THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 54, No. 7, September 1975
Printed in U.S.A.

The Effect of Longitudinal Imbalance on Crosstalk

By G. MILLER

(Manuscript received December 30, 1974)

Multipair cables are carefully constructed to control the coupling between wire pairs to prevent excessive crosstalk. Several types of coupling modes exist between pairs, but the principal effort is devoted to controlling the "metallic-to-metallic" coupling mode because the coupling loss in this path is the most important in ordinary telephonic use of the cable. Over a half-century ago, Campbell showed that crosstalk behavior of this mode could be characterized by measuring a simple function called capacitance unbalance. This paper shows that at voice frequencies the crosstalk characteristics of the "longitudinal-to-metallic" mode can be predicted by measuring a second similar function of the same parameters that are contained in the capacitance unbalance. With the aid of these two functions, it is shown how the longitudinal balance of terminal equipment connected to a cable pair affects crosstalk. It is further shown that a longitudinal balance of approximately 40 dB or more is necessary for any station or terminal equipment used in the telephone network so that it will not significantly increase the small amounts of crosstalk inherent in the careful cable design. Also, a limitation is established for the maximum longitudinal voltages at voice frequency that can be applied without noticeably increasing crosstalk and noise in other cable pairs. This limitation is approximately 40 dB more restrictive than the tariff limitations for metallic voltages.

I. INTRODUCTION

A multipair cable consists of many insulated but unshielded conductors within a protective conducting sheath. The individual conductors are used to form circuits. In one configuration, called a metallic circuit, two conductors are paired and form the circuit. Signals are applied between them. This is called metallic excitation of the circuit, and the signal is said to propagate in the metallic mode. In another configuration, called a longitudinal circuit, two conductors are paralleled and these, plus the conducting cable sheath, form the circuit. Signals are applied between the paralleled conductors and the conducting cable sheath, which is grounded. This is called longitudinal excitation of the circuit, and the signal is said to propagate in the longitudinal mode. It is also possible for one wire pair to be used for both circuits simultaneously and, consequently, for one wire pair to be simultaneously excited in the metallic and longitudinal modes of propagation. This happens when the terminal equipment is longitudinally unbalanced, as will be explained.

Because the conductors are not shielded and are in close proximity to each other, electromagnetic fields generated by current flowing through the conductors cause energy to be coupled from one circuit to another. This is called crosstalk and is undesirable, since it may cause noise in other circuits that can impair the performance of digital and analog systems, or even be intelligible speech that is overheard and leads to loss of privacy.

Crosstalk cannot be eliminated, but several things can be done to reduce it, that is, to increase the crosstalk loss between circuits. First, metallic circuits are used rather than longitudinal circuits, because it was found by experience that the crosstalk loss between two metallic circuits is generally greater than the loss between two longitudinal circuits or between a longitudinal and a metallic circuit. Second, adjacent conductors are paired and often twisted and are used for the metallic circuits because they are less susceptible to inductive noise and the crosstalk loss between twisted pairs is generally greater than between nontwisted pairs. Twisting reduces crosstalk by assuring that each pair of the cable is exposed to opposing couplings by transposing its conductors relative to the disturbing pair. Third, the terminal equipment at both ends of a pair should be longitudinally balanced, i.e., have impedance symmetry with respect to ground, because longitudinal imbalance has the effect of producing longitudinal excitation which consequently can increase crosstalk. Finally, the cable pairs are also constructed to have longitudinal impedance symmetry for the same reason.

Since metallic circuits are usually used, and both cables and terminal equipment are usually constructed to be longitudinally balanced, most crosstalk studies to date have concentrated on what is called metallic-to-metallic crosstalk, i.e., crosstalk between balanced metallic circuits. Much less is known analytically about the crosstalk loss between longitudinal circuits or between a longitudinal and a metallic circuit. For example, to explain crosstalk between balanced metallic circuits, Campbell[1] assumed that all circuits within a cable were longitudinally symmetrical, that the pairs were excited metallically, and that, consequently, the applied metallic signal would not excite any longitudinal voltage in the disturbing or disturbed pairs. Thus, crosstalk would be due to cable characteristics alone. Campbell was then able to show that crosstalk at low or voice frequencies, where inductive coupling is negligible, was very nearly proportional to the capacitance unbalance, which is a function of the four interwire capacitances between two cable pairs, and is now used as a measure of quality of a cable with regard to crosstalk performance.

In another study, Foschini[2] developed an accurate transmission model of cable systems for computing crosstalk which is an extension of Campbell's work. He too assumed longitudinal symmetry and showed that crosstalk coupling losses between metallic circuits can be predicted quite accurately from Campbell's capacitance unbalance. Although his results are valuable in the study of crosstalk for the metallic mode of propagation, they, as well as Campbell's results, do not consider the effects of terminal imbalance on crosstalk loss.

The objectives of this paper are to extend the results of Campbell and Foschini by first removing the constraints of metallic circuits, terminal balance, and pair symmetry; and to construct a model to permit calculating the crosstalk loss between pairs as a function of terminal balance and pair symmetry. The model is used to show why terminal imbalance can greatly increase crosstalk by causing longitudinal excitation of a cable pair and, consequently, why limitations must be imposed on the longitudinal balance of terminal equipment and on the direct application of longitudinal signals. These objectives are accomplished by showing, through numerical solutions and experimental results, that longitudinal excitation couples energy into adjacent wire pairs with much less loss than does metallic excitation.

The paper is divided into four sections. First, the important results on longitudinal balance and longitudinal voltage restrictions are given. Next, the model of crosstalk between two wire pairs in a cable is analyzed using transmission line equations. This model is used to derive a new set of crosstalk coupling coefficients that can be used to relate the crosstalk loss between two metallic circuits, a longitudinal and a metallic circuit, and two longitudinal circuits. Third, average values for these coupling coefficients for a typical cable are obtained, derived from measured characteristics. Using the coupling coefficients the predicted increase in crosstalk resulting from longitudinal excitation is compared with direct measurements of the increase made on another cable. Finally, restrictions on longitudinal balance and longitudinal voltages are established.

II. RESULTS

2.1 Requirement on longitudinal balance

For application to crosstalk performance, it is appropriate to define a longitudinal balance* of terminal equipment as

$$\mathrm{BAL}_{\mathrm{M-L}}(f) = 20 \log_{10} \left| \frac{e_M(f)}{e_L(f)} \right| ,$$

where e_L is the longitudinal voltage produced when a *metallic* voltage e_M is applied at any frequency f. The subscript "M − L" means the conversion from a metallically applied voltage to a longitudinal voltage. This paper shows that a balance of approximately 40 dB or more in the voice frequency region is required for any terminal device to ensure that the level of crosstalk that already exists in the network will not be significantly increased. This requirement is based on measurements of the near-end crosstalk at 1000 Hz of cable with a balanced and unbalanced termination. It is assumed that any metallic signal applied to the telephone network does not exceed the power level specified in Ref. 3. Longitudinal and metallic voltages are defined in Section 3.3.

2.2 Restriction of longitudinally applied voltages

Crosstalk coupling losses decrease with increasing frequency and hence voltage restrictions are frequency dependent. Figure 1 shows the limitations on applied longitudinal voltages established so as to increase the crosstalk energy already present in the telephone network by no more than about 1 dB.

