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The Use of Negative-Impedance Units Inserted Uniformly Into a Transmission Line to Reduce Attenuation

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This paper describes the general properties of a negative-impedance-boosted (NIB) transmission line, an otherwise uniform line to which lumps of negative impedance have been introduced periodically along its length. The purpose of these lumps is to reduce the attenuation of the line, but they also make possible self-oscillation and cause reflections. However, these reflections are cancelled up to a cutoff frequency that has an inverse relationship to the spacing of NIB units. Summaries are included of the latest NIB project; of previous investigations into NIB lines, many of which were not published; and of the first work in the Bell System to reduce transmission-line attenuation through the introduction of loading coils and then unilateral amplifiers. It is shown that many of the differences between properties of NIB lines and properties of a line fitted with unilateral amplifiers come from the different ways the source of energy replenishment is coupled into the line.

It is shown that, under suitable conditions, the addition of small simple NIB units to a transmission line can reduce its loss to a low level in a stable manner and also equalize it. Because of some properties of NIB lines, they cannot be used successfully in all situations; in particular, an environment in which large longitudinal currents are induced in the lines is unfavorable.

I. INTRODUCTION

The use of negative impedances to reduce the attenuation of telephone transmission lines is the subject of this paper. The principal material is in Section V and deals with the general properties of transmission lines to which negative impedances have been added periodically. These properties are compared with those of lines with loading coils and lines having conventional unilateral amplifiers. This use of negative impedances has been investigated a number of times in several ways over the past forty years, but very little of this work has been published. A brief history of this effort is given in Section VI. A short section on the properties of negative-resistance elements is also included.

For perspective in viewing the use of negative impedances in transmission lines, a few highlights are provided from the early development of means to reduce attenuation in the lines of the Bell System. This development from early Bell System work makes a very interesting story in itself, parts of which may be found in Refs. 1 through 8—the account in Ref. 1 being especially good. However, this background information is given here not only for perspective, but because the introduction first of loading coils and then of gain by means of unilateral amplifiers furnishes two important points of reference in discussing the properties of attenuation reduction by means of negative impedance.

The most recent study of the use of negative impedances in transmission lines, begun several years ago by L. A. Meacham, is summarized briefly in Section VII.

II. LOSS REDUCTION BY LOADING COILS AND AMPLIFIERS

The first major advance in pushing back the barrier of distance in telephone transmission came around 1900 with the introduction of loading coils. Through the work of Heaviside and others, it had been known for some years that if the series inductance of a line could be increased, its attenuation would be decreased. But there was no satisfactory way to increase continuously a line's inductance, although this was done to a limited extent in submarine cables later on. Soon after joining the AT&T Laboratories in Boston, Dr. George A. Campbell in 1899 developed a theory of loading for adding inductance in finite lumps to a line.[3] In an unusual coincidence, a similar theory was worked out by Professor Michael Pupin of Columbia University at about the same time. In a patent-interference case, it was decided that Professor Pupin preceded Dr. Campbell by a few days. Campbell's analysis, which was more detailed than Pupin's, gave the relation between kind of line, size of coils, and their spacing and cutoff frequency. These relations were used to begin the introduction of the loading process and are still widely used today.

Campbell's invention of the wave filter came from the results of his analysis of the loading process, which was seen to make a low-pass filter of the line. The making of suitable coils was quite a problem itself in those days, but within about a year, installations of coils were beginning and attenuations were being reduced to about one-half their former values. This was a great advance and made possible a large expansion of the telephone network in distance and use. The theoretical cutoff frequency of these loaded lines was about 2300 Hz, but because of the core material in the coils, the actual line losses began to increase considerably before this cutoff. With later improvements in coils and technique, line losses were often reduced to one-third or one-fourth the nonloaded values.[2] By 1905, with the use of these coils and #8 or #10 wire, the long-distance open-wire network in the East had been extended as far west as St. Louis and Kansas City without benefit of repeaters and by 1911, Denver had been reached.

An important goal at the time was to link the east and west coasts by telephone. To do this, either #5 wire (0.18-inch diameter) had to be used in the line from Denver to San Francisco, or a suitable form of repeater had to be found. The repeater path appeared to be the more economical and so the work already under way on repeaters and their use in transmission lines was expanded.

In fact, between the invention of the telephone and the appearance of the high-vacuum-tube amplifier in 1913, there was a large amount of effort expended, both inside and outside the Bell System, to devise a workable amplifier for telephone signals. During this time, many inventions were submitted to the AT&T Company for possible use as telephone repeaters. Some of these involved rotating electrical machinery or other ponderous mechanical devices that were not suited to high-frequency telephone signals. Others, on analysis, turned out to be simply transformers with no energy source.[4] An amplifier is really a modulator in which an incoming signal, with a small expenditure of energy, modulates a larger local energy source to produce a gain as it repeats the signal. In a vacuum-tube amplifier, the dc plate current supplied by a battery is modulated by the grid voltage. In a transistor amplifier, the collector current is, in effect, modulated by the much smaller base current. In a carbon-button amplifier, the varying resistance of the carbon granules in response to the motions of the receiver armature modulates the dc flowing through the button with a power gain of several hundred. In a parametric amplifier, the source of energy is a local oscillator supplying pump current which is modulated by the signal. And so on.

During the time when a suitable repeater was most actively sought, only two devices showed enough promise to be seriously considered. One of these was a combination of carbon-button transmitter and receiver which, in its best version, had the transmitter and receiver coupled mechanically instead of acoustically. A few commercial installations of this device were made although it had a number of problems, one of which was variability of gain. The other device was a gaseous discharge that fundamentally had the characteristics of a negative resistance rather than a unilateral amplifier.

At this point, the three-electrode vacuum tube appeared on the scene and showed such great promise that the first two devices were soon put aside. The first high-vacuum-type repeater was installed in late 1913, a little less than a year after H. D. Arnold had been shown DeForest's three-electrode tube. By the middle of 1914, three improved repeaters had been successfully installed and operated in the new transcontinental line. The net loss of this line was 20 dB and the 10-dB bandwidth was from 350 Hz to 1250 Hz.[1,4]

III. PROBLEMS AND PROPERTIES OF REPEATERED LINES USING UNILATERAL AMPLIFIERS

The introduction of repeaters into the telephone plant showed up transmission problems that had been obscured before. At any point in the line where its uniformity is disturbed (called an irregularity), part of a wave traveling down the line will be reflected back toward the wave's source. Such a reflected wave is like an echo. When the net loss of the line is 20 dB or more, the returning echoes are weak and hardly noticeable. But when gain was added to such a circuit, the echoes could be very objectionable. In fact, they could limit the amount of usable gain. For example, the net loss in the first transcontinental line in 1914 was limited to 20 dB.

Another factor in the effect of echoes is the transmission time of the line. Because of the inductive loading, the transmission time of the transcontinental line was 70 ms, which is high enough so that if strong echoes occur they will be objectionable. As understanding of these effects grew, the loading was removed, reducing the transmission time to 20 ms. With this change, the loss increased about 100 dB but more gain could be added, reducing the net loss to 12 dB. An extra benefit that came from the removal of the loading was higher-quality speech transmission because of the increased bandwidth.[5] Since the attenuation vs frequency characteristic of open-wire lines is fairly flat, the use of loading restricted the bandwidth. Gradually all loading coils were removed from open-wire lines. The situation is different with cable pairs, which have a considerably higher capacitance. Here the coil loading tended to flatten the attenuation vs frequency curves so that only a little equalization was required at the repeaters. Hence, loading has generally been kept in voice-frequency cable circuits, although the amount of loading has been reduced in long circuits to reduce transmission time.

Another effect of the reflections when repeaters are in the line is the possibility of free oscillation in the line, or "singing." If an amplifier is inserted into a perfectly uniform transmission line in such a way that it does not cause reflections, the gain of the amplifier may be as large as desired. But if there are reflections, the amount of gain that can be inserted will be very definitely limited by either echoes or singing.[5,8–11] Reflections can be caused by an improper termination of the line, by a missing or misplaced loading coil, or by the way amplifiers are coupled into the line.

To obtain bilateral transmission with a unilateral amplifier, a special kind of circuit is required to couple the amplifier into the line. Some form of bridge circuit is the usual means—a Varley loop was first used for telegraph repeaters. Coil arrangements for doing this are called hybrid coils, one of which plus one amplifier make a 21-type repeater that can amplify in both directions. But its losses are high and it sends amplified signals in both directions with the effect of an echo even though the lines in both directions are well-balanced against each other. Hence, they were not used very much. The 22-type repeater, shown in Fig. 1, uses two hybrid coils, two amplifiers, and two line-balancing networks, and gives much better performance. Losses are minimal and reflections come only from imperfections in balance between the line in each direction and its balancing network. This means for coupling amplifiers into a line was invented in 1895 (U. S.

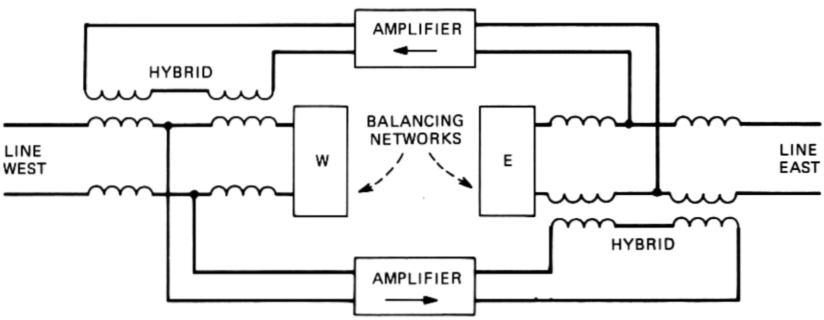


Fig. 1—Diagram for connecting unilateral amplifiers and hybrid coils to make a 22-type repeater.

Patent 542,657) by a Bell System engineer, W. L. Richards, long before there were good amplifiers with which to use it.[4]

After the successful introduction of the 22-type repeater into the system, the AT&T Company continued to have submitted to it inventions that purported to reduce in varying degrees the complexity of the means for coupling unilateral amplifiers into the line. They were mainly variations of the basic bridge circuit or were completely unworkable. No satisfactory substitute for the hybrid coil arrangement of Fig. 1 has appeared except for the very different approach to overcoming line loss by the introduction of negative resistances, which will be discussed below.

As may be seen from Fig. 1, the amount of gain that can be obtained from this repeater configuration before local singing occurs depends on the loss across the hybrids (vertically in the diagram) and is higher for higher losses. This loss is infinite if the network exactly balances its corresponding line and is zero if the line presents a short or open impedance. Thus, to operate these repeaters with considerable gain in two-wire lines requires a fairly high degree of balance between networks and lines. When a number of repeaters is used in tandem in a line, additional possible singing paths are introduced as well as additional echo sources at each departure from uniformity, which from a practical viewpoint is at each repeater. To eliminate these multiple singing paths and echo sources, four-wire transmission was introduced in many toll circuits and is universally used now in all carrier circuits. In this arrangement only two hybrid coils are used, one at each terminal. The upper path in Fig. 1 becomes a two-wire line with a sufficient number of east-west amplifiers approximately uniformly spaced. The lower path is similar but transmits only west to east. The singing problem is greatly reduced with this arrangement because both horizontal transmission paths between the terminal hybrids have very little if any gain since each includes the line loss as well as the amplifier gain. In some respects, the four-wire configuration of repeaters is similar to the use of series negative-impedance boosters, as will be seen below.

If the two wires of the pairs in the two-wire and the four-wire circuits, and the repeater circuits connected to them, are well-balanced against each other, interference currents will largely be drained off to ground away from the repeaters and terminals.

The 22-type repeaters were economically as well as technically successful in toll transmission, but were deemed too expensive and complicated for exchange trunks where gain was needed in the general upgrading of telephone quality in the nineteen twenties. Partly because of continuing study of repeatered systems and partly because of the need for a cheaper repeater, as indicated above, different ways of reducing line attenuation were sought.

One way, quite different from the use of unilateral amplifiers described above, and which appeared to hold considerable promise, was that of introducing negative resistances into the line to cancel, or partly so, the positive line resistance, which, because of good dielectrics, is the principal source of loss in lines. While some thought had been given to such use of negative resistances at about the time the vacuum-tube repeater appeared, as indicated above, a considerable effort in this direction began a little before 1930 and has continued off and on until the present. A review of this work is given below in Section VI.

Now, with our summary of the processes by which gain may be obtained, we are able to see a certain equivalence between an amplifier and a negative resistance. Yet the difference in the ways they must be used in a transmission line, or in the particular form one or the other takes, may make it more desirable to introduce the gain in one of the ways instead of in the other.

The next section will describe some of the properties of negative resistances and devices that exhibit this characteristic.

IV. NEGATIVE RESISTANCES

A positive resistance absorbs energy from connected circuits and dissipates it. A negative resistance delivers energy to connected circuits and so must have within it a source of energy on which it may draw. More precisely, a negative resistance as a device is able to convert the energy of a local source, to which it is connected, into a form suitable for passing on to other circuits. In other words, it must be able to modulate the energy from the local source as an amplifier does. An amplifier, though, is usually a three-terminal device in which the energy stream is controlled externally. A negative resistance, on the other hand, is a two-terminal device utilizing some internal phenomenon that depends on the voltage across it or the current through it to control the energy stream. An early paper by Crisson[12] describes two kinds of negative resistance. He shows also in this paper that either form of negative resistance can be generated by connecting the output of an amplifier to its input either in a series way or a shunt way. This suggests that in general a negative resistance is the result of some kind of feedback process, that is, one in which current or voltage depends on itself in some way, either externally or internally. A good general paper on negative resistance and devices which exhibit it is the one by E. W. Herold.[13]

Some parasitic reactance may be associated with a negative-resistance device and/or some may be intentionally added to it so that, in general, an impedance with a negative resistance component is what is used. The general term negative impedance is applied to such a device. Because of the reactance, the impedance usually does not have a negative resistance component above a certain frequency. It is possible also for negative impedances to be generated in a largely reactive device, such as a varactor, by an internal modulation process. Here the energy source is not dc but the external pump oscillator.

