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Wideband Amplifier Design Using Major Multiloop Feedback Techniques

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Multiloop feedback has heretofore been ignored as a means of obtaining shaped gain amplifiers. In this paper, a theoretical basis is developed for using dual major loop feedback amplifiers to obtain shaped power gain with input and output reflection coefficient constraints. From the theoretical results, practical design procedures can easily be developed and one such procedure is discussed.

The aim of this study was an alternative to the "brute force" termination technique of realizing input and output impedance matches. The development is otherwise unique in that it uses no hybrid transformers for beta circuit coupling or for realization of the reflection coefficient constraints.

I. INTRODUCTION

Wideband feedback amplifier design has heretofore mainly been accomplished by the use of single major loop feedback techniques.[1–4] Major loop feedback implies that the current or voltage on the input to the basic amplifying element is manipulated by the current or voltage that appears on the output of the basic amplifying element. The design concept follows the classical feedback design procedure of assuming a unilateral forward amplifying element of voltage gain μ and a feedback path with voltage gain β. Existing multiloop feedback techniques have been primarily concerned with stability considerations of "tandem" [5] and minor multiloop[2,3] feedback arrangements.

In many applications, input and output impedance matching of the amplifier is necessary. The communications amplifier is one such example, since it requires very low levels of signal interference due to input or output impedance mismatch. The classical single-loop feedback techniques offer little help in designing for the impedance matching constraint. This is due to the fact that the more loop gain in a single-loop feedback circuit, the more extreme (zero or infinite) the input and output impedance becomes.[1–3] Two techniques that are used to circumnavigate this problem are "brute force" terminations and bridge couplings.

The brute force approach obtains the impedance match by placing a resistor in series with the input (or output) for a feedback amplifier with zero input (or output) impedance. A parallel resistor is used for the infinite input or output impedance case.

The use of a balanced resistor bridge is also useful in obtaining an impedance match. This is accomplished by balancing the bridge components with respect to the input (or output) impedance of the feedback amplifier. The use of a resistive bridge is limited, though, due to the excessive resistive losses associated with such a bridge. A useful four-port device, which exhibits the same qualities as a resistive bridge but with much less through loss, is the hybrid transformer.[6] The impedance match with this device is obtained by manipulation of the two unused port impedances.[7]

Since the hybrid transformer is similar to a bridge, one of the two unused ports can be used for the β return path. This technique is theoretically the best alternative mentioned since a property of such a connection is that the impedance match is improved with the amount of loop gain.[2] This technique has been used to advantage on several communications amplifiers.[8,9]

The limitations of the above alternatives of obtaining an impedance match become evident when other design constraints are investigated. For example, the noise figure of an amplifier is degraded by any loss that exists on the input to the amplifier.[10] Thus, the use of brute force or hybrid transformer coupling causes an increase in noise figure. On the output side, a loss increases the power requirement on the last stage of the amplifier. Even if this is no problem, the resultant increase in the distortion may be. This is due to the fact that second-order distortion power increases twice as fast as fundamental power and third-order distortion power three times as fast.[11] Thus, the losses associated with the matching techniques will increase the power requirement and reduce the linearity of the overall amplifier.

The use of the hybrid transformer in the β path may also cause a stability problem. Since the transformer introduces phase shift, due both to the physical length and techniques of construction, their use is limited at very high frequencies.

Investigation into alternative methods of design is therefore desirable. To this end, this paper presents fundamental concepts on the techniques of using major multiloop feedback in amplifier design. The objective is the design of wideband-frequency-dependent gain amplifiers with input and output match constraints. The design procedure does not use hybrid transformers and attempts to minimize brute force termination techniques.

In Section II, the basic amplifier element is introduced. The analysis that follows is applicable to configurations of active devices that can be modeled by this basic amplifier. A circuit form using this basic amplifier is then introduced. Matched impedances and gain relationships are developed for this circuit form in such a way as to make the open loop gain characteristics evident. This paves the way for an initial design approach that is independent of the loop gain characteristics.

In Section III, a second circuit form of shunt-series feedback using the same basic amplifier is introduced. Matched impedances and gain relationships are again developed. The derivations in this section exactly parallel those of Section II.