2.3 Derivation of crosstalk coupling coefficients

Three new capacitive coupling coefficients have been derived that can be used with a simple but reliable computation method to predict the degradation in crosstalk performance for a particular cable when any of its terminations are unbalanced. These coefficients are given in Table I. The coefficients are defined in eqs. (10) to (13), and the interpair capacitances given in the formulas are the capacitances between the pairs shown in Fig. 2.

III. COUPLING BETWEEN TWO WIRE PAIRS IN A CABLE

3.1 Transmission line model

Figure 2 models two wire pairs within a cable of length l. The following assumptions about a cable are made to construct this model:

* A second type of balance for noise immunity purposes is a separate but important consideration for good telephone network performance. It is defined in Section 4.2. However, crosstalk does not enter into establishing its restrictions.

(*i*) The impedance and admittance per unit length of each wire pair, the admittances to ground per unit length, etc., are constant.

(*ii*) The conductance to ground and between wire pairs is negligible, i.e., the admittances are purely capacitive.

(*iii*) The inductive coupling between pairs is negligible at voice frequencies.

(*iv*) The impedances per unit length of all wire pairs are equal.

Let the admittances per unit length between the two circuits be Y_{13}, Y_{23}, Y_{24}, and Y_{14} connected between conductors 1 and 3, 3 and 2, 2 and 4, and 4 and 1, respectively, where conductors 1–2 form one twisted wire pair and 3–4 form the other pair. The impedances per unit length of the four wires are Z_1, Z_2, Z_3, and Z_4 and the admittances of wires to ground are Y_{1g}, Y_{2g}, Y_{3g}, and Y_{4g}. The admittances per unit length of the wire pairs are Y_{12} and Y_{34}, and the voltages and currents are labeled in the figure.

Consider a differential section of the model of length Δx. It is readily seen that the following eight current-voltage relationships hold for this differential section:

$$V_1(x + \Delta x) = V_1(x) - I_1(x)Z_1\Delta x \tag{1a}$$

$$V_2(x + \Delta x) = V_2(x) - I_2(x)Z_2\Delta x \tag{1b}$$

$$V_3(x + \Delta x) = V_3(x) - I_3(x)Z_3\Delta x \tag{1c}$$

$$V_4(x + \Delta x) = V_4(x) - I_4(x)Z_4\Delta x \tag{1d}$$

$$\begin{aligned} I_1(x + \Delta x) = I_1(x) &- \{V_1(x) - V_2(x)\}Y_{12}\Delta x \\ &- V_1(x)Y_{1g}\Delta x - \{V_1(x) - V_3(x)\}Y_{13}\Delta x \\ &- \{V_1(x) - V_4(x)\}Y_{14}\Delta x \end{aligned} \tag{1e}$$

$$\begin{aligned} I_2(x + \Delta x) = I_2(x) &- \{V_2(x) - V_1(x)\}Y_{12}\Delta x \\ &- V_2(x)Y_{2g}\Delta x - \{V_2(x) - V_3(x)\}Y_{23}\Delta x \\ &- \{V_2(x) - V_4(x)\}Y_{24}\Delta x \end{aligned} \tag{1f}$$

$$\begin{aligned} I_3(x + \Delta x) = I_3(x) &- \{V_3(x) - V_4(x)\}Y_{34}\Delta x \\ &- V_3(x)Y_{3g}\Delta x - \{V_3(x) - V_2(x)\}Y_{23}\Delta x \\ &- \{V_3(x) - V_1(x)\}Y_{13}\Delta x \end{aligned} \tag{1g}$$

$$\begin{aligned} I_4(x + \Delta x) = I_4(x) &- \{V_4(x) - V_3(x)\}Y_{34}\Delta x \\ &- V_4(x)Y_{4g}\Delta x - \{V_4(x) - V_2(x)\}Y_{24}\Delta x \\ &- \{V_4(x) - V_1(x)\}Y_{14}\Delta x. \end{aligned} \tag{1h}$$

Dividing through by Δx, taking the limit as Δx approaches zero and recognizing the definition of the derivative, using assumption 3, and writing the resulting eight equations in matrix form, we obtain eq. (2).

$$
\begin{bmatrix} dV_1/dx \\ dV_2/dx \\ dV_3/dx \\ dV_4/dx \\ dI_1/dx \\ dI_2/dx \\ dI_3/dx \\ dI_4/dx \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & -Z_1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -Z_2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -Z_3 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -Z_4 \\
-(Y_{1g} + Y_{12} + Y_{13} + Y_{14}) & Y_{12} & Y_{13} & Y_{14} & 0 & 0 & 0 & 0 \\
Y_{12} & -(Y_{2g} + Y_{12} + Y_{23} + Y_{24}) & Y_{23} & Y_{24} & 0 & 0 & 0 & 0 \\
Y_{13} & Y_{23} & -(Y_{3g} + Y_{34} + Y_{13} + Y_{23}) & Y_{34} & 0 & 0 & 0 & 0 \\
Y_{14} & Y_{24} & Y_{34} & -(Y_{4g} + Y_{34} + Y_{14} + Y_{24}) & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}
\tag{2}
$$

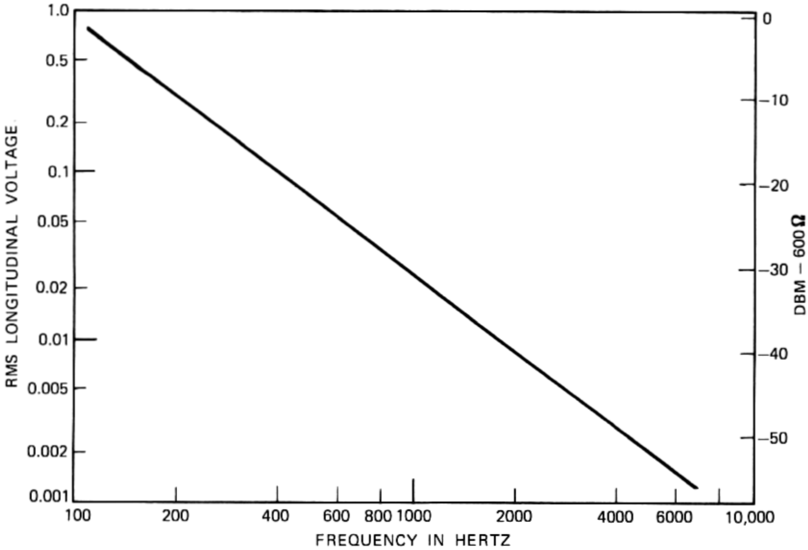

Fig. 1—Longitudinal signal limitations.