When a negative resistance is generated by connecting the output of an amplifier to its input, the voltage-current characteristic is a line with negative slope extending from the origin to where the amplifier begins to overload. Here, in effect, the true energy source is concealed within the device. However, in many negative-resistance devices, the energy comes to the device from the source through a direct current that also flows through the two terminals of the device. Over a certain range of values of this current, the device exhibits a negative resistance and so is able to convert energy from the source to a form suitable for use in the connected circuits. All known negative resistance devices of this type have either one or the other of two shapes of voltage-current characteristic, as illustrated in Fig. 2. The series type is shown in Fig. 2a, and the shunt type in Fig. 2b. Between points A and B, the slope dV/dI is negative and, for current variations in this range, the device presents a negative resistance to external circuits. The magnitude of the negative resistance is this slope expressed in ohms.

To get an understanding of the effect of a negative resistance in a circuit, consider the diagrams of Fig. 2c, where a positive resistance R_o has been added to the negative resistance R and the resulting overall voltage-current curve drawn. When the added positive resistance just balances the negative component, the V-I curve of the combination has a slope of zero between A and B (Fig. 2c). That is, if, as indicated in the figure, a varying signal current is superimposed on the bias of direct current supplied by the energy source, the voltage drop across the combination of positive resistance and negative resistance device is zero as long as the variations remain within the range A to B. This means that all the positive signal energy dissipated by the positive resistance R_o comes from the negative resistance R and none from the signal generator e. This is an indication of how the negative resistance

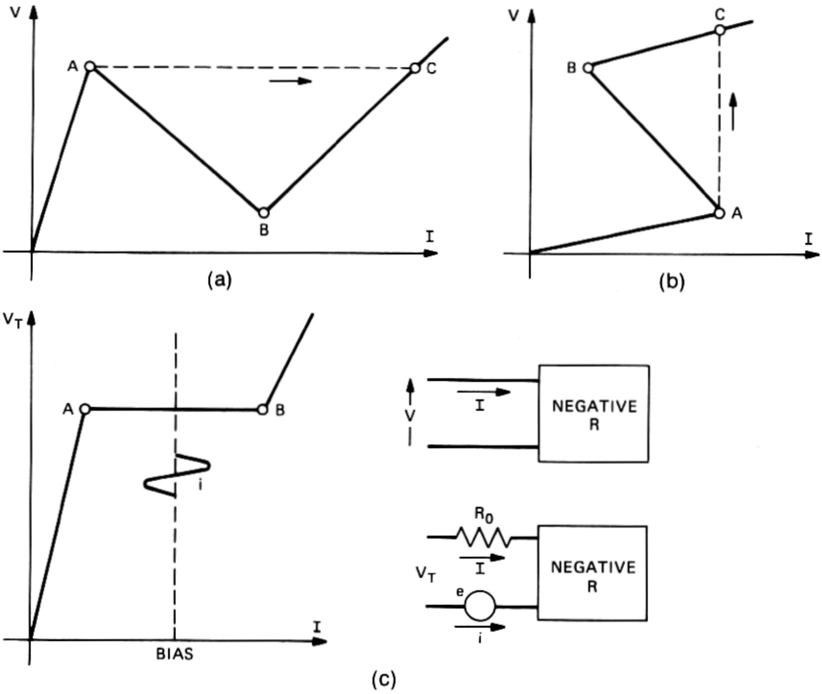


Fig. 2—Voltage-current curves for negative resistance.

supplies energy to the connected circuits by appearing to cancel or nullify the positive resistance, or a part of it.

Of course the borderline case just discussed could not really exist because the signal generator e, being zero, is not in control. To understand this incipient instability, let R_o be less than R so that the slope of the V-I curve between A and B is now negative as in Fig. 2a, and consider also the small parasitic inductance l which would be in series with the resistances. Then the signal current i would be given by the expression

$$i = e/(sl - R + R_o),$$

where s is complex frequency $\sigma + j\omega$. If the magnitude of R is greater than that of R_o, the impedance of the circuit has a root in which s has a positive real part and so is unstable. If, in addition, there is a resonance of reactances associated with the circuit, sustained oscillation will occur at the frequency of resonance. Thus, there is an instability barrier that prevents the negative resistance from supplying any more energy to connected circuits than just enough to cancel the losses in the external positive resistance.

Because of the instability conditions just described, the V-I characteristics of Fig. 2 cannot be obtained directly from measurements on the device alone. If the voltage in Fig. 2a or the current in Fig. 2b is increased from 0 until point A is reached, the current in Fig. 2a or the voltage in Fig. 2b will appear to jump to point C, skipping over the intermediate part of the characteristic. Actually this is a very fast transient with a positive real exponent as indicated above. This instability will not occur if a positive resistance a little larger than the magnitude of negative resistance is connected in series with the series device, nor will it occur if a positive conductance a little larger than the negative conductance is shunted across the shunt device. Hence, the names, series and shunt, are given to the two types of negative resistance. The characteristics, Figs. 2a and 2b, are deduced by measurements of the combination circuit, as shown in Fig. 2c. The slope of the negative-resistance region can be measured quite accurately in such a circuit by adjusting R_o to obtain a null of signal voltages across the series combination of R and R_o.

One of the earliest negative-resistance devices is the carbon arc (or other gaseous discharge). This is the series type and is generated when the ionization in the discharge begins to increase by means of the current of the discharge itself. The avalanche process in semiconductors is the modern counterpart of this. The dynatron invented by A. W. Hull[14] is of the shunt type and depends on secondary emission, which becomes appreciable when a voltage reaches a certain value. Other devices that can exhibit negative resistances are thermistors, avalanche diodes and transistors, tunnel diodes, IMPATT diodes,[15] and Gunn-effect diodes.[16] The paper on IMPATT diodes by K. D. Smith[15] is a particularly good description of this device and of the general properties of such negative resistances. As the physical dimensions are reduced, the parasitic reactances are reduced and the device can show a negative resistance at higher frequencies. The IMPATT and Gunn-effect diodes differ from the others in that they show a negative resistance only in a narrow frequency band, depending on their sizes.

Other sources of negative resistance are made by combining several devices such as two vacuum tubes or two transistors and, in this form, they are often called negative-impedance converters.[17] Vacuum tubes were used in the first E-type negative-impedance repeaters and transistors in the latest ones and also in the work described below.

When negative resistances are made in this way, the opportunity exists of modifying the variations of the negative resistance with frequency and/or current by adding passive circuit elements to the configuration in order to serve particular needs. They also can be made in integrated-circuit form for small size.

V. GENERAL PROPERTIES OF TRANSMISSION LINES WITH NEGATIVE-RESISTANCE LUMPS ADDED PERIODICALLY

It was shown in Section IV how a negative-resistance device supplies energy at signal frequencies to circuits in which it is connected, acting as if it cancelled the effect of positive resistance in the same circuit. Hence, it is natural to think of this as a way to reduce the attenuation of a transmission line. This is not a new idea. It was mentioned as a possibility in the 1919 repeater paper[4] referred to above. The term booster was applied to negative-impedance devices used in this way in that paper and also in another.[12] This name was revived by Meacham in his negative-impedance work.[18] In quite a bit of the literature, the process of adding negative impedances to a transmission line is called negative-impedance loading, drawing a parallel to the familiar practice of coil loading. The term boosting seems to the author to be a better description of the process and will be used here.

This section deals with some of the general properties of negative-impedance-boosted (NIB) lines. This will be done without reference to any particular form of negative impedance, as far as is possible. However, to discuss the particular shapes of the resulting transmission curves or the particular details of stability, it is necessary to completely specify the properties of the negative impedance used. It is a fact also that whether or not the general idea of boosting is attractive may depend on the existence of a booster with certain properties. Thus, while the possibility of using boosters to overcome attenuation was recognized in the Jewett paper[4] of 1919, as mentioned above, no very suitable devices were available then.

5.1 Coupling negative impedance into the line

The simplest method of utilizing negative-impedance boosters (NIB) in a line is to connect the series-type units in series with both wires of the line and uniformly spaced along it, as shown in Fig. 3. In this form, the boosters are simply two terminal devices through which the line currents flow. A direct current, which flows through boosters and line along with signal currents, supplies the necessary energy to the boosters. The unit may consist of a single special element or a combination of several circuit elements. Experimental versions of

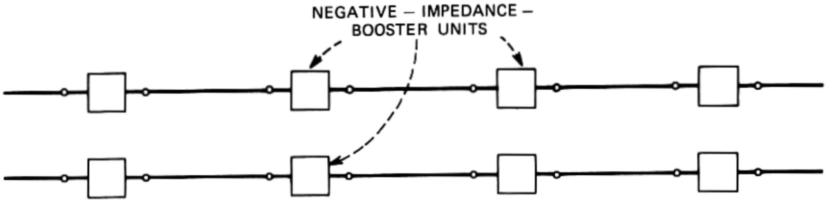


Fig. 3—A NIB line made by the periodic series insertion of negative-impedance units into a transmission line.

the latter arrangement have been made very small in size using hybrid integrated-circuit techniques. In the E-type negative-impedance repeaters,[19,20] transformers are used to couple the boosters into the line, and these repeaters are used only at central offices. Thus, the manner of using negative-impedance boosters is quite different from the manner of using unilateral amplifiers.

While most of the descriptions in this paper are in terms of boosters added in series with the line, reduction of attenuation may be obtained also by adding suitable negative resistances in shunt to the line. Or a combined series and shunt connection may be used. In the latter circumstance, the combination booster may be made to present to the line an impedance that approximates the characteristic impedance of the line. This has been done in some of the E-type negative-impedance repeaters[21] discussed in Section 6.3. Reflections in this case are small and the behavior of the line in this respect is more like one equipped with conventional repeaters.

5.2 Bilateral transmission

As may be seen from the discussion about Fig. 2c in the previous section, the performance of the negative resistance is just the same whether generator e is on the right or the left. And so another general property of boosted-transmission lines is that transmission over the line is bilateral and symmetrical, as it is for the uniform nonboosted line. However, this is true only over a certain range of current through the line, i.e., the range within which the booster presents a negative resistance, as shown in Fig. 2. This property exhibits a fundamental difference between the negative-resistance energy-conversion device and the unilateral-amplifier energy-conversion device.

5.3 Necessity of using lumps of negative resistance

If the negative resistance could be added in infinitesimal bits of ohms, the line resistance would be neutralized continuously and an ideal line with propagation constant $j\omega\sqrt{LC}$ and image impedance $\sqrt{L/C}$ would result. But, for the present at least, negative resistance is available only in sizable finite lumps and so this is what must be used. There are several consequences. First, adding lumps of negative resistance does not yield the same propagation constants and image impedance that adding the same total amount of negative resistance continuously does. Formulas are given in Section 5.5 below. Second, the added lumps cause reflections, the effects of which are described in Section 5.4. Third, the consideration of even very small lumps of pure negative resistance is nonphysical, since this implies an infinite energy source to supply the infinite band of frequencies. Of course, parasitic reactances in real devices prevent this conceptual difficulty from arising in practice. But there is a further restriction. In Section 5.9, it may be seen that, because of transmission-line properties, the negative resistance must be reduced as frequency increases to avoid instability.

5.4 Reflection effects and cutoff frequency

The addition of these lumps of negative resistance to the line introduces reflections, because each negative resistance presents a discontinuity to traveling waves and so each is a source of reflections. But, as shown by G. A. Campbell in the case of added series inductance,[3] if the added series impedances are spaced uniformly, the sum of all the reflections is zero up to a certain cutoff frequency. This cutoff occurs at the frequency for which the spacing of the added impedances is one-half wavelength of the boosted line. Thus, an infinitely long boosted line behaves like a smooth line up to the cutoff frequency. Another consequence of the process is that the cutoff frequency is related in an inverse way to the spacing of the added impedances, as shown in Fig. 4. More detailed relations are given in Ref. 22.

Another general property, then, is that a boosted line has a cutoff frequency because of the reflections introduced by the boosters and that this cutoff is related to the spacing between boosters in a more or less inverse way, and that up to the cutoff, all the reflections cancel so that the line looks like a smooth line for these frequencies.

5.5 Propagation formulas for NIB lines

Adding circuit elements in finite lumps to an otherwise uniform transmission line results in a line with different properties than one to which the same total amount of elements has been added continuously.

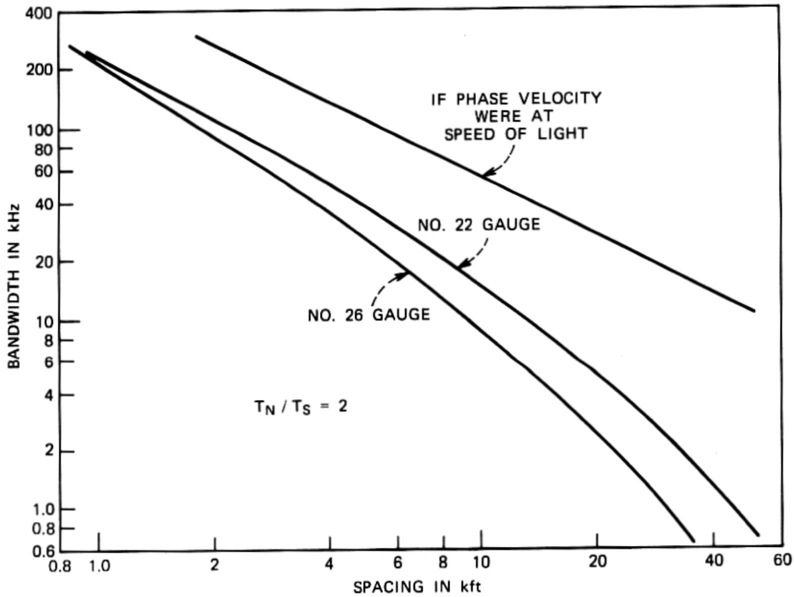
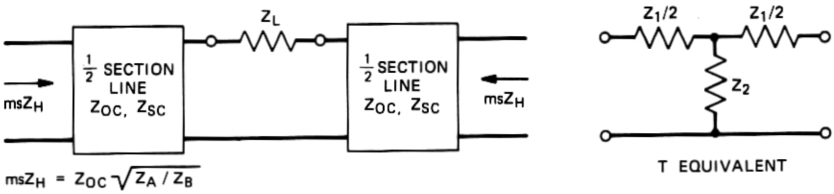
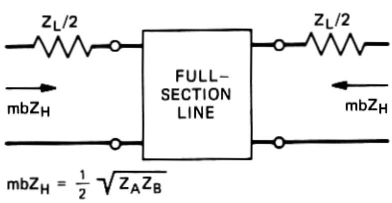


Fig. 4—Relationship between bandwidth and spacing in NIB lines.