In Section IV is given the results of the two previous sections to demonstrate the procedure used to obtain an initial circuit design for a practical amplifier configuration. The configuration treated is that of a cascade of N common emitter transistor stages. It is shown that for N odd, the results of Section II can be used, and for N even, the results of Section III apply. One numerical example is supplied for each case. Two appendices provide the calculations used to derive the results in Sections II and III.

II. SHUNT TRANSADMITTANCE: SERIES TRANSIMPEDANCE FEEDBACK

Each dual-loop feedback amplifier discussed in this paper contains three major components: two feedback networks and one amplifying element. Each major component is assumed to be made up of any number of passive and active elements. Characteristics of importance for the amplifying element component are given in Fig. 1; this abbreviated model is designated a basic amplifier. In this figure, z_x is the input impedance and I_s is a current-controlled current source. I_s is given by the product of a frequency-dependent variable k and the current through z_x.

In Fig. 2, the first multiloop feedback circuit form is given. Series feedback voltage source aI_o sums up the most important characteristic

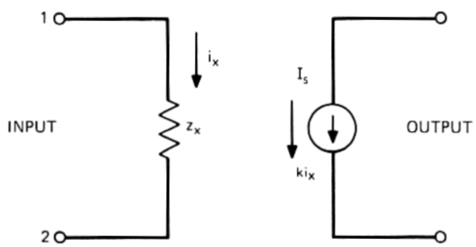


Fig. 1—Basic amplifier.

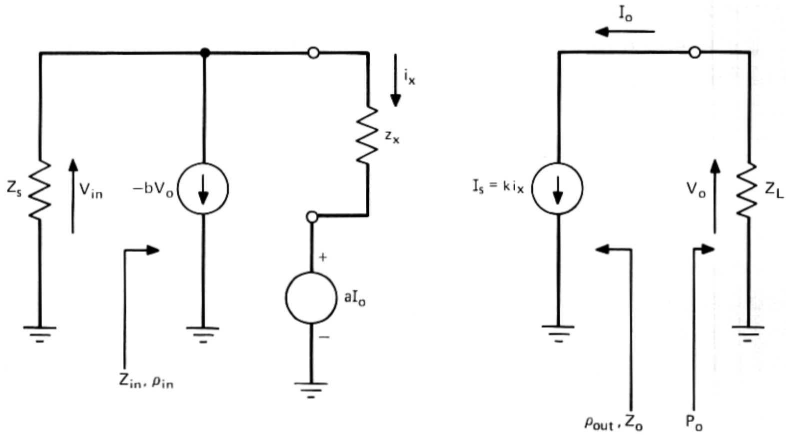


Fig. 2—First feedback form.

of one feedback network. Shunt current source $-bV_o$ likewise is the important characteristic of the second feedback network. Since the series feedback voltage source is dependent upon output current, it represents a transimpedance feedback. Similarly, the shunt current source is dependent on the output voltage yielding a transadmittance feedback. Source and load impedances, Z_s and Z_L, summarize amplifier interaction with the driving circuitry and the loading circuitry, respectively.

2.1 Input and output impedance

Z_{in} and Z_{out}, the input and output impedance, are desired to be matched to Z_s and Z_L, respectively. Thus, Z_{in} and Z_{out} are needed and are given by

$$Z_{\text{in}} = \frac{z_x + ka}{1 + kbZ_L}, \tag{1}$$

$$Z_{\text{out}} = \frac{z_x + Z_s + ka}{kbZ_s}. \tag{2}$$

If the amplifier gain k is large, then Z_{in} and Z_{out} become

$$Z_{\text{in}} = \frac{ak}{bkZ_L} = \frac{a}{bZ_L}, \tag{3}$$

$$Z_{\text{out}} = \frac{ak}{bkZ_s} = \frac{a}{bZ_s}. \tag{4}$$

For the matched condition, $Z_{\text{in}} = Z_s^*$ and $Z_{\text{out}} = Z_L^*$. Using these conditions in eqs. (3) and (4) yields

