -37 dB; this is approximately equal to the isolation of two beams spaced θ_1 degrees apart. The isolations for typical beam spacings are included in Table I. In the earth-station example, the minimum beam spacing is 0.6°, but the corresponding isolations, 37 and 43 dB at 4 and 6 GHz, respectively, are too small for allowable adjacent-satellite interference.[12] These isolations can be increased to 45 and 49 dB, respectively, by increasing the beam (and satellite) spacing to 1°. The increased beam spacing also allows room between feed horns, so they can be moved individually to track small errors in satellite positioning.

IV. AREA EFFICIENCY

Suppose an off-axis plane wave is incident on the main-reflector aperture shown in Fig. 1. The rays intercepted and reflected by the main reflector are displaced laterally with respect to those from an on-axis beam. But if the subreflector surface is sufficiently broadened, each of these rays will be intercepted and focused to a new point that is displaced laterally with respect to focal point f. To accommodate off-axis beams in the horizontal plane, the subreflector width is increased; similarly, for beams in the vertical plane, the height is increased, as indicated by the dashed line in Fig. 1.

The lateral displacement of the focus, corresponding to an off-axis beam at an angle θ, is equal to θ times the equivalent focal length, F. It is assumed that a separate corrugated feed horn is optimally positioned about the focus of each off-axis beam, i.e., each feed is pointed such that the original on-axis amplitude distribution is maintained across the main-reflector aperture, and each feed is longitudinally positioned to minimize aberrations.

The phase center of a corrugated horn can be calculated as a function of frequency.[9] This in turn allows the longitudinal position of the feed to be optimized for broadband performance.

Assuming the foregoing precautions are observed, each beam of the antenna in Fig. 1 has a computed gain about 1 dB less than that obtainable from an aperture with a uniform amplitude distribution. The underlying reasons for the good area efficiency, 80 percent, are (*i*) the main reflector does not have to be enlarged to accommodate off-axis beams, and (*ii*) the F/D ratio of a Cassegrainian antenna is fairly large.

V. POLARIZATION CROSS-COUPLING

T. S. Chu and R. H. Turrin have shown that the cross-coupling of an offset reflector is a function of (*i*) the angle between the feed axis and the reflector axis and (*ii*) the half-angle subtended at the focus by the reflector.[13] In an offset Cassegrainian antenna with a moderate F/D ratio, these angles are fairly small; thus, the cross-coupling is very small. In particular, in Fig. 1, $\psi = 14°$ and $\gamma = 8.5°$. For linearly polarized excitation, the cross-polarized lobes have a peak value of -45 dB. It is anticipated that, in beams with small off-axis angles, as in Table I, the cross-coupling will be about the same.

VI. MULTIPLE-BEAM ANTENNA PARAMETERS

The off-axis beam parameters and corresponding feed-horn dimensions of an offset Cassegrainian antenna fed with corrugated horns can be calculated once the main-aperture diameter and operating

Table I — Multiple-beam antenna parameters

	Earth Station at 4/6 GHz	Satellite at 20/30 GHz
Aperture diameter, D	30 meters	3 meters
Wavelength, λ	7.5 cm/5 cm	1.5 cm/1 cm
Beamwidth, β	0.165°/0.11°	0.33°/0.22°
Primary focal length, F'	30 meters	3 meters
Off-axis beam angle, θ	5°	4°
No. of beamwidths, $N = \theta/\beta$	30/45	12/18
Off-axis parameter, X'	30/45	12/18
Main-reflector offset, Y_2	25 meters	2.5 meters
Subreflector offset, Y_1	5 meters	0.5 meter
Equivalent focal length, F	100 meters	10 meters
F/D ratio	3.33	3.33
Coma aberration, $\Delta\phi$	18°/27°	14°/21°
Off-axis parameter, X	3.0/4.5	1.2/1.8
Feed-horn length, L	3.8 meters	76 cm
Feed-horn diameter, $2a$	1.03 meters	20.5 cm
Beam spacing θ_1	0.6°	1.2°
Isolation at θ_1 spacing	37 dB/43 dB	37 dB/43 dB
Isolation at 1° spacing	45 dB/49 dB	—
No. of available beams	16 (in a row)	18 (within U. S.)

wavelengths are specified. Typical results for an earth-station antenna at 4 and 6 GHz and a satellite antenna at 20 and 30 GHz are given in Table I. Similar results for other diameters and wavelengths can be found by following the text and performing the calculations in the order listed in Table I.

VII. CONCLUSIONS

An offset Cassegrainian antenna fed with corrugated horns is expected to have well-isolated multiple beams that are broadband and dual-polarized. The antenna has good area efficiency and is relatively compact. This combination of properties makes the antenna well-suited for earth stations and satellites.

VIII. ACKNOWLEDGMENT

The author wishes to thank R. F. Trambarulo and T. S. Chu for stimulating discussions, and T. S. Chu for the envelope profile shown in Fig. 2. The author also wishes to thank M. J. Gans for his calculation of area efficiency and his off-axis radiation patterns, which show that the further-out side lobes are not affected by coma aberration.

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