3.2 Transmission line equations

Equation (2) above can be written in matrix notation as

$$\frac{d\boldsymbol{V}}{dx} = -\boldsymbol{ZI} \tag{3a}$$

$$\frac{d\boldsymbol{I}}{dx} = -\mathbf{Y}\boldsymbol{V}, \tag{3b}$$

where

$$\boldsymbol{V} = \begin{bmatrix} V_1(x) \\ V_2(x) \\ V_3(x) \\ V_4(x) \end{bmatrix}, \quad \mathbf{Z} = \begin{bmatrix} Z_1 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 \\ 0 & 0 & Z_3 & 0 \\ 0 & 0 & 0 & Z_4 \end{bmatrix}, \quad \boldsymbol{I} = \begin{bmatrix} I_1(x) \\ I_2(x) \\ I_3(x) \\ I_4(x) \end{bmatrix}$$

Table I — Average crosstalk coupling coefficients for a multipair cable

Coefficient	Average Magnitude (picofarads)	Formula
$C_{M_2M_1}$	7.5	$C_{13} - C_{14} - C_{23} + C_{24}$
$C_{L_2M_1}$	65.2	$2\{C_{13} + C_{14} - C_{23} - C_{24}\}$
$C_{L_1M_2}$	64.0	$2\{C_{13} - C_{14} + C_{23} - C_{24}\}$
$C_{L_2L_1}$	9076.0	$4\{C_{13} + C_{14} + C_{23} + C_{24}\}$

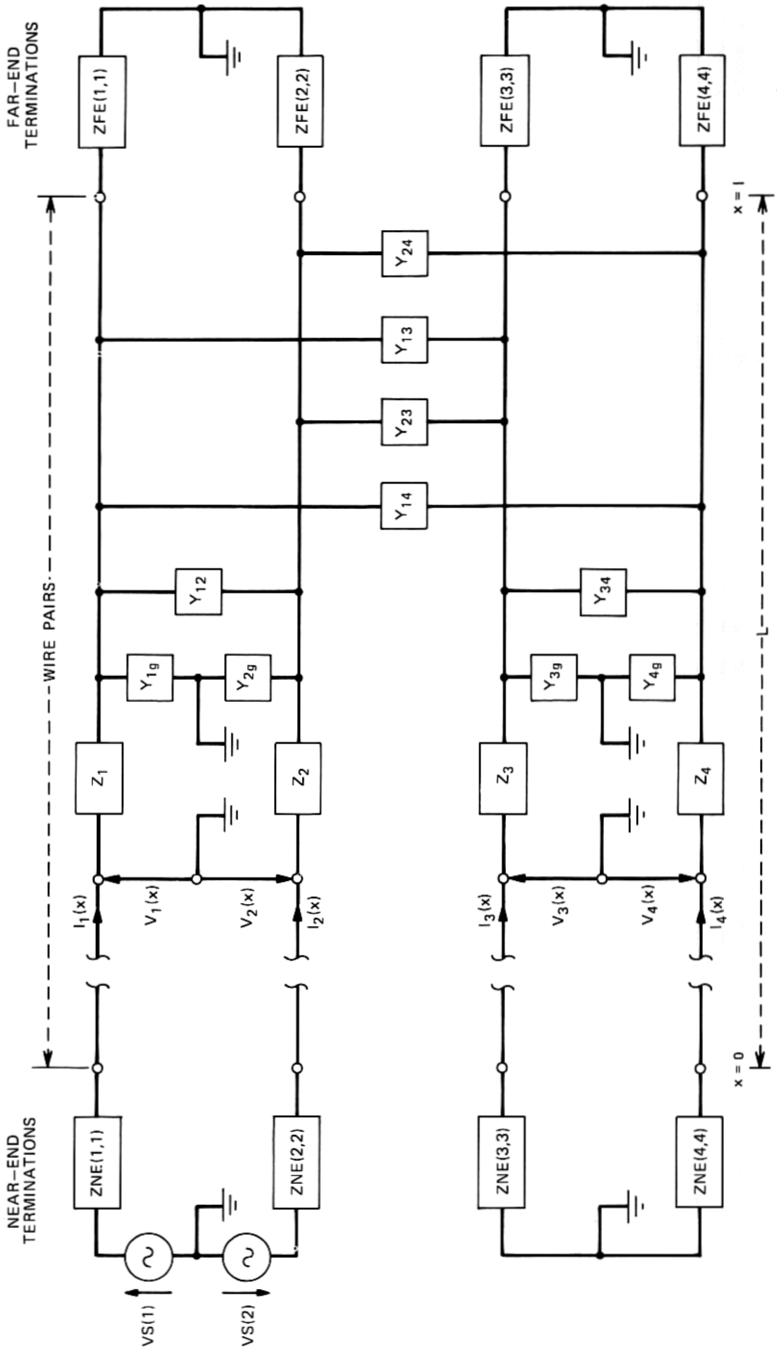

Fig. 2—Model of wire pairs and terminations used to analyze crosstalk coupling.

are the voltage, impedance, and current matrices, respectively, and, using assumption (*ii*),

$$
\mathbf{Y} = j\omega \begin{bmatrix} C_{1g} + C_{12} + C_{13} + C_{14} & -C_{12} & -C_{13} & -C_{14} \\ -C_{12} & C_{2g} + C_{12} + C_{23} + C_{24} & -C_{23} & -C_{24} \\ -C_{13} & -C_{23} & C_{3g} + C_{34} + C_{13} + C_{23} & -C_{34} \\ -C_{14} & -C_{24} & & C_{4g} + C_{34} + C_{14} + C_{24} \end{bmatrix}
$$

is the admittance matrix. Equations (3a) and (3b) are basic transmission line equations describing the voltage-current relationships between wire pairs. We will use them to calculate crosstalk coupling between wire pairs. They are more conveniently written as a matrix differential equation:

$$
\frac{d}{dx}\begin{bmatrix} V \\ I \end{bmatrix} = -\begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix}\begin{bmatrix} V \\ I \end{bmatrix}, \tag{4}
$$

where $\mathbf{O}$ is a 4×4 null matrix. The solution of eq. (4) is straightforward and is discussed in Appendix A.

3.3 Crosstalk coefficients for various coupling modes

Since there is negligible inductive coupling at voice frequencies, the insight to crosstalk coupling can be obtained from the transmission line equations involving the admittance matrix only. Rewriting eq. (3b) explicitly, we have

$$
\begin{bmatrix} dI_1/dx \\ dI_2/dx \\ dI_3/dx \\ dI_4/dx \end{bmatrix} = -j\omega \begin{bmatrix} C_{1g} + C_{12} + C_{13} + C_{14} & -C_{12} & -C_{13} & -C_{14} \\ -C_{12} & C_{2g} + C_{12} + C_{23} + C_{24} & -C_{23} & -C_{24} \\ -C_{13} & -C_{23} & C_{3g} + C_{34} + C_{13} + C_{23} & -C_{34} \\ -C_{14} & -C_{24} & -C_{34} & C_{4g} + C_{34} + C_{14} + C_{24} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}. \tag{5}
$$

It is clear from eq. (5) that coupling between wire pairs 1–2 and 3–4 could not possibly occur if the four interpair capacitances C_{13}, C_{14}, C_{23}, and C_{24} were all zero regardless of the longitudinal imbalance at the terminations. Furthermore, inspection of eq. (5) shows that the coupling between the wire pairs is some function of these four inter-wire capacitances. This is seen by observing the contributions to dI_1/dx and dI_2/dx from V_3 and V_4, and the contributions to dI_3/dx and dI_4/dx from V_1 and V_2.