$$Z_s^* = Z_{\text{in}} = \frac{a}{bZ_L}, \tag{5}$$

$$Z_L^* = Z_{\text{out}} = \frac{a}{bZ_s}. \tag{6}$$

Substituting $(Z_L^*)^* = Z_L$ from eq. (6) into (5) yields

$$Z_s^* = \frac{a}{b(a^*/b^*Z_s^*)}, \tag{7}$$

which implies

$$\frac{a^*}{b^*} = \left(\frac{a}{b}\right)^* = \frac{a}{b}. \tag{8}$$

Thus, conjugate matching yields the requirement that the ratio of a to b (or more generally ka to kb) must be real. Given this fact, eqs. (5) and (6) are identical, i.e.,

$$Z_s^* Z_L = Z_s Z_L^* = \frac{a}{b}. \tag{9}$$

The imaginary part of $Z_L\, Z_s^*$ is therefore constrained by

$$\text{Im}\,\{Z_L\}\ \text{Re}\,\{Z_s\} - \text{Re}\,\{Z_L\}\ \text{Im}\,\{Z_s\} = 0. \tag{10}$$

A necessary condition for an amplifier to be absolutely stable is that Z_{in} and Z_{out} be passive.[12] This is satisfied when the real parts of Z_s and Z_L are positive. Thus, the imaginary part of Z_s and Z_L have the same sign, implying that if the matched load impedance is capacitive (inductive), then the matched source impedance is capacitive (inductive).

If the reflection coefficient [reflection coefficient ρ is defined as $(Z - Z_{\text{ref}}^*)/(Z + Z_{\text{ref}})$][13,14] at the input is evaluated (assuming $Z_s = a/bZ_L^*$), the following is obtained:

$$\rho_{\text{in}} = \rho_{\text{in}_0}\left(1 + \frac{2ab\ \text{Re}\ (Z_L)}{z_x bZ_L^* + a}\,k\right)^{-1}. \tag{11}$$

In eq. (11) ρ_{in_0} is the input reflection coefficient when $k = 0$.

Evaluating the return ratio T (Ref. 2) of the circuit in Fig. 2 with respect to the output dependent current source gives

$$T = \frac{-2ab\ \text{Re}\ (Z_L)}{z_x bZ_L^* + a}\,k. \tag{12}$$

Return difference F (Ref. 2) is defined as $1 - T$; thus, eq. (11) can be rewritten as

$$\rho_{\text{in}} = \rho_{\text{in}_0}\frac{1}{F}. \tag{13}$$

Therefore, the return loss ($20 \log 1/|\rho|$) is improved with increasing return difference F. Since the output reflection coefficient is given by

$$\rho_{out} = \rho_{out_0} \frac{1}{F}, \tag{14}$$

it also realizes the same improvement with increased return difference.

2.2 Gain equations

The transducer gain for the circuit in Fig. 2 can be calculated when it is assumed that $Z_s = a/bZ_L^*$:

$$|S_{21}|^2 = \frac{1}{|ab|} \frac{|T|^2}{|1 - T|^2}. \tag{15}$$

Again, T is the return ratio and is given by eq. (12).

In eq. (15), T is proportional to k. Thus, for large k, $|S_{21}|^2$ goes to $1/|ab|$. Therefore, eq. (15) can be rewritten as

$$|S_{21}|^2 = |S_{21\infty}|^2 \frac{|T|^2}{|1 - T|^2}, \tag{16}$$

where

$$|S_{21\infty}|^2 = \frac{1}{|ab|}. \tag{17}$$

2.3 Design procedure

In the derivations given thus far, a definite effort has been made to separate the dependence of k. This was done for two reasons: (*i*) to allow an initial design to be effected with k not a variable, and (*ii*) to allow definitive statements to be easily made concerning the effects of k. The former can easily be implemented by assuming $k = \infty$.

In this case of $k = \infty$, eqs. (8), (9), and (17) are relevant. These equations are repeated for convenience:

$$\left(\frac{a}{b}\right)^* = \frac{a}{b} \tag{8}$$

$$Z_s Z_L^* = \frac{a}{b} \tag{9}$$

$$|S_{21\infty}|^2 = \frac{1}{|ab|}. \tag{17}$$

It should be noted that eq. (8) implies that a/b is real, but a and b can be complex. $|S_{21\infty}|^2$ in eq. (17) is the maximum available gain since it is obtained with the input and output matched.

In summary, the design procedure given below could be used when the desired gain g and impedance matches are known.

(*i*) Choose an arbitrary starting Z_s and Z_L such that $Z_s Z_L^*$ is real.