(a) ONE FULL NIB SECTION (SPLIT LINE)



$Z_l/2 = Z_{OC}Z_A / Z_B$, $Z_2 = Z_{OC}(Z_B - Z_A) / 2Z_B$

$Z_A = 2Z_{SC} + Z_L$

$Z_B = 2Z_{OC} + Z_L$

(b) ONE FULL NIB SECTION (SPLIT BOOSTER)

Fig. 5—Two forms of NIB line sections with equivalent circuits and image impedances. Z_L is the total impedance of a pair of NIB units.

The formulas for propagation constant and characteristic impedance of a uniform line cannot be modified by replacing the old element value with a new average value. The method used to derive formulas for the boosted line was first suggested by Campbell,[3] in his calculations on the addition of loading coils. Many of the formulas appear in unpublished notes written around 1940 by Bullington[23] and around 1950 by Van Wynen,[24] both of Bell Laboratories. To obtain these formulas, the appropriate line sections are combined with the negative-impedance unit Z_L in either of two ways shown in Fig. 5 and an equivalent T or π network derived. These complete NIB sections, in either of the two forms, may then be cascaded to form a NIB line of the desired length. The midsection and midbooster image impedances of the boosted line are

$$msZ_H = Z_{oc}\sqrt{Z_A/Z_B}$$

and

$$mbZ_H = \tfrac{1}{2}\sqrt{Z_A Z_B},$$

where

$$Z_A = 2Z_{sc} + Z_L$$
$$Z_B = 2Z_{oc} + Z_L,$$

and Z_{sc} and Z_{oc} are the short- and open-circuit impedances of one-half a line section and Z_L is the booster impedance, as indicated on Fig. 5. The propagation factor per section of the boosted line, P, is given by

$$\sinh\,(P/2) = \sqrt{Z_A/(Z_B - Z_A)}.$$

Another method of analysis, used in the work of A. L. Hopper,[22] is to represent each of the three cascaded parts of the NIB section by a matrix. The A form transmission matrix for the half-section of line is

$$M \text{ line} = \begin{pmatrix} \cosh p & Z_o \sinh p \\ \dfrac{\sinh p}{Z_o} & \cosh p \end{pmatrix},$$

where p is the propagation factor for the half-section of line and Z_o is its characteristic impedance. The matrix for the series booster is

$$M \text{ booster} = \begin{pmatrix} 1 & Z_L \\ 0 & 1 \end{pmatrix}.$$

By multiplication of the three appropriate matrices, a new matrix for the full NIB section of Fig. 5a is found. The elements of this new A

matrix are

$$A_{11} = \cosh 2p + \frac{1}{2}\frac{Z_L}{Z_o}\sinh 2p = \cosh P$$

$$A_{12} = Z_o \sinh 2p + Z_L \cosh^2 p = Z_H \sinh P$$

$$A_{21} = \frac{\sinh 2p}{Z_o} + \frac{Z_L}{Z_0^2}\sinh^2 p = \frac{\sinh P}{Z_H}$$

$$A_{22} = A_{11}.$$

Here, $2p$ is the propagation factor for the full section of line only. In the final column above, the matrix elements are expressed in terms of the parameters of the full boosted section, P being the propagation factor and Z_H the image impedance. These may be calculated from the matrix coefficients. By multiplying in sequence n matrices of the last form, the equivalent transmission matrix of a cascade of n NIB sections may be obtained and its properties determined.

5.6 *Image impedance*

Another general property of boosted lines is that the image impedance is fairly close to being resistive and fairly close to being flat with frequency. This property is described best by the curves of Fig. 6 computed for the particular realization of negative impedance devised by L. A. Meacham and described in more detail below in Section VII. The parameter R_{NET} or Δr is introduced here. It is the sum of line resistance plus booster resistance, at very low frequencies, per section or per mile or for a whole line, as designated. The figure shows the image impedance Z_H for a particular NIB line configuration along with the Z_o of the nonboosted line. While Z_H is smaller than Z_o over much of the frequency range, it is not as small as $\sqrt{L/C}$, which it would be if the negative resistance had been added continuously. However, its property of being fairly close to resistive and flat with frequency is a great improvement over the Z_o of nonboosted lines. Its smaller magnitude reduces crosstalk considerably.

As will be seen in Section 5.10, the image impedance at very low frequencies may not be just as shown in Fig. 6. The frequency scale would have to be considerably expanded to show this.

5.7 *Transmission properties*

5.7.1 *Infinite line*

The next general property (related to the previous one, of course) is that the transmission characteristics of the boosted line are much better than those of the nonboosted line. As mentioned above, the shape

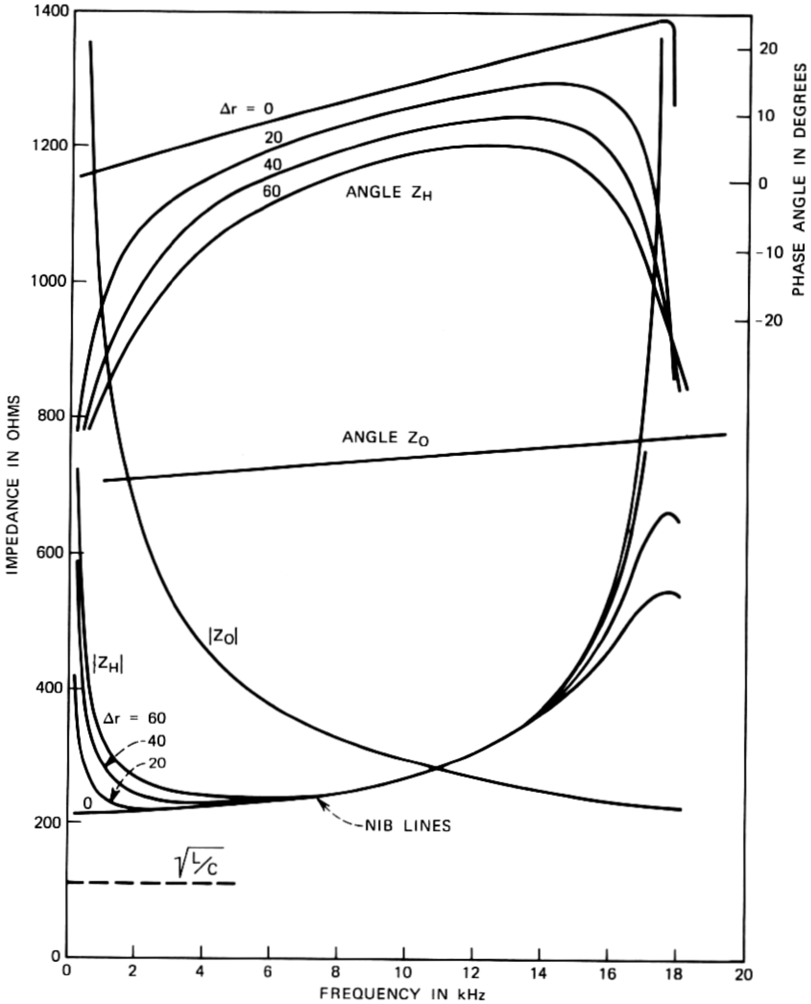


Fig. 6—Image impedance vs. frequency for a 26-gauge NIB line with 6000-foot booster spacing. Parameter Δr is the net resistance at very low frequencies per section of NIB line. The characteristic impedance Z_o of the line before the insertion of NIB units is also shown.

of these characteristics may be chosen by adjustment of the booster parameters. When the choice is for as flat an attenuation curve as possible, the two parts of the propagation constant are plotted in Figs. 7, 8, and 9 for the same particular line configuration used to describe Z_H, along with propagation of the nonboosted line.

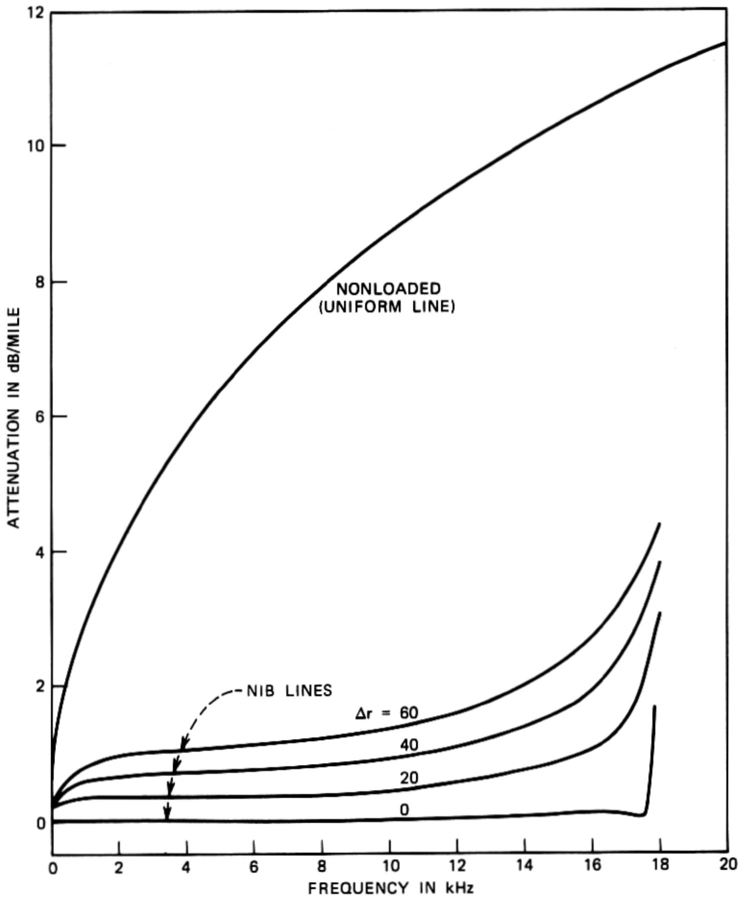


Fig. 7—Attenuation vs frequency for a NIB line and the uniform nonboosted line.

To further highlight the transmission properties of boosted lines, a comparison with coil-loaded lines is made. The boosting process is a much more powerful one because it adds energy at signal frequencies. In a coil-loaded line, the reduction in attenuation is directly related to the amount of inductance added. Hence, reduction in loss is related to cutoff frequency and spacing because of the relation between these two and inductance. In the boosted line, the attenuation reduction can be varied by means of the parameter Δr, independently of the booster spacing and cutoff frequency. The relation between these two latter parameters is shown in Fig. 4.

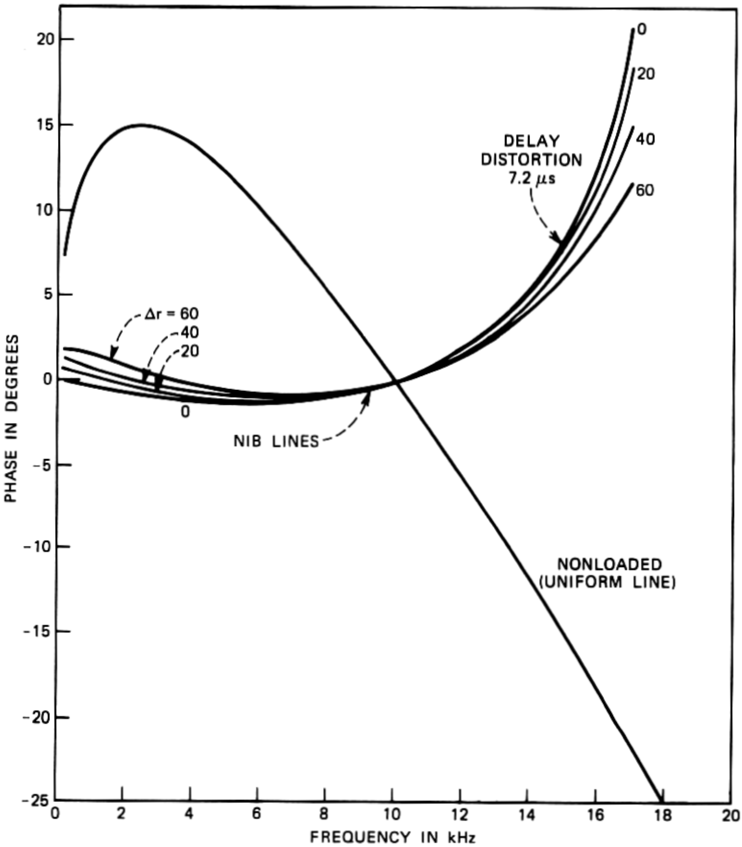


Fig. 8—Deviation of phase from linearity for a NIB line (26-gauge wire and 6000-foot booster spacing) and the uniform line.

The comparatively sharp cutoff of the coil-loaded line is accompanied by large phase distortion, roughly 10 or more times that of the boosted line. The curves of Fig. 9 show that the phase velocity for coil-loaded lines is very low while that of NIB lines is high. However, the introduction of coil loading has been very beneficial in improving and extending the telephone plant.