The insight needed to understand why longitudinal excitation and longitudinally unbalanced terminations increase crosstalk is obtained when eq. (5) is transformed and expressed in terms of the longitudinal and metallic voltages and currents, rather than in terms of the conductor currents and conductor-to-ground voltages. This transformation is easily made because the longitudinal and metallic voltages and currents are linearly related to the conductor voltages and currents. If wire pair 1–2 is now denoted as circuit one and wire pair 3–4 is denoted as circuit two, then the metallic voltages and currents on the two circuits are defined to be

$$V_{1m} = V_1 - V_2, \qquad I_{1m} = \frac{I_1 - I_2}{2}$$

and

$$V_{2m} = V_3 - V_4, \qquad I_{2m} = \frac{I_3 - I_4}{2}.$$

The longitudinal voltages and currents on the two circuits are

$$V_{1L} = \frac{V_1 + V_2}{2}, \qquad I_{1L} = I_1 + I_2$$

and

$$V_{2L} = \frac{V_3 + V_4}{2}, \qquad I_{2L} = I_3 + I_4.$$

Expressed in matrix form, these eight equations become

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 1 \\ 0 & 0 & -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} V_{1M} \\ V_{1L} \\ V_{2M} \\ V_{2L} \end{bmatrix} \tag{6}$$

and

$$\begin{bmatrix} I_{1M} \\ I_{1L} \\ I_{2M} \\ I_{2L} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}. \tag{7}$$

Now, by using eqs. (6) and (7), eq. (5) can be expressed in terms of the longitudinal and metallic voltages and currents as

$$\begin{bmatrix} dI_{1M}/dx \\ dI_{1L}/dx \\ dI_{2M}/dx \\ dI_{2L}/dx \end{bmatrix} = -j\omega \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\times \begin{bmatrix} C_{1g}+C_{12}+C_{13}+C_{14} & -C_{12} & -C_{13} & -C_{14} \\ -C_{12} & C_{2g}+C_{12}+C_{23}+C_{24} & -C_{23} & -C_{24} \\ -C_{13} & -C_{23} & C_{3g}+C_{34}+C_{13}+C_{23} & -C_{34} \\ -C_{14} & -C_{24} & -C_{34} & C_{4g}+C_{34}+C_{14}+C_{24} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & 1 & 0 & 0 \\ -\frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 1 \\ 0 & 0 & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} V_{1M} \\ V_{1L} \\ V_{2M} \\ V_{2L} \end{bmatrix}, \tag{8}$$

or, performing the matrix multiplications,

$$\begin{bmatrix} dI_{1M}/dx \\ dI_{1L}/dx \\ dI_{2M}/dx \\ dI_{2L}/dx \end{bmatrix} = -\frac{j\omega}{4} \begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ q_{41} & q_{42} & q_{43} & q_{44} \end{bmatrix} \begin{bmatrix} V_{1M} \\ V_{1L} \\ V_{2M} \\ V_{2L} \end{bmatrix}, \tag{9}$$

where the elements of the 4×4 symmetric matrix $\mathbf{Q}$ in eq. (9) are given in Appendix B.

Much useful information can be obtained by simple inspection of some elements of $\mathbf{Q}$. First, the derivative of the metallic current in circuit one due to the metallic voltage in circuit two is proportional to q_{13}, i.e., the crosstalk coupling loss between the metallic circuits is directly related to q_{13}. Thus, the coupling between two metallic circuits, i.e., the metallic-to-metallic coupling, is proportional to

$$C_{M_2M_1} = -q_{13} = C_{13} - C_{14} - C_{23} + C_{24}. \tag{10}$$

This is the capacitance unbalance term first derived by Campbell[1] and used today as one measure of cable quality. Referring again to eq. (9), we see that the derivative of the metallic current in circuit one due to the longitudinal voltage in circuit two is proportional to q_{14} and that the derivative of the metallic current in circuit two due to the longitudinal voltage in circuit one is proportional to q_{32}. In other words, the crosstalk coupling from a longitudinal to a metallic circuit is proportional to

$$C_{L_2M_1} = -q_{14} = 2(C_{13} + C_{14} - C_{23} - C_{24}) \tag{11}$$

or

$$C_{L_1M_2} = -q_{32} = 2(C_{13} - C_{14} + C_{23} - C_{24}). \tag{12}$$

The subscript L_2M_1 means "from the longitudinal mode in circuit two to the metallic mode in circuit one." Also, we can readily see that the derivative of longitudinal current in circuit one resulting from the longitudinal voltage in circuit two is proportional to q_{24}. In other words, the crosstalk coupling between two longitudinal circuits is proportional to

$$C_{L_2L_1} = -q_{24} = 4(C_{13} + C_{14} + C_{23} + C_{24}). \tag{13}$$

Using these four coupling coefficients, it is now possible to compare the difference in crosstalk loss between two metallic circuits, a longitudinal and a metallic circuit, and two longitudinal circuits. This comparison was made for one cable and the results are discussed in the next section.

3.4 Comparison of crosstalk using the coupling coefficients

One good feature of the four coupling coefficients given in eqs. (10) through (13) is that they are easily measured. Hence, they provide a simple method for comparing the difference in crosstalk between two metallic circuits, a longitudinal and a metallic circuit, and two longitudinal circuits. To make such a comparison, it is necessary to have data on the interwire capacitances, C_{13}, C_{14}, C_{23}, and C_{24}, for real cable. Such measurements were made in 1968 on a 22-gauge, pulp-insulated cable manufactured by Western Electric. These measurements were made on many different 50-pair binder groups.* The data on interwire capacitances were taken for random samples out of the 1225 possible sets† of interwire pair combinations within each binder group.

Using these data, the average value of the four coupling coefficients were calculated and are given in Table I. These show that, on the average, the coupling between two metallic circuits is significantly less than the coupling between a longitudinal and a metallic circuit, and that the coupling between two longitudinal circuits is by far the greatest. *Hence, the fundamental reason why terminal longitudinal imbalance increases crosstalk is that longitudinal imbalance causes excitation of the longitudinal circuit.*

Comparison of the values of the coupling coefficients made so far does not provide any quantitative estimate of the amount of the differences in crosstalk losses to be expected. Such an estimate can be obtained by using the coupling coefficients for individual wire-pair combinations to construct distributions of 1000-Hz near-end crosstalk

* A binder group is a unit of 12, 16, 20, 25, 50, or 100 twisted wire pairs bound together within a cable.

† For a 50-wire pair cable there are $n(n-1)/2 = 50(49)/2 = 1225$ possible two-wire pair combinations. The sample sizes ranged from 200 to 600 pair combinations.

loss. This was done for the metallic-to-metallic, longitudinal-to-metallic, and longitudinal-to-longitudinal crosstalk loss distributions by using the formula given in Ref. 4,

$$N_l = 20 \log_{10} \left[\frac{j\omega C_u Z_0}{8} \right],$$

where C_u is the capacitance unbalance, i.e., $C_{M_2M_1}$, $C_{L_2M_1}$, or $C_{L_2L_1}$, ω is the radian frequency in Hertz, and Z_0 is the characteristic impedance of 22-gauge pulp. The inductive contribution is neglected. The distributions are shown in Fig. 3. The rms crosstalk loss corresponds to that loss which would result in the average crosstalk power in watts. Consequently, crosstalk power transferred between two circuits with crosstalk loss equal to the rms value would be the average crosstalk power. The rms values are 105.2-dB loss between metallic circuits and