(*ii*) Substitute a from eq. (9) into eq. (17), yielding

$$g = |S_{21\infty}|^2 = \frac{1}{Z_s Z_L^* |b|^2}. \tag{18}$$

Synthesize b such that

$$|b| = \frac{1}{\sqrt{Z_s Z_L^* |g|}}. \tag{19}$$

There are no constraints upon the phase of b except those that may result from stability considerations.

(*iii*) Synthesize a such that

$$a = b Z_s Z_L^*. \tag{20}$$

(*iv*) The value of k is now obtained by considering the practical active devices used to simulate the ideal amplifying element. With k known, the return ratio T [eq. (12)] can be calculated; this yields the obtainable impedance match and gain deviation, eqs. (13), (14), and (16). If the design objectives are not met, the previous calculations should make the necessary changes evident, e.g., lower Z_s or a higher value of k.

III. CURRENT TRANSFER SHUNT; VOLTAGE TRANSFER SERIES FEEDBACK

The last multiloop feedback circuit to be considered is shown in Fig. 3. In this case, the series feedback voltage source is dependent upon the output voltage and thus represents a voltage transfer feedback. Similarly, the shunt current source is a current transfer feedback. The voltage source is given by aV_0 and the current source by $-bI_0$, otherwise Figs. 2 and 3 are identical.

3.1 Input and output impedance

The input and output impedances, when evaluated, are given by

$$Z_{\text{in}} = \frac{z_x - akZ_L}{1 - bk}, \tag{21}$$

$$Z_{\text{out}} = \frac{Z_s + z_x - kZ_s b}{-ka}. \tag{22}$$

For large k, eqs. (21) and (22) become

$$Z_{\text{in}} = \frac{a}{b} Z_L, \tag{23}$$

$$Z_{\text{out}} = \frac{b}{a} Z_s. \tag{24}$$

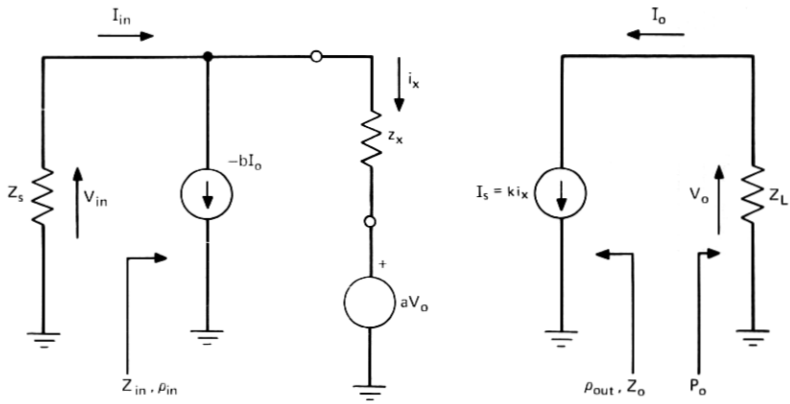


Fig. 3—Second feedback form.

The conditions of input and output matches yield

$$Z_s^* = Z_{\text{in}} = \frac{a}{b} Z_L = \frac{a}{b} (Z_{\text{out}})^* = \frac{a}{b} \left(\frac{b}{a}\right)^* Z_s^*. \tag{25}$$

Thus,

$$\left(\frac{a}{b}\right)^* = \left(\frac{a}{b}\right), \tag{26}$$

and

$$\frac{Z_s^*}{Z_L} = \frac{Z_s}{Z_L^*} = \frac{a}{b}. \tag{27}$$

Since a/b is real and the real part of Z_s and Z_L are nonnegative, then eq. (27) implies that if Z_s is capacitive (inductive) then Z_L must be inductive (capacitive).

Input and output reflection coefficients can be evaluated along with the return ratio and return difference. The results are shown below for Z_s and Z_L, satisfying eq. (27).

$$T = \frac{2kab \text{ Re } (Z_L)}{bz_x + aZ_L^*}. \tag{28}$$

$$F = 1 - T. \tag{29}$$

$$\rho_{\text{in}} = \rho_{\text{in}_0} \frac{1}{F}; \qquad \rho_{\text{in}_0} = \frac{bz_x - aZ_L}{bz_x + aZ_L^*}. \tag{30}$$

$$\rho_{\text{out}} = \rho_{\text{out}_0} \frac{1}{F} = \frac{1}{F}; \qquad \rho_{\text{out}_0} = 1. \tag{31}$$

Thus, as in the case of the first circuit form, the reflection coefficients are improved by the return difference.