5.7.2 *Finite lines*

All the discussion so far has been in terms of the properties of the infinite line. These can be approached in the finite line if the terminations are close to the image impedance, otherwise there are differences.

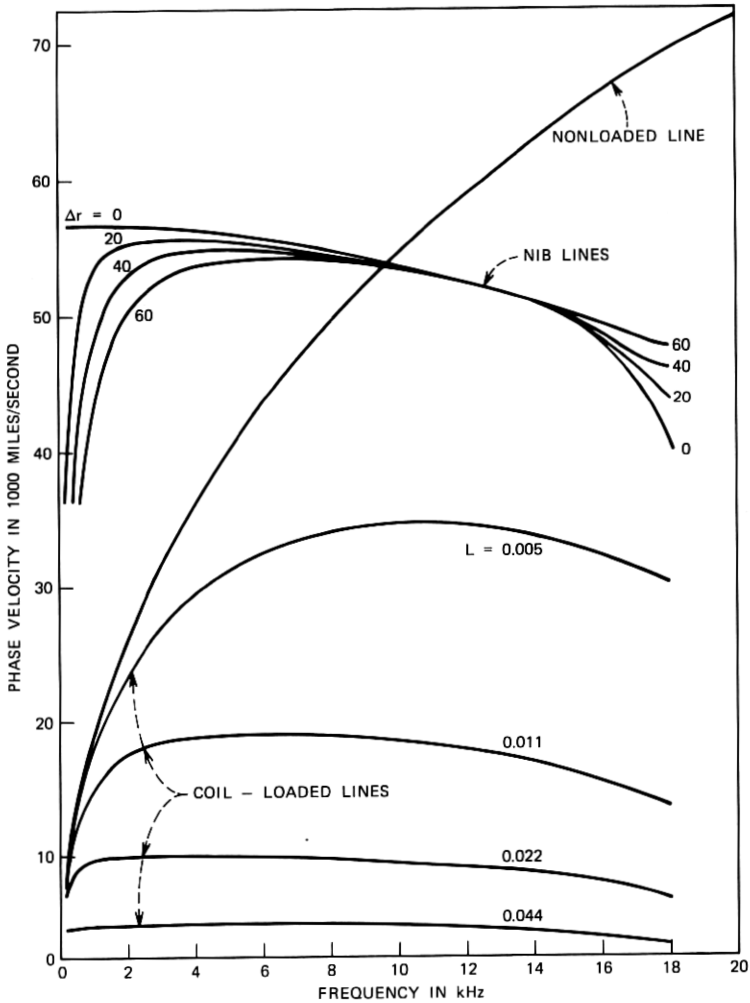


Fig. 9—Phase velocity vs frequency for a NIB line and for several coil-loaded lines.

We saw above that all the reflections from the NIB units cancel in the infinite line. But when the terminations of a finite line differ from the image impedance, all these reflections no longer cancel and the input impedance of such a line is not smooth like the Z_H curves. There will be one ripple for each boosting point and the magnitude of the ripples will be directly related to the magnitude of the difference between the actual termination and the image impedance. The same applies to the insertion-loss curves.

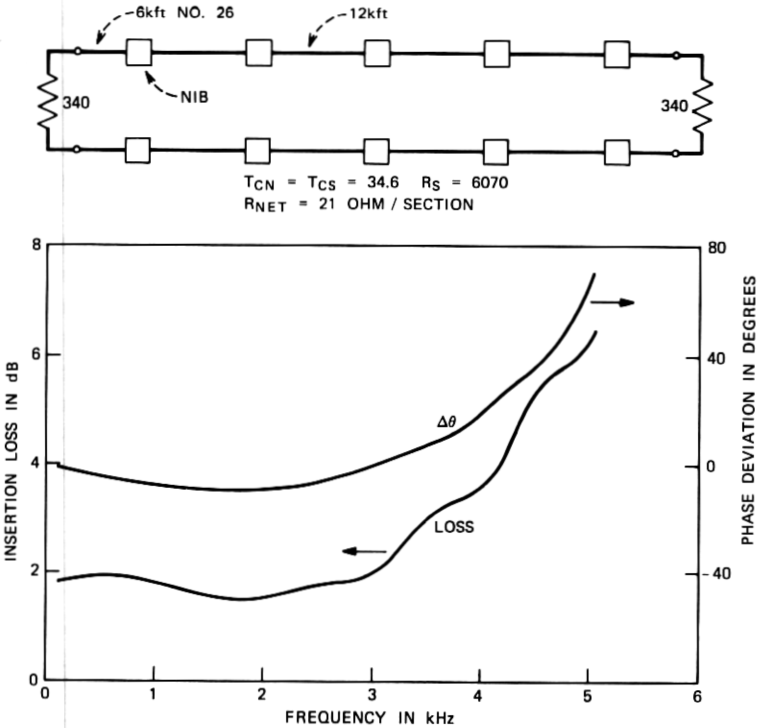


Fig. 10—Insertion loss (in dB and deviation of phase angle from linearity) of a five-section NIB line terminated with resistances.

An example of this effect and of the performance of a NIB line of finite length having resistance terminations, which are simple approximations to the image impedance but not the best, is given in Fig. 10. It should be noticed that the spacing of NIB units and, hence, the frequency band for this line are not the same as in Figs. 6, 7, 8, and 9. Where it is desired to have the return loss as high as possible, a better approximation to the image impedance should be made. See Section 6.2 for a possible method.

The ripples in the transmission curves are considerably larger when the line ends with half-NIB units rather than half-line sections as in Fig. 10.

5.8 Limit of gain

Another general property of boosted lines, in common with all circuits to which negative resistance or gain is added, is the possibility of self-oscillation, or instability. While, under certain conditions of termination, a small amount of insertion gain may be introduced by a boosted line, in most applications, the boosters should go no further than to cancel, or nearly cancel, the line losses in order to maintain stability, as suggested in Section IV. How closely this limit can be approached depends largely on the line terminations and on the margin against instability in the infinite line used in the choice of booster parameters. A similar limitation exists for a uniform line equipped with unilateral amplifiers when provision is made for two-way transmission. A net loss of zero can be approached as all reflections from imperfect terminations approach zero. Of course, a one-way transmission line can, in principle, have as high a gain as desired if terminations are perfect and there are no return paths for the signal. In some ways the NIB line is like the transmission portion of a four-wire repeatered line in which the overall net loss is near zero. The complex subject of stability of NIB lines will be dealt with in more detail in Section 5.10, below.

5.9 Restriction on the shape of negative resistance vs frequency

There is a further restriction on finite lumps of negative resistance used as boosters at finite spacings in a transmission line. If the total amount of negative resistance added to a line approximately cancels the series resistance of the line wires at low frequencies, then the magnitudes of the negative resistances must be reduced at higher frequencies to avoid self-oscillation. This is a consequence of the wide range of impedances that a finite length of transmission line can present for various frequencies and terminations. How the restriction arises will be seen in the next part. If the reduction of negative resistance at higher frequencies is made with careful consideration, the transmission characteristics can be shaped in a number of desirable ways, and stability is achieved. This will be discussed below in the sections on particular realization. Incidently, another consequence of the wide range of line impedances is that a series negative resistance, often referred to as open-circuit stable, can cause instability in an open-circuited transmission line.

5.10 Stability in NIB lines

When lumped negative impedances are inserted into a transmission line periodically, self-oscillation can occur; that is, the line can be unstable. While it is the presence of the negative impedances that brings about the possibility of instability, it is pointless to try to talk about the stability of the negative impedances. What needs to be investigated is the whole circuit since, with the same negative impedance unit, stability will exist for some values of connected passive elements and instability for others.

5.10.1 *Infinite line*

First consider the infinite line with uniformly spaced NIB units since this is simpler to deal with than the finite terminated line and also is the initial condition from which the latter is derived. The following material is presented as reasonable description rather than precise, rigorous analysis.

The boosted line is characterized by a propagation constant and image impedances. The two principal image impedances, as with coil-loaded lines, are the midsection one, msZ_H, and the midbooster one, mbZ_H, as indicated in Fig. 11. Calculation and measurement of boosted lines show that when the negative resistances nearly cancel the line resistance, msZ_H becomes very large and mbZ_H becomes very small near the frequency for which the spacing between NIB units is one-half wavelength. This is illustrated in Fig. 12. The line impedance will have values intermediate between these extremes at other points of the section. Also, all sections will be alike in this because of the assumed uniformity. Since the midbooster image impedance can become very small, it appears to be a likely parameter to consider in investigating the stability of the line. To carry this out, imagine that, as indicated in Fig. 11, a booster has been divided into two equal parts and separated so that a generator e can be inserted into the line there. The current that flows at this point will be

$$i = e/(2mbZ_H).$$

This would be the same at any one of the boosters because of the

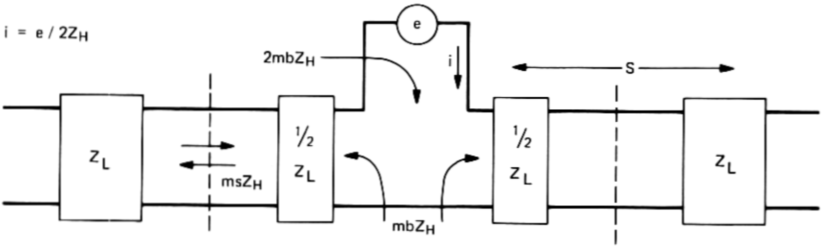


Fig. 11—Representation of NIB line for studying stability. S is space between NIB units of impedance Z_L as in Fig. 5.

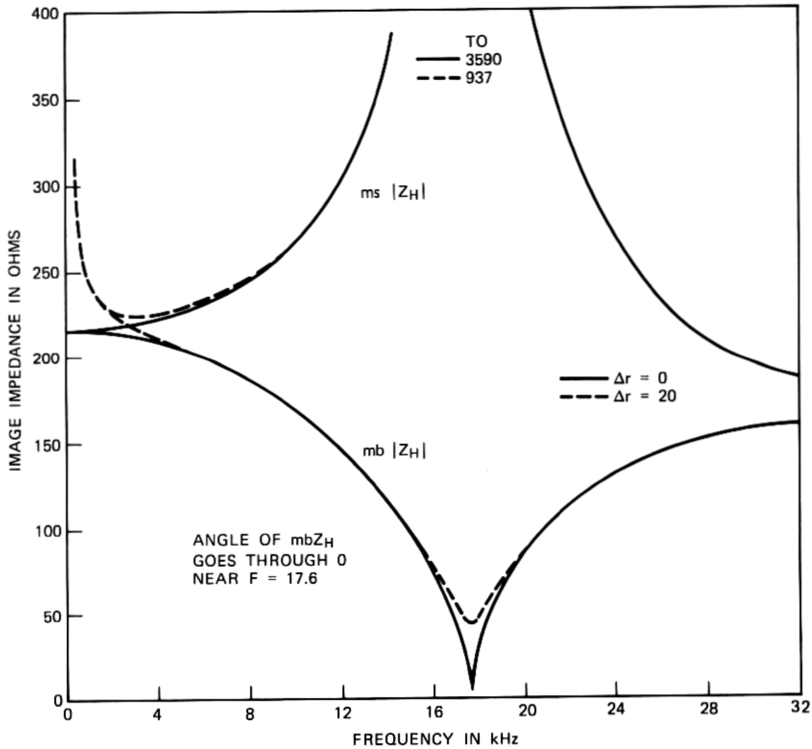


Fig. 12—The two image impedances Z_H of a NIB line.

uniformity. As shown in Section 5.5 above, the midbooster image impedance may be calculated from the formula

$$mbZ_H = \tfrac{1}{2}\sqrt{Z_A Z_B},$$

where

$$Z_A = 2Z_{sc} + Z_L$$
$$Z_B = 2Z_{oc} + Z_L,$$

and where Z_{sc} and Z_{oc} are the impedances of one-half section of line that has been short-circuited then open-circuited at the far end, and Z_L is the impedance of the NIB unit. Thus, Z_A is the impedance seen looking in at the test point of Fig. 11 when the two adjacent midsection points are shorted and Z_B is the impedance seen when they are opened.

The irrational nature of Z_H makes it difficult to analyze in the usual way. However, if Z_H becomes zero under certain circumstances, we can be fairly sure that instability will accompany those circumstances. First consider separately Z_A and Z_B as plotted in Figs. 13 and 14 from calculations of a particular NIB line. In each of these is a separate enlarged plot of the region near $Z_A = 0$ and $Z_B = 0$. Only in these enlarged plots do the differences caused by giving Δr the three values, -1.0, 0.0, and $+1.0$, show up. But these differences are very significant for the question of stability. Extensions of these plots for negative frequencies would yield mirror images of the curves actually plotted reflected about the resistance axis. In the case of Z_A, it is seen that the curve extends into quadrants II and III to the left of the origin, that is, into the region of negative resistance, at very low frequencies when $\Delta r < 0$, and otherwise stays in the region of positive resistance. The curve of Z_B extends into quadrants II and III to the left of the origin for frequencies near 17.6 kHz when $\Delta r < 0$.