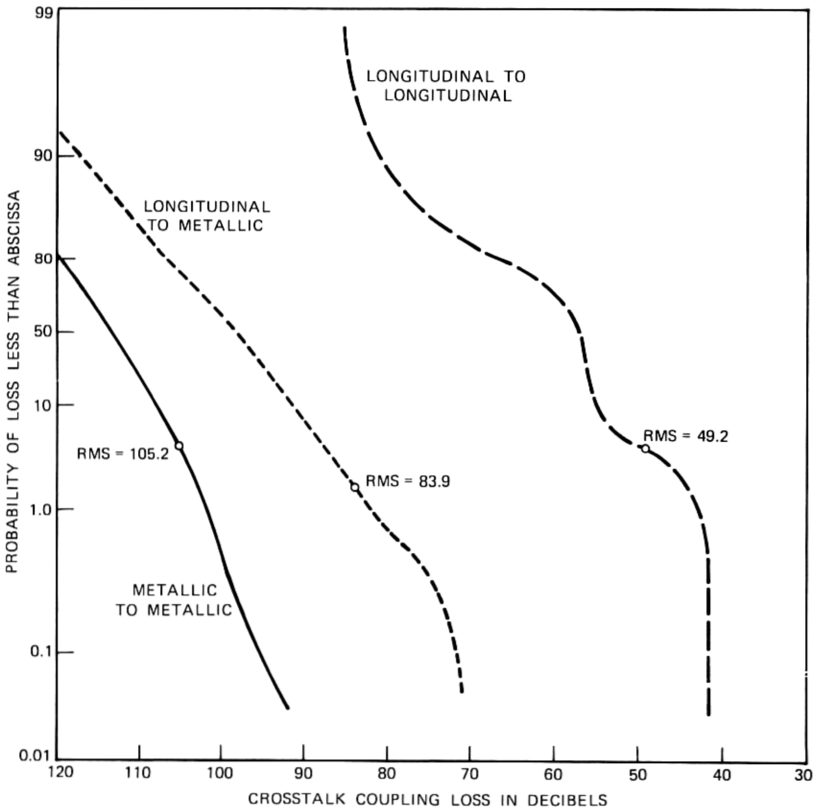

Fig. 3—Computed 1000-Hz near-end crosstalk based on the interwire capacitances of 416 pair combinations of 1319 ft of 22-gauge pulp cable.

only 83.9-dB loss between a longitudinal and a metallic circuit. Hence, the new coefficient $C_{L_2M_1}$ predicts that longitudinal excitation of a wire pair in the cable measured causes 21.3 dB (105.2 − 83.9) more average crosstalk power in a metallic circuit than metallic excitation. Similarly, the rms loss between two longitudinal circuits, predicted by $C_{L_2L_1}$, is 49.2 dB, which is 56 dB (105.2—49.2) less than the metallic-to-metallic loss. These results are compared in Section 3.6 to crosstalk loss measurements made on another cable. In Section 3.5, these results are compared to results obtained from computer solution of the transmission line equations (9), i.e., by simulation of the cable.

3.5 Numerical solutions

A second, more difficult method of calculating the crosstalk between the various modes is direct solution of the transmission line equations on a computer with an appropriate set of boundary conditions.

A computer program has been written to solve these equations that simulates a cable of the same length and identical characteristics of the Western Electric cable used to obtain the coupling coefficients. Two conditions of interest were simulated on the computer. First, metallic excitation by a balanced 1000-Hz signal generator in series with a 600-ohm resistance was applied to a pair, denoted the disturbing pair, and all other pairs were terminated metallically with 600-ohm resistors from tip to ring. Second, the same conditions applied except one wire of the disturbing pair was grounded. This resulted in a degradation of 14.5 dB in the rms value of the balanced near-end crosstalk loss distribution.

To compare the degradation in crosstalk obtained by the two methods, i.e., coupling coefficients versus numerical solutions, it is necessary to note that grounding a wire connected to a signal generator produces a longitudinal voltage that is one-half the value of the applied metallic voltage. This follows directly from the definitions of longitudinal and metallic voltages in terms of the voltage from each wire pair conductor to ground, eq. (6) with $V_2 = 0$. Hence, an approximate 6-dB adjustment must be made when using the longitudinal-to-metallic coupling coefficients $C_{L_1M_2}$ and $C_{L_2M_1}$, which predict a 15.3-dB degradation in the near-end rms crosstalk loss at 1000 Hz due to grounding, as compared to 14.5 dB predicted by the numerical computation. This good agreement suggests that the new capacitive coupling coefficients do provide a simple but reliable method of predicting the degradation in crosstalk performance for a particular cable when its terminations are unbalanced.

3.6 Comparison with measured data

In 1962, measurements of the degradation of near-end crosstalk loss caused by grounding one conductor of the disturbing or disturbed pairs on randomly selected pair combinations in a 7500-ft length of a 22-gauge multipair pulp-insulated exchange grade trunk cable were made by Bell Laboratories. This was a real working cable between Oceanside and Vista, California. All load coils in the section under test here were first removed and the cable ends spliced. The results of these measurements are shown in Fig. 4. The results reveal, at 940 Hz,* a degradation of about 19.4 dB in the near-end rms crosstalk loss when a ground was applied to one wire of either the disturbing or the disturbed pairs.

Since grounding one conductor causes a longitudinal voltage excitation that is one-half the metallic voltage, a 6-dB numerical adjustment was made on the measurements to predict that the rms crosstalk loss between a longitudinal and a metallic circuit is 25.4 dB worse than the rms loss between metallic circuits. This is compared to 21.3 dB obtained using the coupling coefficients for the cable discussed in the previous section. This 4.1-dB difference may be due to the fact that the two cables were not the same, each having different value parameters characterizing them as well as different lengths.

When one conductor of both disturbing and disturbed pairs were grounded, the measured rms crosstalk loss was 32.6 dB, as shown in Fig. 4. This is the loss between the two longitudinal circuits† and, as can be seen, it is 61.5 dB less than this rms loss between the metallic circuits. This measured difference compares favorably to the calculated difference of 56 dB as shown in Fig. 3. The 5.5-dB difference may be due to cable differences. In conclusion, direct crosstalk measurements on another cable substantially support the analytical method for calculating crosstalk using the coupling coefficients or computer simulations.

3.7 Metallic-to-longitudinal conversion because of wire pair imbalance

So far, we have analyzed the effect of direct longitudinal excitation of wire pairs on crosstalk between pairs. This excitation results when longitudinally unbalanced terminations are used. However, now we discuss how longitudinal excitation can also result because of "pair longitudinal imbalance," which is defined as any lack of symmetry

* 940 Hz is close enough to 1000 Hz to permit direct comparison with calculated results.

† Referring to the definitions of longitudinal and metallic voltages, it is simple to show that the coupling loss for the longitudinal-to-longitudinal mode is the same as for both pairs grounded.

in the wire pairs with respect to ground or with respect to each other. Such asymmetry can cause part of the metallic signal to be converted to a longitudinal excitation even when there is perfect longitudinal balance at the terminations.

To understand the causes of wire pair longitudinal imbalance, again refer to eq. (9). Perfect pair balance is the condition that exists whenever a metallic signal does not excite the longitudinal modes in either the disturbing or disturbed wire pair. This requirement can be met if and only if

$$q_{21} = q_{23} = q_{41} = q_{43} = 0.$$

These last four conditions are satisfied if

$$C_{1g} = C_{2g} \tag{14a}$$

$$C_{3g} = C_{4g} \tag{14b}$$

$$C_{14} = C_{23} \tag{14c}$$

$$C_{13} = C_{24}. \tag{14d}$$

Equations (14a) and (14b) are necessary since, for example, if C_{1g} were not equal to C_{2g}, there would be a lack of longitudinal symmetry in wire pair one even if the terminations were all perfectly balanced. Equations (14c) and (14d) imply that equal and opposite currents are coupled (metallic-to-metallic crosstalk) from each of the wires in the disturbing pair to the disturbed pair preserving the pair symmetry.