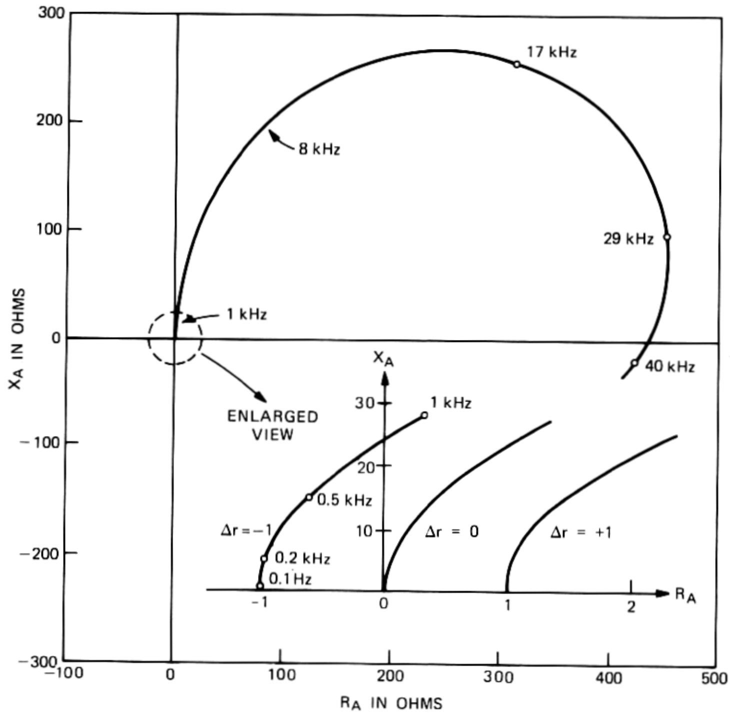


Fig. 13—Reactance vs resistance plot of impedance Z_A as frequency varies. Z_L is impedance of a pair of NIB units and Z_{sc} is the open circuit impedance of a half section of uniform line.

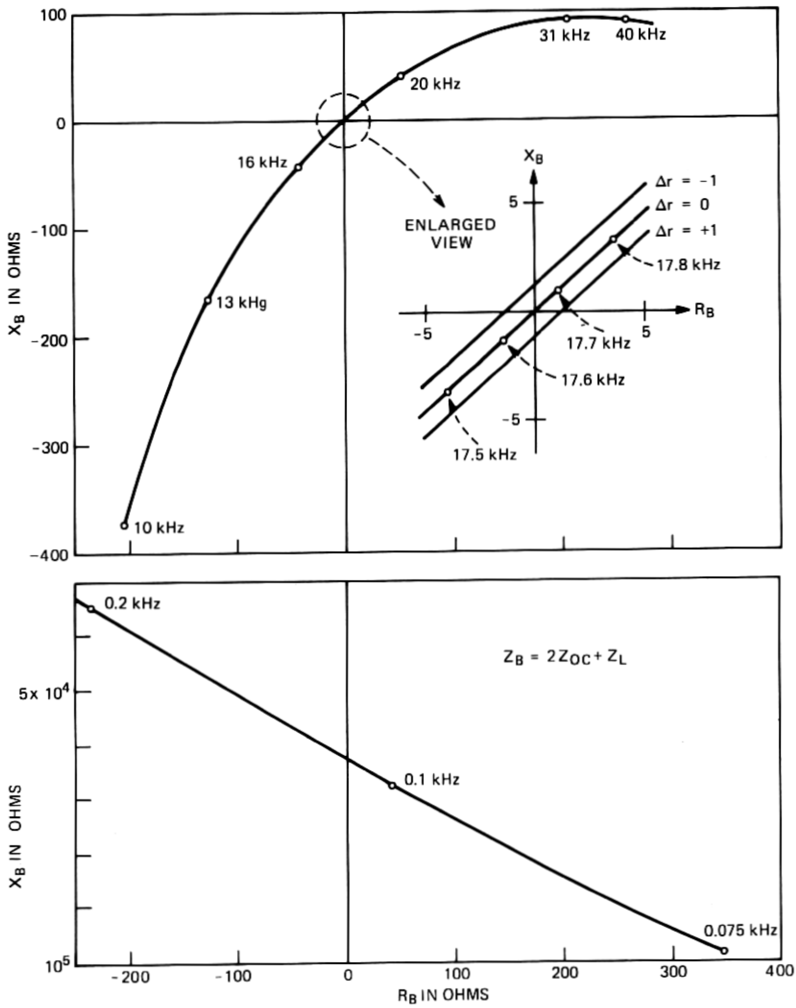


Fig. 14—Reactance vs resistance plot of impedance Z_B in which Z_{oc} is the open-circuit impedance of a half section of uniform line.

Thus, when $\Delta r < 0$, the product $Z_A Z_B$ can be zero at very low frequencies by means of Z_A and at a considerably higher frequency by means of Z_B. The latter is responsible for the zero at 17.6 kHz in mbZ_H, shown in Fig. 12, and arises through a resonance of the reactance of the booster and that of the open-circuit section of line. It is impossible to show the effects of the zero in Z_A on $Z_A Z_B$ in Fig. 12 because

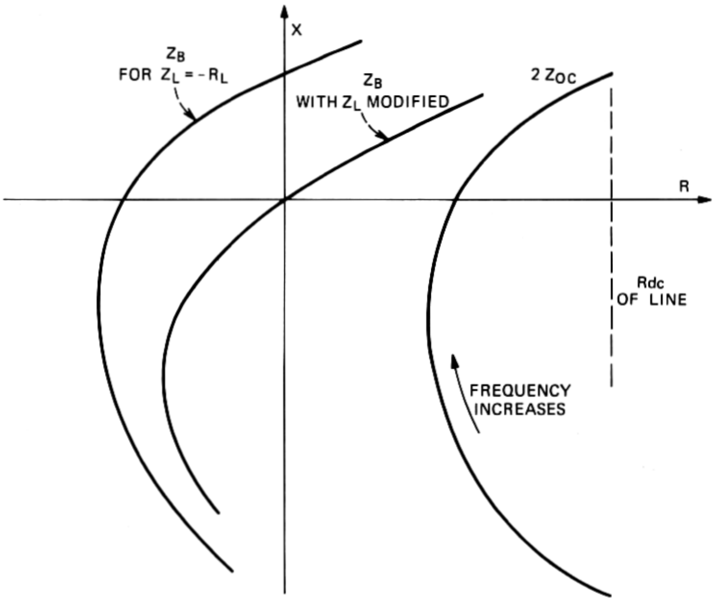


Fig. 15—Reactance vs resistance plots of Z_B and Z_{oc}.

of the very low frequencies (see Fig. 13) at which they occur. Plots of the impedance $Z_H = \frac{1}{2}\sqrt{Z_A Z_B}$ similar to those of Figs. 13 and 14 touch the origin at a very low frequency or loop around it at the zero in Z_B when $\Delta r < 0$. A lumped network, whose impedance may be expressed as a rational function of complex frequency s, would have a similar plot looping around or touching the origin if one of its roots occurred at a complex frequency with positive real part. In these circumstances the network is unstable. From this we may infer that the NIB line can be unstable in the two ways shown when $\Delta r < 0$. The way involving Z_B depends on other circumstances as well. The booster parameters for Figs. 13 and 14 were chosen with respect to the line parameters so that a just-stable condition exists for $\Delta r = 0$. If these parameters were different, e.g., for a larger time constant in the booster, Δr could be negative by a limited amount without instability at the higher frequency. There is no qualification about the zero through Z_A. This always arises in the infinite line for $\Delta r < 0$.

Now we may see why the lumps of negative resistance must be reduced with increasing frequency to avoid instability. Consider the X vs R diagram of Fig. 15 in which Z_B and $2Z_{oc}$ are plotted. If the booster impedance Z_L is a pure resistance, $-R_L$, which cancels all the line resistance, the diagram for Z_B would be that at the left which loops around the origin and would be unstable as we have seen. But if the negative resistance component of Z_L is reduced in magnitude as frequency increases, we may obtain the Z_B curve shown in the middle, which is just stable. R_B must be positive where $X_B = 0$. Reducing $|R_L|$ for this purpose may be done in such a way as to also shape the transmission properties of the NIB section in some desirable way, as indicated above.

These conditions may be summarized by the diagrams of Fig. 16, which are sketches made from calculated values of $Z_A Z_B$ but are not plots of actual data. In the top diagram, two conditions, $\Delta r < 0$ and $\Delta r > 0$, are shown, but the booster capacitance is sufficiently large that a zero in Z_B does not occur. In the bottom left diagram, the same two conditions are shown, but here the booster capacitance is sufficiently small that the zero in Z_B occurs even when $\Delta r > 0$. In the bottom right diagram, no zero occurs even though R_B may be negative at some frequencies. These diagrams may be interpreted as Nyquist diagrams for testing the stability of impedances,[23,25] as the encirclements of the

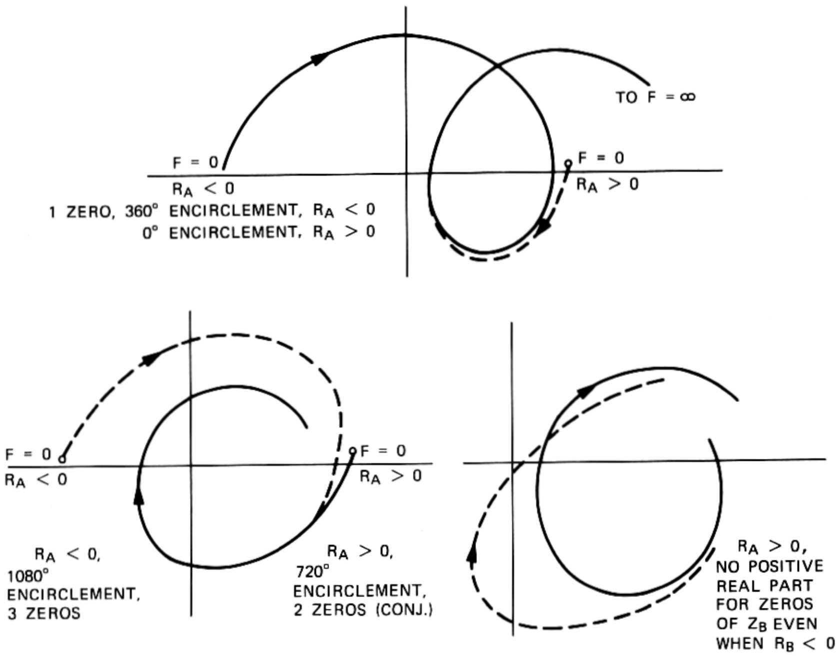


Fig. 16—Nyquist plots of Z_H^2.

origin in the various cases agree or not with the rules for stability, as noted. Only the positive frequency parts of the plots are shown in the diagrams for simplicity.

It can be realized from Figs. 13 and 14 that whether or not a zero occurs in Z_B depends on the details of the booster involved and cannot be determined in general circumstances.

While the above discussion of stability in the infinite NIB line is not a rigorous mathematical one, it is the result of much study, calculation, and experiment and is believed to be a good description of the situation. These conclusions are also in agreement with a computer investigation of the nature of the roots of the input impedance of a NIB section in which the line was approximated by a rational network and the section terminated in various ways.

5.10.2 Finite line

The stability of a NIB line consisting of a finite number of sections is considerably more complicated than the stability of the infinite line because the terminations (if different from Z_H) must be considered also. Because details of the configuration of line and booster, as well as the terminations, are important in these problems, they are more appropriately discussed in connection with particular arrangements. Two particular situations will be described briefly here.

The extreme condition for line stability with any passive terminations whatsoever requires that

$$(R_{11}^2 - R_{12}^2) > 0 \text{ for all frequencies.}$$

In this, R_{11} and R_{12} are the real parts of Z_{11} and Z_{12}, the impedance parameters of the matrix for the cascade of NIB sections under consideration. This condition, first derived by Gewertz,[26] has also been derived by Llewellyn[27] and others. A simple neat derivation is given by Walter H. Ku.[28] To satisfy this condition, the net resistance, Δr, of line and booster must be fairly positive. Thus, what has sometimes been a goal in NIB work, namely, a line with zero net loss and stable for *all* terminations, is an impossibility. Of course, we cannot expect a line equipped with 22-type repeaters to be stable at zero net loss for all terminations either.

On the other hand, when resistance terminations near the low-frequency magnitude of Z_H are used, the line can be stable even when the net resistance, Δr, is negative. This is illustrated in Fig. 17, which shows some stability relations for a four-section NIB line. A number of other configurations and numbers of sections have been studied, but

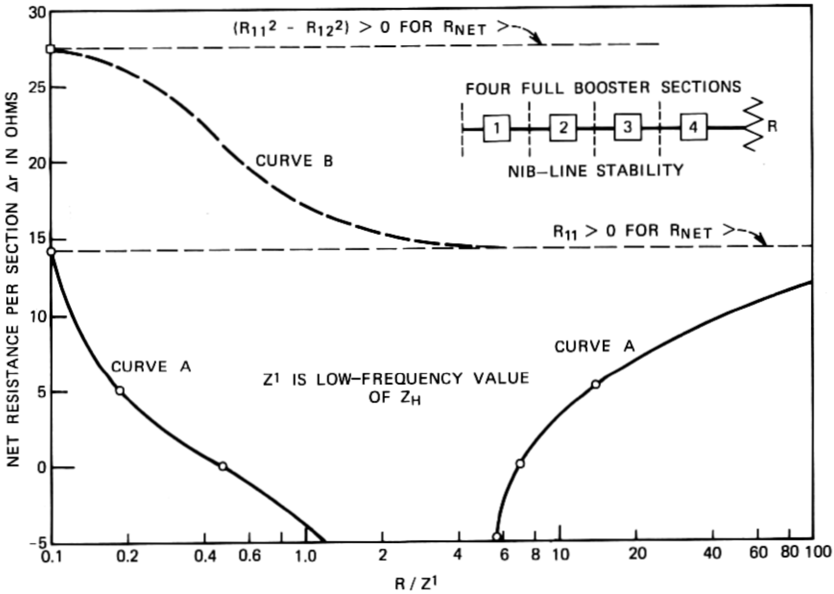


Fig. 17—Boundary curves between stability and potential instability in finite NIB lines terminated with resistance.

only this one is presented here. In this figure, the ordinate is the net resistance Δr per section of line, as already discussed, and the abscissa is the resistance R (normalized) which terminates the line. The two parts of the solid curve A would meet if more negative values of Δr had been plotted. For values of Δr and R that lie above this curve, the input resistance of the line is positive at all frequencies; below it the input resistance is negative at some frequency. This curve is, thus, a boundary between stability above and potential instability below. Instability will exist in fact if, in addition, the input reactance is zero, or if it is cancelled by external reactance at the frequency for which the input resistance is negative. The dashed line at the top of the figure indicates the value of Δr necessary for the line to be stable for any termination. The worst termination is a reactance. If some resistance R is added to this worst reactance, the dashed boundary curve B is obtained, separating stability (above) from potential instability (below). Above the dashed line near the middle of the figure, the NIB line is stable for any resistance termination.

While the curves of Fig. 17 were obtained by means of a rather elaborate computing process, they were verified on a laboratory setup.

Additional verification was obtained from a different computing process. The two-line sections of a single-section NIB line were approximated by lumped-element networks. Then the roots of the characteristic equation of the whole NIB line section, including termination, were found. The conditions of instability or stability, as determined from the nature of these roots, was in agreement with those corresponding to Fig. 17.

It is seen from this figure that, for a small range of terminating resistance, stability prevails even for negative values of the net resistance Δr. However, as the number of sections in the line increases, the amount of negative Δr that can be tolerated becomes smaller, approaching none for the infinite line, as already determined. Actually, for a line of 16 sections, Δr cannot be less than zero.

The boundary curve A of Fig. 17 is one section of a boundary surface over the half-plane of a general terminating impedance.

It should be noted that the $\Delta r = 0$ dividing line discussed here is a result of other NIB parameters having been chosen so that line attenuation is zero and as flat as possible over the appropriate frequency band. If these other NIB parameters had been chosen in other ways, the dividing line could have been at some positive or negative value of Δr.

The NIB configuration in which the line ends with half-NIB units instead of half-line sections requires a considerably higher value of Δr for the same degree of stability.

5.11 Determining the booster impedance Z_L

Two methods that have been used in determining the required booster impedance Z_L are described briefly.

The first method was originally described by K. Bullington,[23] and is a calculation made to determine the booster impedance Z_L that will give some desired propagation factor P. This may be done using the A_{11} matrix coefficient for one NIB section given in Section 5.5, above. If the two forms of this coefficient are equated and solved for Z_L, we get

$$Z_L = 2(Z_o/\sinh 2p)(\cosh P - \cosh 2p),$$

where $2p$ and P are the propagation factors for one section of plain line and boosted line, respectively.

Then a network which approximates the computed impedance Z_L as closely as possible is devised. This may be done formally, by using a "negative-impedance converter," as indicated by John Linvill[17] and others, to convert a positive impedance, $-Z_L$, into the negative impedance, Z_L. Or it may happen that a large part of Z_L is supplied by some special device and only an additional "trimming" network is needed. Small differences between the real network and the calculated Z_L can result in stability problems. These should be investigated with the real network.

The second method is to take a particular network configuration having a negative resistance and vary its parameters and/or add some trimming elements to get desired transmission properties and stability for the resulting NIB line.

The calculated performance curves shown in Figs. 6, 7, 8, and 9 were obtained from NIB sections derived by the second method using the booster circuit (Fig. 23) devised by L. A. Meacham.[18] Two of these curves are replotted in Fig. 18.

To compare with these, one example using the first method is given also. For this, the third-degree maximally flat characteristic was taken as the desired transmission curve. If this is expressed in complex frequency form, i.e.,

$$Y = 1 + 2s + 2s^2 + s^3,$$

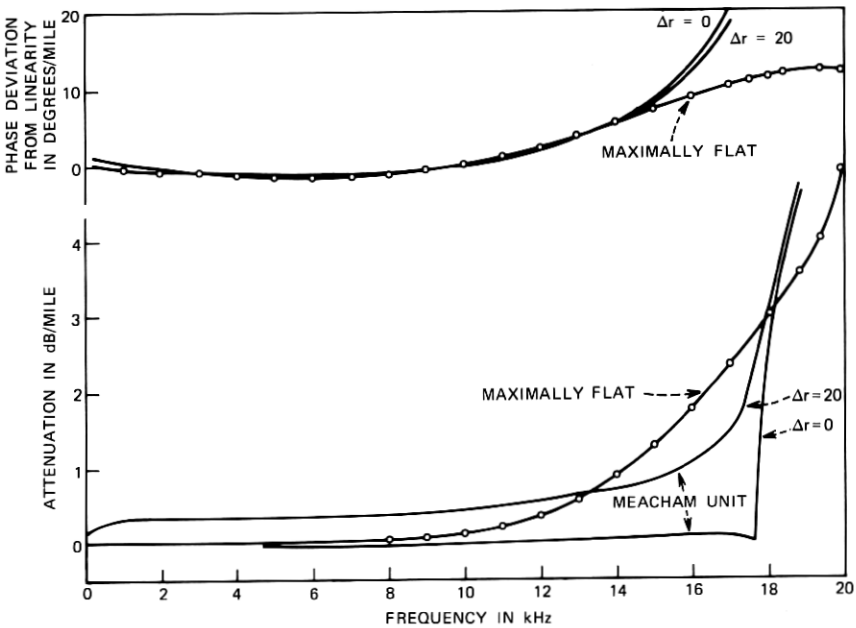


Fig. 18—Attenuation and phase curves resulting from two forms of NIB units in NIB lines.

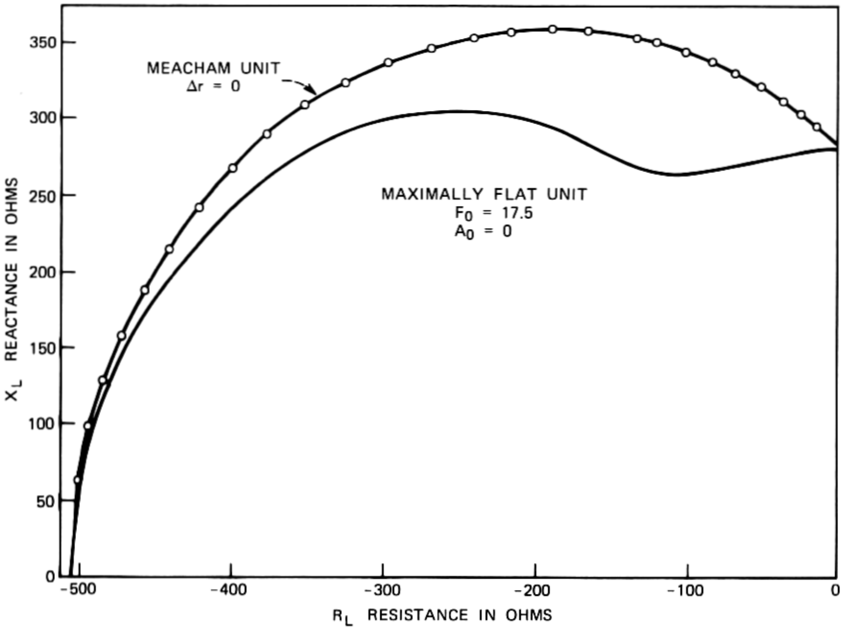


Fig. 19—Reactance vs resistance plots of the two forms of NIB units used in Fig. 18.

then compatible amplitude and phase curves are realized. The transmission curves for the NIB section designed in this way are shown in Fig. 18, designated maximally flat.

Curves, X_L vs R_L, describing the negative impedances Z_L, derived in the two ways, are plotted in Fig. 19.

A good computer program is essential to the successful use of either method. How these two methods have been used in actual work will be mentioned in the section on history, which follows.

5.12 Pulse transmission

In the transmission curves of Fig. 18 for the Meacham unit, the booster parameters were chosen so that the attenuation curve of the NIB line would be as flat as possible. The parameters can also be chosen so that the phase curve of the line is as linear as possible. This makes for a minimum of distortion in pulse transmission as indicated in oscillograms shown in Ref. 18. Photographs of pulse waveforms received through NIB lines formed in both of these ways are shown in Fig. 24. NIB parameters were chosen for flat attenuation for Fig. 24a and b and for the linear phase in Fig. 24c and d.

5.13 Temperature compensation

In most transmission lines, it is desirable that compensation be made for the variation in attenuation caused by temperature variations. Several methods have been tried. When the NIB unit is the Meacham one, the compensation may be accomplished quite readily in many situations, as mentioned in Ref. 18, by using a temperature-sensitive resistor as a part of one of the NIB resistors. This method is particularly simple and effective for underground cable when the temperature-sensitive resistor can be underground also at essentially the same temperature as the cable.

5.14 Interference susceptibility

One problem which may arise with the use of NIB lines, especially when series units are used, is that of interference currents. Even if the line is well balanced so that longitudinal-induced interfering currents do not appear in the metallic path, they must pass through the negative-impedance units. If these currents are large compared with the signal current, they may carry the total current beyond the negative-impedance region of the unit and so cause distortion in the signal (see Fig. 2). To avoid this, a much larger current capacity in the NIB unit than would be necessary for the signal alone must be provided, or operation in a region of high interference must be avoided.

VI. HISTORY OF NEGATIVE-IMPEDANCE BOOSTING

6.1 Crisson

It was mentioned above that the possibility of using negative-resistance boosters to reduce the attenuation of transmission lines was recognized before 1919.[4] But the first theoretical and experimental consideration of the use of such boosters in telephone lines was described by Crisson in his 1931 paper in *The Bell System Technical Journal*.[12] However, it is believed that this consideration was of the occasional or single addition of a negative-resistance unit to a line rather than of their periodic addition.[23]

6.2 Bullington and Edwards

The first appearance of the concept of introducing lumps of negative resistance periodically into a line to obtain a new "uniform" line with desirable transmission properties seems to be in the work of K. Bullington[23] in 1940 and 1941. This effort was stimulated partly by the availability of a quite small thermistor unit which had a negative-resistance region in its voltage-current curve in the audio-frequency range. At that time, it was recognized that the use of a net negative resistance at all frequencies would cause instability. However, it was also thought that if each negative-impedance section could meet the Nyquist stability requirement for impedances (see Refs. 23 and 25, and Fig. 19), any desired number of such sections could be connected in tandem with overall stability. While these point contact units were mechanically fragile, a stable net gain of 2 to 3 dB over the frequency range of 200 to over 3000 Hz was demonstrated on a 54,000-foot section of 19-gauge cable pair. Five negative-impedance series-type units were inserted at 12,000-foot intervals with 3000-foot end sections. The application intended was for voice-frequency toll circuits. Considerable analyses were made[23] that are quite complete and very readable. The discussion covers several of the general properties of Section V, particularly Sections 5.4, 5.5, 5.6, 5.7, and 5.10, that is, reflection effects and cutoff, propagation formulas, image impedance, and stability. The first method (see Section 5.11) of choosing the booster impedance Z_L was used, assuming zero loss and linear phase in the transmission band for a start. The thermistor impedance provided a negative resistance that decreased with frequency and also a positive reactance. Both of these were a major part of the desired Z_L. The remainder was made up by a simple added network.

An interesting suggestion, which does not seem to have been attempted, was made by Bullington for the termination of finite NIB lines. His calculations indicated that if the line were ended with 0.7-line sections instead of 0.5-line sections, the correct terminating impedance would be a very nearly constant resistance plus an inductance, which would be simpler to construct than the image impedance, particularly near the cutoff frequency. This particular project was ended by the more pressing demands of World War II.

6.3 Merrill and the E repeater

The next stage of effort produced the first E-type negative impedance repeaters, in the development of which the work of J. L. Merrill, Jr.[19,20] occupies a large part. Merrill had done some work before the war in pursuing the work begun by Crisson. As a part of this effort, the analysis of K. G. Van Wynen, referred to above,[24] dealt with the impedance and transmission properties of networks equivalent to the boosted sections. While the use of an E repeater introduces a negative resistance in series with the line to reduce its loss, the concept of use is quite different from that of a line with a considerable number of negative-impedance units added periodically along its length. The E repeaters are used only in central offices and usually with only one or two in a given line. Hence, they disturb the uniformity of the line and cause reflections. However, the latest versions of these repeaters have been installed at a great rate and are filling a real need in making gain available for the trunks of the exchange plant. One of the important factors in the simplicity of the use of E repeaters is that the dc continuity of the circuit is maintained and so dc and very-low-frequency signaling currents go through without rearrangement or new apparatus. Also, if the repeater fails, the unrepeatered line is still available. How this comes about may be seen from Fig. 20 which shows a block diagram of the E1 series repeater and how it is connected. The negative impedance is coupled into both conductors of the line by means of the transformer. The negative impedance is generated by a device called a negative-impedance converter which, when a passive impedance Z_n is connected to two of its terminals, presents an impedance of approximately $-Z_n$ at its other two terminals. The "simplicity" factor mentioned above emphasizes the fact that one of the principal differences between conventional amplifying and the use of negative resistance lies in the manner in which the source of energy replenishment for the signal is coupled into the line.

The first model of the E repeaters, the E1, was introduced in 1948.[19] The converter used a twin triode vacuum tube with positive feedback coupling, and the Z_n network could be adjusted for various amounts of negative impedance, or loss reduction. The repeater can be used in coil-loaded lines or nonloaded lines.

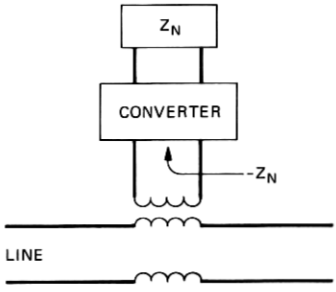


Fig. 20—Method of coupling E-type negative-impedance repeater into transmission line.