In other words, if the conditions of eqs. (14) are met and all the terminations are balanced, then all the currents are strictly confined to the metallic circuits. This is not to say that crosstalk cannot occur. It means that only one of the three kinds of coupling can occur, i.e., from metallic circuit to metallic circuit. In fact, the crosstalk will then be proportional to Campbell's capacitance unbalance expression which simplifies to

$$C_{M_2M_1} = C_{13} + C_{24} - C_{14} - C_{23} = 2(C_{13} - C_{14}). \tag{15}$$

Cable data reveal that the capacitances to ground for wire pairs are nearly equal, their differences on the average being less than 2 percent of their magnitude. The percent differences in the interwire capacitances are larger (e.g., 10 percent), but they are much smaller than the capacitances to ground. This suggests that metallic-to-longitudinal conversion of signals due to the cable characteristics alone is small. Computer simulation of wire pairs, using eq. (9) and assuming balanced terminations, supports this suggestion. To put it another way, the high quality of manufactured multipair cable used in the Bell System ensures excellent pair longitudinal balance. The small imbalance in

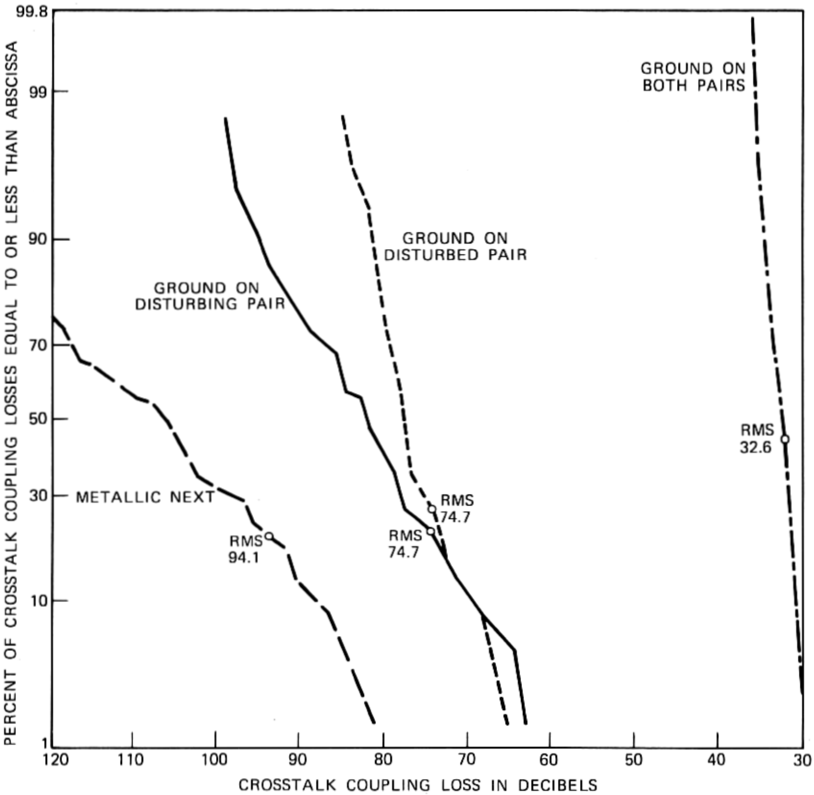

Fig. 4—940 cycles near-end crosstalk coupling loss measured on 60 pair combinations within a 51-pair unit of a DSAC-202 cable (cable length 7500 ft). All grounds are applied at measure end.

the pairs is rarely a significant factor in the contribution to longitudinal voltages that degrade crosstalk. Substantial conversion to longitudinal modes does occur when there is imbalance at the terminations, as revealed by our analysis and, consequently, it is necessary to place limits on permissible terminal longitudinal imbalance.

IV. LONGITUDINAL BALANCE REQUIREMENTS

Crosstalk energy can reach the metallic mode in the disturbed circuit, circuit two, from an applied metallic signal in the disturbing pair, circuit one, in three different ways.

(*i*) Direct coupling from a metallic signal in circuit one to a metallic disturbance in circuit two.

(*ii*) Conversion of the metallic signal of circuit one to a longitudinal signal in circuit one because of an unbalanced termination* in that circuit, then coupling of the longitudinal signal in circuit one to a metallic disturbance in circuit two.

(*iii*) Conversion of the metallic signal in circuit one to a longitudinal signal in circuit one because of an unbalanced termination, coupling of the longitudinal signal in circuit one to a longitudinal signal in circuit two, and, finally, conversion of the longitudinal signal of circuit two back to a metallic disturbance in circuit two due to an unbalanced termination on circuit two.

The crosstalk described in (*i*) above is independent of the imbalance at the terminations. It is the result of the capacitance unbalance $C_{M_2M_1}$ between the individual wire pairs and there is little more that can practically be done to circuits to reduce it. *The important thing is to make sure that any equipment that is connected at the terminations of the cable does not degrade the low levels of crosstalk that currently exist by introducing longitudinal excitations.*

4.1 Longitudinal balance requirement

The data on the vulnerability of cable to longitudinal imbalance have been obtained by measurements made on two different cables.† Measurements on the cable in California, with the 6-dB numerical adjustment, revealed that longitudinal signals, on the average, crosstalk into adjacent wire pairs with 25.4-dB less coupling loss for that cable than do metallic signals. The data on the Western Electric reel of cable, used in the newly derived capacitance unbalance formulas, showed 21.3-dB less coupling loss for longitudinal signals.

The definition of longitudinal balance, for application to crosstalk performance, is repeated:

$$\mathrm{BAL}_{\mathrm{M-L}}(f) = 20 \log_{10} \left| \frac{e_M(f)}{e_L(f)} \right| ,$$

where e_L is the longitudinal voltage produced when a *metallic* voltage e_M is applied at any frequency f. The measurements made on the cable in California establish the more stringent longitudinal balance requirement, and it shall therefore be assumed that rms longitudinal-to-metallic crosstalk loss is 25 dB less than metallic-to-metallic cross-

* Conversion because of imbalance in the cable itself can be neglected, as discussed in Section 3.7.

† Subsequent to the beginning of this investigation, measurements on the vulnerability of one other cable to crosstalk because of longitudinal imbalance have been made. These measurements do not alter the conclusions reached by using the data on the first two cables only.

talk loss. This implies that, if a network which is sending a metallic signal has a balance of about 25 dB, the longitudinal signal developed because of this imbalance may contribute the same amount of crosstalk power to nearby cable pairs as the direct metallic signal applied to it. It is not known exactly how the two components of the crosstalk produced by metallic and longitudinal signals will add, that is, on a voltage basis, a power basis, or somewhere in between. However, a longitudinal signal developed because of imbalance is likely to be correlated to the metallic signal causing it. Hence, it will be assumed that the signals add approximately on a voltage basis.

What is needed is a balance such that the contribution to crosstalk power because of imbalance is small compared to the crosstalk that exists when a metallic signal is applied. For illustrative purposes, it is assumed that an increase of 1.0 dB is not too noticeable and is thus a permissible contribution. In Fig. 5, which shows how two voltages expressed in dB are added, it is seen that, in order for the power in a signal to be increased by no more than 1.0 dB because of the presence of a second signal, the voltage difference must be over 17 dB. Thus, a longitudinal balance of approximately 42 dB is required (we will use 40 dB) to ensure that crosstalk is increased by no more than this amount, due to the longitudinal-to-metallic coupling path, type (*ii*), described at the beginning of Section IV.

We now show that the crosstalk resulting from the coupling path described as type (*iii*) is less severe and has no bearing in determining the balance requirement. To do this requires discussing a second measure of balance, that known as longitudinal-to-metallic balance.