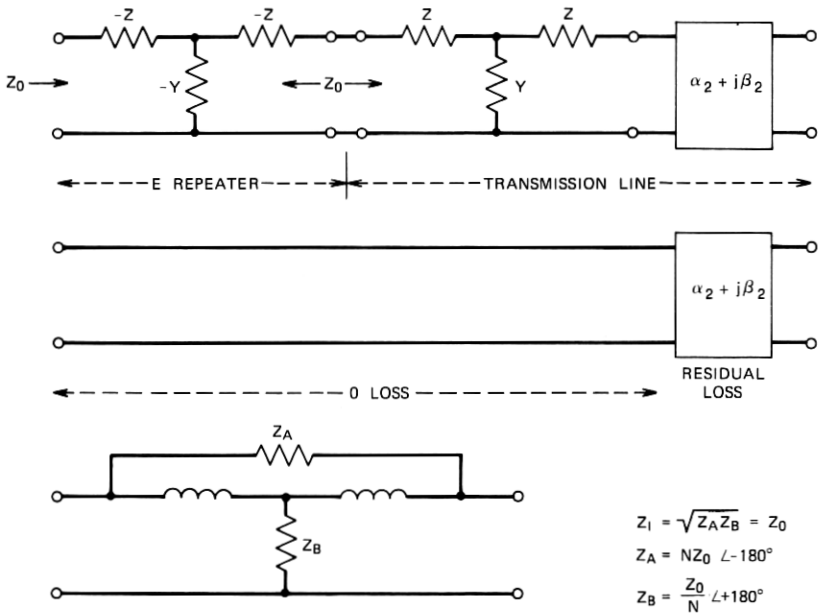


Fig. 21—Equivalent circuits for E-type negative-impedance repeaters.

As discussed above, the introduction of these impedance lumps into a line causes reflections or echoes when the same line Z_o termination is retained. Hence, the principal use of E1 repeaters has been in interoffice trunks rather than in toll trunks, where, because of the longer delays involved, echoes can be much more objectionable. Where there is only one repeater in the line, as in the majority of these cases, the amount of gain and the amount of reflection are directly proportional to the magnitude of the negative resistance used.

As mentioned above, if series and shunt negative impedances of appropriate magnitude are combined properly, not only will losses be reduced, but the characteristic impedance of the line may be matched approximately. This was done in the E23 repeater[20] to obtain much smaller reflections than those that accompany the E1 repeater, and so be usable in toll-connecting trunks. This process is indicated in the diagrams of Fig. 21 taken from Ref. 20. It shows how the negative-impedance repeater cancels a part of the line loss leaving a residual propagation factor of $\alpha_2 + j\beta_2$, while also matching the characteristic impedance Z_o of the line. It also shows that by means of the bridged-T structure, only two instead of three negative-impedance units need be built, and how the two are proportioned. The coils provide coupling into the line as well as being part of the bridged-T structure.

A recent version of the E repeaters, the E6, is like the E23 in that it matches the line, but it is different in some other respects.[21] It was introduced about 1960. It uses transistors instead of vacuum tubes in the basic converter circuit devised by R. L. Wallace and J. G. Linvill.[17] In the physical embodiment, the amount of negative resistance, or gain, adjustment is separated from the line impedance-matching function to simplify the operation of the repeaters.

The question may be asked, since two amplifiers are used to generate the two negative impedances, why not use them to make a 22-type repeater? There seems to be no simple direct answer to this question. In some situations or at some times, one of the methods may provide an advantage over the other and vice versa. For example, when a simple way of providing continuity for dc signaling along with gain is needed, there is an advantage in using a negative-impedance repeater. The advantage may lie with the 22-type repeater in other circumstances.

The E repeaters provide a simple way to add small amounts of gain to exchange trunks. But in their present conception, they do not provide the broad possibilities that can be envisioned with the use of periodically added negative impedances to obtain various desirable transmission characteristics.

6.4 Schott and Wallace

During the period 1953 to 1956, L. O. Schott did some design and experimental work on NIB lines. Two particular situations were considered. In the first, six negative-impedance units were spaced one-third mile apart in 22-gauge cable to give a signal band from about 7 kHz to 100 kHz for possible carrier applications. The negative-impedance unit shown in Fig. 22a was similar in form to the E1 repeater, but with the converter using the transistor circuit of Linvill.[17]

A considerably different negative-impedance-unit circuit, Fig. 22b, suggested by R. L. Wallace, was used in the second situation. Instead of the two like transistors used earlier, one npn, and one pnp were used in the new arrangement. With this change, the coupling transformer was eliminated and the current for powering the unit allowed to flow through the line and units in series. While this requires two units, one for each conductor, it brings a great simplification to the circuit with the opportunity for miniaturization. These units were applied to six 3900-foot sections of 32-gauge cable, providing a transmitting band

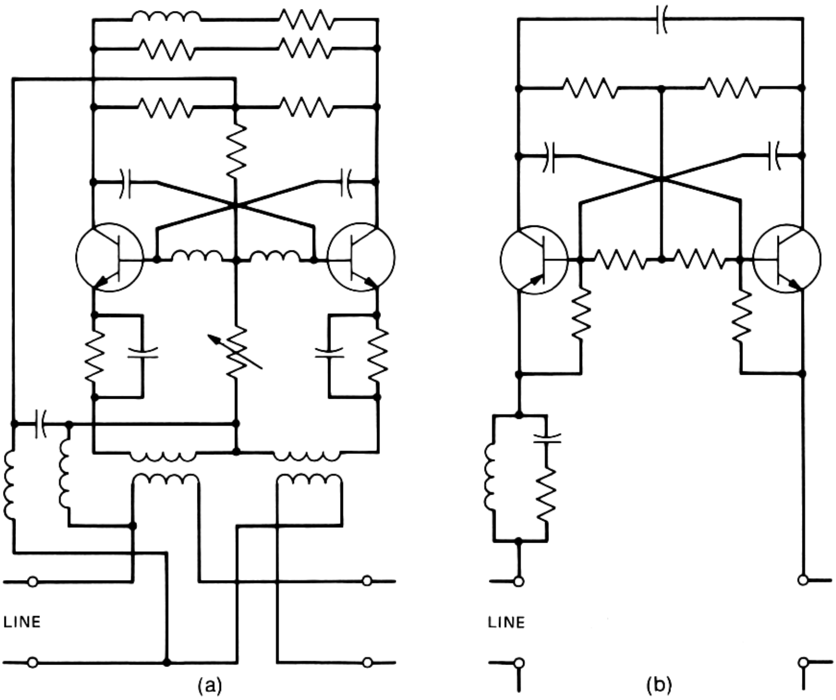


Fig. 22—Circuit diagrams for two negative-impedance converters: (a) Linvill circuit, and (b) Wallace-Schott circuit.

from 0.1 kHz to 7.5 kHz. The *RLC* network in series with the unit was added to provide a margin against instability.

6.5 Hoth and Ross

During the period from about 1954 to 1957, a study in considerable detail was made by D. F. Hoth and W. L. Ross concerning the possibility of using negative-impedance boosting to provide gain in exchange trunks. Their main conclusion was that, though their complete experimental NIB system operated successfully and appeared to offer small cost savings, the E repeaters for voice and the T1 system for carrier seemed to hold more promise in the exchange trunk area. This conclusion appears to have been influenced, in part at least, by some unsatisfactory aspects of the negative-impedance unit used.

This unit was an avalanche transistor with one resistor and one capacitor connected externally. The negative resistance is generated by the avalanche discharge process within the transistor and is the series type. Some of the unsatisfactory properties were lack of uniformity of the transistors, nonlinearity of the negative resistance developed, and comparatively high voltage required at each unit.

A real system was set up consisting of three 9000-foot sections of 26-gauge cable, with provision being made for signaling, powering, and matching of the line to office impedance. The line had an image impedance of about 300 ohms and a sharp cutoff near 8 kHz. The low image impedance is a distinct advantage for reduced crosstalk. Regulation of net loss was by means of a 40-Hz pilot tone. The change of amplitude of this tone in response to cable-temperature change was used to shift the dc line current, which in turn would change the amount of negative resistance because of the nonlinearity of the NIB units.

The shape of the booster impedance Z_L was chosen by the first method described in Section 5.11 and the actual unit made to approximate this fairly well over the transmission band. Instead of giving each NIB section sufficient loss so that the system would be stable with any passive termination, they designed the sections for zero net loss and then put terminating networks at each end so that the system was stable for any passive termination beyond these networks. This is equivalent to restricting the termination of the actual NIB sections to lie in a certain impedance range.

Considerable analysis was done on the problem of stability, particularly on the conditions required for stability with any passive termination.

6.6 Other projects and general comments

While there have been quite a number of projects, beyond these listed above, in which the use of negative impedances in transmission lines has been studied, the list is representative except for the work of L. A. Meacham dealt with separately in Section VII.

A handicap in many of the projects was the goal of a line with zero net loss which at the same time could tolerate terminations of any passive impedance whatsoever. We have seen that this is an impossible goal. The closer we approach zero net loss, the more restricted the range of terminating impedance becomes until, at zero, termination must be perfect except in the few special situations indicated in Section 5.10.2. But similar restrictions apply to other ways of reducing line loss also. It was shown clearly in the early years of repeatered lines[8–10] that irregularities in the line and imperfect terminations severely limited the amount of gain that could be used in a line. A paper, "Some Fundamental Properties of Transmission Systems,"[27] was written by F. B. Llewellyn in part to show that the restrictions that are necessary for transmission circuits to be stable are essentially the same whether unilateral amplifiers or negative impedances are used to reduce the losses.

VII. MEACHAM'S WORK

In 1963, a new look at the use of negative-resistance lumps placed uniformly in a line was begun.

7.1 Meacham NIB unit

The point of departure here was a fairly simple realization of a negative-resistance circuit. The generation of negative resistance was not accomplished in a single device, as in some previous situations, but by the cross coupling of two transistors in a simple way. Though derived independently, it is similar to the Wallace circuit of Fig. 22b in that it uses an npn and a pnp transistor. But the Meacham circuit does not use coupling capacitors. Two advantages come from this: the negative resistance is usable down to dc, and the physical bulk of the capacitors is eliminated. The circuit could be realized also by a single pnpn device. Later in the investigation, the negative resistance was made highly linear by the addition of two diodes, and the transmission shaping possibilities were enlarged by adding a resistance and capacitance in shunt.[18] The Meacham negative-impedance circuit and its voltage-current characteristic are shown in Fig. 23. These negative impedances were used for the transmission curves of Figs. 6, 7, 8, and 9. And while these were plotted from calculated data, measured performances of the real physical lines are always in very good agreement with those calculated. Some work was done to demonstrate that the NIB units could be built in very small integrated-circuit form.

7.2 Field tests

Two field tests were successfully completed and numerous laboratory demonstrations made. The lines constructed for these tests provided excellent transmission with low phase and nonlinear distortion for a variety of signals. Lines in both tests were about 32 miles long. One was a single circuit in a suburban exchange area, part aerial cable and part underground cable.[18] The other test involved three circuits using three order wire pairs in the L4 spur cable from Netcong to Newark, New Jersey. Extensive measurements of these were made over a period of about a year. They were operated at a net loss of about 3 dB and had very low noise and a satisfactory return loss. The longitudinal interfering currents were negligible. Signals from a 202D Data Set system (frequency-shift keying at 1200 bits/second) with its

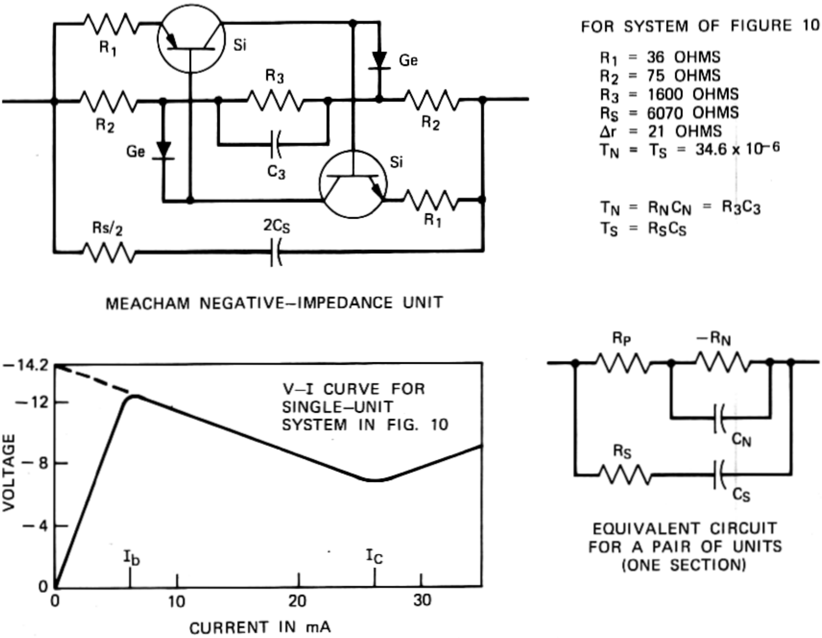


Fig. 23—Circuit diagram for Meacham negative-impedance unit with equivalent circuit and voltage-current curve.

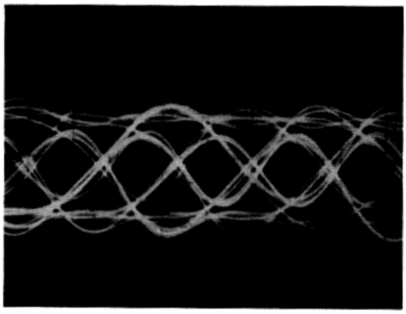
equalizers removed were sent through one of these lines having a bandwidth of about 5 kHz with no noticeable distortion (see Fig. 24a). With a slight rearrangement at the terminals, baseband rectangular binary pulse signals at a 12-kHz rate were easily transmitted, as may be seen from the eye diagram of Fig. 24b. These lines were designed for essentially flat attenuation curves. If the NIB unit parameters are chosen to have the most linear phase curve instead, pulse transmission is even better. This is shown in Figs. 24c and 24d for such a line consisting of nine 6000-foot sections. Figure 24c is the response to a single 50-μs rectangular pulse and Fig. 24d is the response to a pseudo-random-pulse train at 14 Kbauds and 16 levels.

Suitable means were worked out for precise measuring of the NIB units and for measuring the overall net resistance of the operating line.

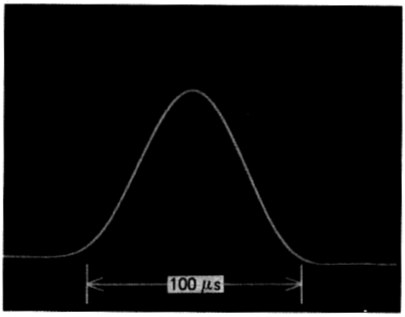
7.3 Papers published

Two papers have been published on parts of the work, one by L. A. Meacham[18] and the other by A. L. Hopper.[22] The purposes of the present paper are: (*i*) to present a comprehensive general view of the

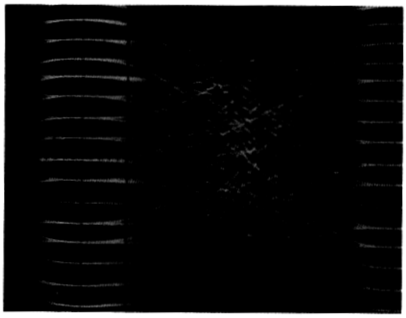
(a) 202D DATA SET SOURCE EQUALIZERS REMOVED 1.2 kBIT/s



(b) BASEBAND TRANSMISSION - 12 kHz BINARY



(c) RESPONSE TO SINGLE 50 μs RECTANGULAR PULSE



(d) RESPONSE TO PSEUDO-RANDOM TRAIN — 14 kBAUD,16 LEVELS

Fig. 24—Pulse transmission in NIB lines. Parameters chosen for flat attenuation in (a) and (b) and for linear phase in (c) and (d).

properties of NIB lines (Section V), *(ii)* to describe the earlier work on negative-impedance boosting, and *(iii)* to present additional results not given in the two previous papers, notably the work on stability outlined in Section 5.10. The author of the present paper began work with this project late in 1968.

The paper by Meacham is a general one describing the negative-impedance unit, how it is used in the line, and the improved transmission which results in various situations. The one by Hopper is on the design of NIB lines following the viewpoint mentioned above, namely, varying the parameters of the NIB unit to achieve desired transmission properties in a NIB line made up of these units added periodically to a particular uniform line.

In all the work on this project, the approach was not to try to build a NIB unit to have a certain impedance Z_L, but to vary the parameters of this particular NIB circuit configuration to obtain certain desirable transmission properties of the combination of transmission line plus booster. This provides advantages as well as restrictions.

7.4 Design of NIB lines

The use of the high-speed digital computer was essential in carrying out this process. Details of this method and design results for a number of uniform lines and various bandwidths are given in Hopper's paper. The curves of Fig. 4 on the relation between spacing and bandwidth are one result of this work. These designs are for zero net loss and are just stable. Some backing away from this condition is necessary for practical operation. The transmission objective in most of these examples is as flat an attenuation curve as possible. However, it is possible to make a somewhat different choice of parameters to get as nearly linear phase curve as possible. Some work, not published, was also done in choosing parameters so that some measure of distortion in pulse transmission would be a minimum. In this work of designing NIB lines, the two NIB units in the two wires of one section of line are represented by the equivalent circuit shown in Fig. 23.

7.5 Choosing element values for the NIB unit

The designs for the lines are expressed in terms of the parameters of the equivalent circuit. How these are translated into the element values of the real physical circuit (see Fig. 23) is described in unpublished notes by D. M. Chapin and is outlined below.

An important parameter of the Meacham NIB unit is the voltage V_K, which is the difference between the emitter-base voltage for a transistor and the voltage drop across its associated diode. The diodes were added to counteract the nonlinearity of the transistor emitter-base junction. Most of the units were built with silicon transistors and germanium diodes that provide a value of about 0.5 V for V_K. For smaller values of V_K, the current range over which the negative resistance appears is less, and disappears for $V_K = 0$. The circuit will operate as a negative resistance without the diodes, but will not be as linear.

The first element to be chosen is the resistor R_2. There are three ways this may be chosen, but the most direct is the following:

$$R_2 = \frac{\alpha}{2\alpha - 1}\frac{R_p}{2} + \frac{V_K}{(2\alpha - 1)\mathrm{D}},$$

where α is the α of the transistor, V_K the parameter mentioned above, R_p is the positive resistance in the equivalent circuit, and D is the current range of the negative resistance $I_c - I_b$ in the V-I curve of Fig. 23. The parameter R_p is

$$R_p = \frac{4R_1R_2}{R_1 + R_2},$$

and, as mentioned above, applies to the two units, whereas the element values are being derived for a single unit. From studies of the NIB line process, a very good value for R_p which is used in most designs is

$$R_p = 100 \text{ ohms}.$$

The above choice for R_2 is nearly the same as that which would have been obtained to minimize the total dc voltage drop across units and line per section. Or it could have been done by

$$R_2 = V_K/I_b,$$

where I_b is the value of current at which the negative resistance begins in the V-I curve.

Then, R_1 is determined by

$$R_1 = \frac{R_2R_p}{4R_2 - R_p}.$$

It may be shown that

$$\frac{R_n}{2} = R_3 \frac{(2\alpha - 1)R_2 - R_1}{R_1 + R_2}$$

and

$$C_n = \frac{C_3}{2} \frac{R_1 + R_2}{[(2\alpha - 1)R_2 - R_1]}.$$

If R_L is the total (both conductors) resistance of the nonboosted line per section, we have $\Delta r = R_p - R_n + R_L$ with which to determine the required R_n, where Δr is the net low-frequency resistance per section as used in Section 5.10. From these we have for R_3

$$R_3 = \frac{(R_p + R_L - \Delta r)(R_1 + R_2)}{2[(2\alpha - 1)R_2 - R_1]}.$$

If the circuit is operated without the diodes, V_K is replaced by V_{eb}.

The choice of R_s and C_s is discussed in Hopper's paper, Ref. 22. This completes the translation of the equivalent-circuit parameters into the elements of the real NIB circuit.

To keep the net resistance Δr close to its desired value, it is necessary that negative resistance $(R_n - R_p)/2$ be adjusted quite precisely.

The simplest way to do this is to build the units with elements of reasonable tolerances except for R_3. Then, with a standard variable resistor connected in place of R_3, this variable resistor is adjusted until the measured value of $(R_n - R_p)/2$ reaches its desired value. The variable resistor is then replaced by a fixed one of this value.

The measurement is simply made by superimposing a small low-frequency current on the bias current, which is $(I_c + I_b)/2$. A standard variable positive resistance in series with the NIB unit is then adjusted for a null in voltage across the combination which indicates equality of positive and negative resistance.

7.6 Temperature compensation

Compensation for temperature variations in the lines of the two field tests was made by replacing part of the fixed resistor R_3 in Fig. 23 by a resistor wound with a nickel alloy having a suitable positive temperature coefficient. Thus, the negative resistance of the NIB unit, R_n, increases as the cable resistance R_L increases. This method is particularly effective and simple when the temperature-sensitive resistor can be in the same temperature environment as the line.

VIII. CONCLUSIONS

Knowledge about the properties of NIB transmission lines, which are lines into which negative-impedance units have been inserted periodically, has been advanced considerably by this latest investigation. The distinctive properties of these lines stem largely from the way the sources of energy replacement, that is, the negative-impedance units, are coupled into the line. The cost and bulk of coil or resistance hybrids, necessary when unilateral amplifiers are used, are avoided and bilateral transmission is provided directly. The reflections caused by the insertion of these units into the line cancel each other up to a certain cutoff frequency when the units are uniformly spaced and the line is terminated approximately in its image impedance. There is an approximately inverse relationship between this cutoff frequency and the spacing of the units.

Two separate field test lines, each about 32 miles long, were constructed and operated for some time. One of these was in a suburban exchange area and used partly aerial and partly underground cable. The other used order wires in the L4 spur cable (underground) between Netcong and Newark, New Jersey. Both of these demonstrated, in agreement with analysis, that these NIB lines can be operated stably, with a margin, at a net loss of about 2 dB and that they can provide high-quality transmission with low-phase and nonlinear distortion for speech or pulse signals. Because the NIB units are effective down to dc, baseband transmission of pulse signals is possible and was demonstrated on the test lines.

Two problems have held back the use of these lines. The first has to do with the effect of temperature variations of the transmission lines. A simple method for the correction of these variations which has been effective in the test lines is not universally applicable. It requires the temperature of the booster to be simply related to that of the line. The second is the susceptibility of the line to certain kinds of interfering currents and is more fundamental, as no satisfactory way to overcome it has been found. In well-balanced conventional transmission lines, the effect of induced longitudinal interfering currents is negligible. In NIB lines, these currents must flow through the NIB units and if the interfering currents are large compared with the signal currents, they may cause the total dynamic current to go outside the region of negative resistance and, hence, cause large distortion. This effect limits the use of NIB lines to environments where interfering currents are small. The interfering currents were negligible in the two test lines, and there are probably many such situations. Nevertheless, it is apparent that the use of the negative-impedance method to reduce attenuation is worthy of further consideration in certain situations, as for example in those which are similar to the Netcong-Newark field test, even though because of the above difficulties the method may not be suitable for use in all situations.

IX. ACKNOWLEDGMENT

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REFERENCES

1. T. Shaw, "The Conquest of Distance by Wire Telephony; A Story of Transmission Development From the Early Days of Loading to the Wide Use of Thermionic Repeaters," B.S.T.J., *23*, No. 4 (October 1944), pp. 337–421.
2. T. Shaw, "Evolution of Inductive Loading for Bell System Telephone Facilities," B.S.T.J., *30*, Nos. 1, 2, 3, and 4 (January, April, July, and October 1951).
3. G. A. Campbell, "On Loaded Lines in Telephonic Transmission," Phil. Mag., Sixth Series, *5* (January-June 1903), pp. 313–330. Also in *Collected Papers of George A. Campbell*, a commemorative volume published by AT&T in 1937.
4. B. Gherardi and F. B. Jewett, "Telephone Repeaters," Trans. AIEE, *38* (1919), pp. 1287–1345.

5. A. B. Clark, "Telephone Transmission Over Long Cable Circuits," B.S.T.J., *2*, No. 1 (January 1923), pp. 67–94.
6. A. B. Clark, "The Development of Telephony in the United States," Trans. AIEE, Part I, Com. and Elec., *71* (November 1952), pp. 348–363.
7. F. B. Jewett, "Dr. George A. Campbell," and "Dr. Campbell's Memoranda of 1907 and 1912," B.S.T.J., *14*, No. 4 (October 1935), pp. 553–572.
8. G. Crisson, "The Limitation of the Gain of Two-Way Telephone Repeaters by Impedance Irregularities," B.S.T.J., *4*, No. 1 (January 1925), pp. 15–25.
9. G. Crisson, "Irregularities in Loaded Telephone Circuits," B.S.T.J., *4*, No. 8 (October 1925), pp. 561–585.
10. L. G. Abraham, "Certain Factors Limiting the Volume Efficiency of Repeatered Telephone Circuits," B.S.T.J., *12*, No. 4 (October 1933), pp. 517–532.
11. L. G. Abraham, "Circulating Currents and Singing on Two-Wire Cable Circuits," B.S.T.J., *14*, No. 4 (October 1935), pp. 600–631.
12. G. Crisson, "Negative Impedances and the Twin 21-Type Repeater," B.S.T.J., *10*, No. 3 (July 1931), pp. 485–513.
13. E. W. Herold, "Negative Resistance and Devices for Obtaining It," Proc. IRE, *23*, No. 10 (October 1935), pp. 1201–1223.
14. A. W. Hull, "Description of the Dynatron," Proc. IRE, *6*, No. 1 (February 1918), pp. 5–35.
15. K. D. Smith, "The IMPATT Diode—A Solid-State Microwave Generator," Bell Laboratories Record, *45*, No. 5 (May 1967), pp. 144–148.
16. R. S. Englebrecht, "Bulk Effect Devices for Future Transmission Systems," Bell Laboratories Record, *45*, No. 6 (June 1967), pp. 192–198.
17. J. G. Linvill, "Transistor Negative Impedance Converters," Proc. IRE, *41*, No. 6 (June 1953), pp. 725–739.
18. L. A. Meacham, "Negative Impedance Boosting," B.S.T.J., *47*, No. 6 (July-August 1968), pp. 1019–1041.
19. J. L. Merrill, Jr., "Theory of the Negative Impedance Converter," B.S.T.J., *30*, No. 1 (January 1951), pp. 88–109.
20. J. L. Merrill, Jr., A. F. Rose, and J. O. Smethurst, "Negative Impedance Telephone Repeaters," B.S.T.J., *33*, No. 5 (September 1954), pp. 1055–1092. J. Gammie and J. L. Merrill, Jr., "Stability of Negative Impedance Elements in Short Transmission Lines," B.S.T.J., *34*, No. 2 (March 1955), pp. 333–360.
21. A. L. Bonner, J. L. Garrison, and W. J. Kopp, "The E6 Negative Impedance Repeater," B.S.T.J., *39*, No. 6 (November 1960), pp. 1445–1504.
22. A. L. Hopper, "Computer-Aided Analysis and Design of Negative Impedance Boosted Transmission Lines," IEEE Trans. on Comm. Technology, *19*, No. 4 (August 1971), pp. 501–516.
23. K. Bullington, "U. S. Patent 2,360,932, Negative Resistance Loading," April 25, 1942.
24. K. G. Van Wynen, unpublished notes, 1950.
25. H. W. Bode, *Network Analysis and Feedback Amplifier Design*, Chapter 8.
26. C. Gewertz, "Synthesis of Networks," J. Math. and Phys., M.I.T., January 1933.
27. F. B. Llewellyn, "Some Fundamental Properties of Transmission Systems," Proc. IRE, *40*, No. 3 (March 1952), pp. 271–283.
28. W. H. Ku, "A Simple Derivation for the Stability Criterion of Linear Active Two-Ports," Proc. IEEE, *53*, No. 3 (March 1965), pp. 310–311.