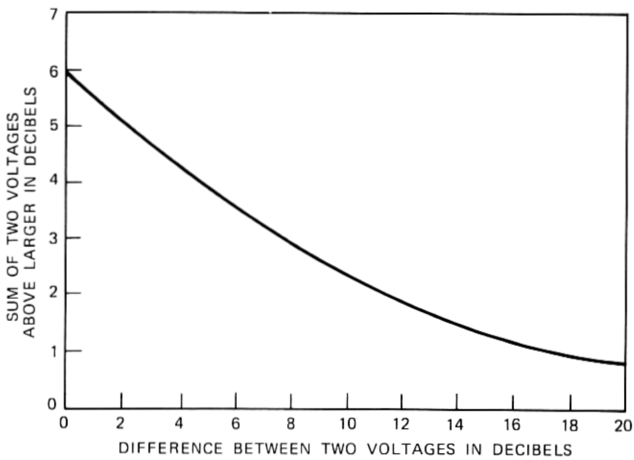

Fig. 5—Sum of two voltages expressed in decibels.

4.2 Longitudinal-to-metallic balance

In addition to the possibility of severely degrading the crosstalk levels already occurring in the telephone network, there is a second important reason why high longitudinal balance at the terminations must be maintained. This is to reduce the metallic noise resulting from power line induction. Here, the disturbing signals are longitudinal in nature and, to assure good performance of the user's circuit, the conversion loss from a longitudinal noise signal on his circuit to a metallic signal on his circuit must be large. A measure of this conversion loss, defined as longitudinal-to-metallic balance, is:

$$\mathrm{BAL}_{\mathrm{L-M}}(f) = 20 \log_{10} \left| \frac{e_L(f)}{e_M(f)} \right| ,$$

where e_L is the applied longitudinal voltage source and e_M is the resulting metallic signal. The balance subscript "L − M" means a conversion from an applied longitudinal signal to a metallic signal. It is important to note that the two measures of balance, $\mathrm{BAL}_{\mathrm{M-L}}$ and $\mathrm{BAL}_{\mathrm{L-M}}$, are not equal, i.e., reciprocity does not necessarily apply. Moreover, they are not necessarily correlated.

Generally, to assure good performance, the minimum balance $\mathrm{BAL}_{\mathrm{L-M}}$ of a termination is well in excess of 40 dB over the voiceband. Consequently, we use 40 dB as a lower bound on the $\mathrm{BAL}_{\mathrm{L-M}}$ of terminations, keeping in mind that this in no way establishes 40 dB as the necessary performance minimum. Discussion of that topic is outside the scope of this paper.

Using the assumed minimum bound on $\mathrm{BAL}_{\mathrm{L-M}} = 40$ dB of terminations on the disturbed circuit, it is now shown that the crosstalk coupling loss path (*iii*) is at least 20 dB less than the metallic-to-metallic path and, hence, is not a factor. It is also assumed that the balance $\mathrm{BAL}_{\mathrm{M-L}}$ of the terminations on the disturbing circuit is 40 dB, determined from the crosstalk requirement because of the coupling path of type (*ii*). Now, since there is approximately 60 dB less crosstalk loss between two longitudinal circuits than between two metallic circuits, the difference in the losses between type (*i*) and type (*iii*) paths is $\mathrm{BAL}_{\mathrm{M-L}}$ (disturbing circuit) −60 dB + $\mathrm{BAL}_{\mathrm{L-M}}$ (disturbed circuit), or at least 20 dB.

V. REQUIREMENTS ON LONGITUDINALLY APPLIED SIGNALS

So far, we have considered how longitudinally unbalanced terminations can cause increased crosstalk, because they longitudinally excite a wire pair. We have recognized that it is this longitudinal excitation that is the fundamental cause of the increased crosstalk, and we have recommended a metallic-to-longitudinal balance, BAL_{M-L}, limit for termination of 40-dB minimum to restrict the amount of longitudinal excitation produced. However, it is also possible to longitudinally excite wire pairs directly from a voltage source connected between the tip and ring of a wire pair and the cable sheath, or ground. Such direct excitation must also be limited, because it too causes crosstalk in a disturbed metallic circuit in two ways:

(*i*) Direct coupling from the longitudinal mode in circuit one to the metallic mode in circuit two.
(*ii*) Coupling of the longitudinal mode in circuit one to the longitudinal mode in circuit two and conversion of energy in the longitudinal mode of circuit two to the metallic mode in circuit two because of an unbalanced termination in circuit two.

Since the effect of directly applying longitudinal signals is the same as longitudinal signals arising from metallic-to-longitudinal imbalance, and since the rms crosstalk loss for this type of signal is on the average 25 dB less than metallic signals, longitudinal voltage limits should be 40 dB more restrictive than metallic voltage limits. Figure 1 shows the restriction on longitudinally applied voltages as a function of frequency. It is based on the restrictions already placed on metallic voltages determined by a previous study at Bell Laboratories* and the 40-dB restriction determined here.

VI. SUMMARY

The following has been accomplished in this paper:

(*i*) Three new capacitive coupling coefficients have been derived that provide a simple but reliable method of predicting the degradation in crosstalk performance for a particular cable when its terminations are unbalanced.
(*ii*) It has been demonstrated that a metallic-to-longitudinal balance requirement of 40 dB or more for any terminations connected into network should not noticeably increase the low levels of crosstalk that are already present.
(*iii*) A requirement has been established on longitudinally applied signals that if met should not degrade crosstalk performance.

VII. ACKNOWLEDGMENTS

The author would like to thank J. R. Rosenberger and the reviewers for their helpful comments.

---

* Restrictions on metallic voltages are given in terms of permissible power into 600 ohms in Ref. 3.

APPENDIX A

In eq. (4), let $T(x)$ be an 8×1 column matrix where

$$\boldsymbol{T} = \begin{bmatrix} \boldsymbol{V}(x) \\ \boldsymbol{I}(x) \end{bmatrix} \tag{16}$$

and let $\mathbf{A}$ be an 8×8 constant matrix (not a function of x),

$$\mathbf{A} = \begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix}. \tag{17}$$

Then

$$\frac{d\boldsymbol{T}(x)}{dx} = -\mathbf{A}\boldsymbol{T}. \tag{18}$$

The solution to this matrix differential equation[5] is known to be

$$\boldsymbol{T}(x) = \exp(-\mathbf{A}x)\boldsymbol{T}(0).$$

Since the parameters that characterize the line are independent of x, it is readily seen by solving eq. (4) that

$$\begin{bmatrix} \boldsymbol{VFE} \\ \boldsymbol{CFE} \end{bmatrix} = \exp\left\{-\begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix} l\right\}\begin{bmatrix} \boldsymbol{VNE} \\ \boldsymbol{CNE} \end{bmatrix}, \tag{19}$$

where the far-end voltages and currents where $x = l$ are

$$\boldsymbol{VFE} = \begin{bmatrix} V_1(l) \\ V_2(l) \\ V_3(l) \\ V_4(l) \end{bmatrix}, \qquad \boldsymbol{CFE} = \begin{bmatrix} I_1(l) \\ I_2(l) \\ I_3(l) \\ I_4(l) \end{bmatrix}$$

and the near-end voltages and currents are

$$\boldsymbol{VNE} = \begin{bmatrix} V_1(0) \\ V_2(0) \\ V_3(0) \\ V_4(0) \end{bmatrix}, \qquad \boldsymbol{CNE} = \begin{bmatrix} I_1(0) \\ I_2(0) \\ I_3(0) \\ I_4(0) \end{bmatrix}.$$

With the eight equations given in (19) and a knowledge of the boundary relations at the terminations, we can characterize the model of the system at each point in space (x) by a vector pair of voltages $\boldsymbol{V}(x)$ and pair currents $\boldsymbol{I}(x)$.

The matrix exponential,

$$\exp\left\{-\begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix} l\right\},$$

may be evaluated in closed form. With a closed-form representation, the voltages and currents can be expressed in closed form and more complex structures such as spliced cable systems can be simulated. The way to explicitly determine the matrix exponential is to use the fact that

$$\mathbf{Q} = -\begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix}$$

satisfies its own characteristic equation (Cayley-Hamilton Theorem[5]). Then

$$\exp \mathbf{Q} = \sum_{k=0}^{7} \alpha_k \mathbf{Q}^k.$$

Replacing $\mathbf{Q}$ by a diagonal matrix consisting of the eight eigenvalues of $\mathbf{Q}$ enables us to solve for α_i. However, for fairly short unspliced cable systems we may use the first few terms in a power series, i.e.,

$$\exp[-\mathbf{Q}l] = \mathbf{I} - l\mathbf{Q} + \frac{l^2}{2}\mathbf{Q}^2 - \cdots,$$

where $\mathbf{I}$ is the identity matrix, to accurately approximate the matrix exponential as was done for the cable in the numerical solutions section.

A.1 Boundary conditions

For any two wire pairs within a cable, four sets of current-voltage relationships exist at the wire terminations. Referring back to Fig. 2, we define the near end to be the subscriber side of the loop with its termination where $x = 0$, and the far end, where $x = l$, to be the other termination, possibly a central office. The disturbing pair will always be designated wire pair 1–2 with a generator of some kind at the near end, and the disturbed pair will be designated 3–4. Suppose the generator is two voltage sources each grounded at one end and in series with an impedance and the remaining terminations consist each of two complex impedances to ground shown in Fig. 2. Then we have the eight relations at the boundaries

$$V_1(0) = VS(1) - ZNE(1, 1)I_1(0) \tag{20a}$$

$$V_2(0) = VS(2) - ZNE(2, 2)I_2(0) \tag{20b}$$

$$V_3(0) = -ZNE(3, 3)I_3(0) \tag{20c}$$

$$V_4(0) = -ZNE(4, 4)I_4(0) \tag{20d}$$

$$V_1(l) = ZFE(1, 1)I_1(l) \tag{20e}$$

$$V_2(l) = ZFE(2, 2)I_2(l) \tag{20f}$$

$$V_3(l) = ZFE(3, 3)I_3(l) \tag{20g}$$

$$V_4(l) = ZFE(4, 4)I_4(l), \tag{20h}$$

which may be written in matrix form as

$$\boldsymbol{VNE} = -\mathbf{ZNE}\,\boldsymbol{CNE} + \boldsymbol{VS} \tag{21a}$$

$$\boldsymbol{VFE} = \mathbf{ZFE}\,\boldsymbol{CFE}, \tag{21b}$$

where

$$\mathbf{ZNE} = \begin{bmatrix} ZNE(1, 1) & 0 & 0 & 0 \\ 0 & ZNE(2, 2) & 0 & 0 \\ 0 & 0 & ZNE(3, 3) & 0 \\ 0 & 0 & 0 & ZNE(4, 4) \end{bmatrix},$$

$$\mathbf{ZFE} = \begin{bmatrix} ZFE(1, 1) & 0 & 0 & 0 \\ 0 & ZFE(2, 2) & 0 & 0 \\ 0 & 0 & ZFE(3, 3) & 0 \\ 0 & 0 & 0 & ZFE(4, 4) \end{bmatrix}.$$

and*

$$\boldsymbol{VS}^T = [VS(1), VS(2), 0, 0].$$

We can solve the 16 equations (19) and (21) and determine ***VNE*** and ***CNE***. Now we have the model completely characterized by the vector pair of voltages $\boldsymbol{V}(x)$ and $\boldsymbol{I}(x)$ via the equation

$$\begin{bmatrix} \boldsymbol{V} \\ \boldsymbol{I} \end{bmatrix} = \exp\left\{-\begin{bmatrix} \mathbf{O} & \mathbf{Z} \\ \mathbf{Y} & \mathbf{O} \end{bmatrix} x\right\} \begin{bmatrix} \boldsymbol{VNE} \\ \boldsymbol{CNE} \end{bmatrix}.$$

It should be pointed out that the terminations are not always simple impedances to ground. For instance, for a second type of termination, where an ordinary telephone set is connected to a wire pair, tip, and ring, there is no direct conducting path to ground. If the impedance to ground from the tip and ring is assumed to be infinite, then we cannot write a simple impedance matrix relating the current to voltage as in eqs. (21a) or (21b). As a result, there is a rather tedious but straightforward rearrangement of eqs. (19) and (21). Finally, a third type of termination could be a central office that will also require modification of the impedance matrix. All three of these types of terminations have been simulated in computer programs.

---

* The superscript T means transpose.

APPENDIX B

The elements of the admittance matrix in eq. (13) are given below.

$$q_{11} = \{C_{1g} + C_{2g} + 4C_{12} + C_{13} + C_{14} + C_{23} + C_{24}\}$$
$$q_{12} = q_{21} = 2\{(C_{1g} - C_{2g}) + (C_{13} + C_{14} - C_{23} - C_{24})\}$$
$$q_{13} = q_{31} = -(C_{13} - C_{14} - C_{23} + C_{24})$$
$$q_{14} = q_{41} = -2(C_{13} + C_{14} - C_{23} - C_{24})$$
$$q_{22} = 4\{(C_{1g} + C_{2g}) + (C_{13} + C_{14} + C_{23} + C_{24})\}$$
$$q_{23} = q_{32} = -2(C_{13} - C_{14} + C_{23} - C_{24})$$
$$q_{24} = q_{42} = -4(C_{13} + C_{14} + C_{23} + C_{24})$$
$$q_{33} = \{C_{3g} + C_{4g} + 4C_{34} + C_{13} + C_{14} + C_{23} + C_{24}\}$$
$$q_{34} = q_{43} = 2\{(C_{3g} - C_{4g}) + (C_{13} - C_{14} + C_{23} - C_{24})\}$$
$$q_{44} = 4\{(C_{3g} + C_{4g}) + (C_{13} + C_{14} + C_{23} + C_{24})\}.$$

REFERENCES

1. "Dr. Campbell's Memoranda of 1907 and 1912," B.S.T.J., *14*, No. 4 (October 1935), pp. 558–572.
2. G. J. Foschini, "Crosstalk in Outside Plant Cable Systems," B.S.T.J., *50*, No. 7 (September 1971), pp. 2421–2448.
3. "Transmission Specifications for Private Line Metallic Circuits," Bell System Technical Reference, PUB 43401, December 1971.
4. G. S. Eager, Jr., L. Jachimowicz, I. Kolodny, and D. E. Robinson, "Transmission Properties of Polyethylene-Insulated Telephone Cables at Voice and Carrier Frequencies," Transaction of the AIEE, Communications and Electronics, *78*, Part 1, November 1959, pp. 618–640.
5. L. A. Zadeh and C. A. Desoer, *Linear System Theory: The State Space Approach*, Appendix C, New York: McGraw–Hill, 1963.