3.2 Gain equation

When the load and source impedances satisfy eq. (27), the transducer gain for the circuit in Fig. 3 is given by

$$|S_{21}|^2 = |S_{21\infty}|^2 \frac{|T|^2}{|1 - T|^2}, \tag{32}$$

$$|S_{21\infty}|^2 = \frac{1}{|ab|}. \tag{33}$$

This is the same form as was given in eqs. (16) and (17); thus, the same statements apply to the above equations concerning improvement with feedback.

3.3 Design procedure

Initial circuit design can proceed in a manner similar to the first case. The term k again is assumed equal to infinity; this yields the germane equations summarized below.

$$\left(\frac{a}{b}\right)^* = \frac{a}{b}. \tag{26}$$

$$\frac{Z_s^*}{Z_L} = \frac{a}{b}. \tag{27}$$

$$|S_{21\infty}|^2 = \frac{1}{|ab|}. \tag{33}$$

The four design steps outlined previously apply except as follows.

(*i*) Choose Z_s and Z_L such that Z_s^*/Z_L is real.

(*ii*) Substitute eq. (27) into (33) so that

$$g = |S_{21\infty}|^2 = \frac{1}{\frac{Z_s^*}{Z_L}|b|^2}. \tag{34}$$

Synthesize b such that

$$|b| = \frac{1}{\sqrt{\frac{Z_s^*}{Z_L}|g|}}. \tag{35}$$

(*iii*) Synthesize a such that

$$a = \frac{Z_s^*}{Z_L} b. \tag{36}$$

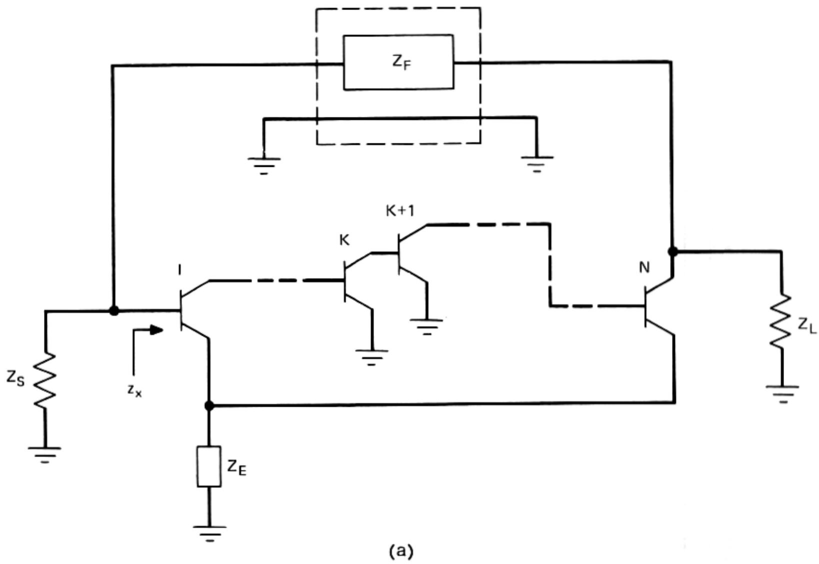
(*iv*) With k known, the return ratio is obtained from eq. (28). Equations (30), (31), and (32) then yield the obtainable impedance matches and gain deviation, respectively.

IV. DESIGN EXAMPLES

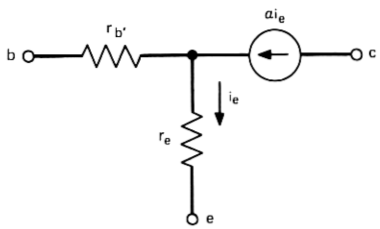
Results obtained in the last two sections will now be applied to a basic amplifier consisting of a cascade of N common emitter transistor stages. Transistors will be assumed to be used in a frequency range well below cutoff. The first case to be treated is for N odd.

4.1 N odd

Consider the circuit given in Fig. 4a. In this circuit, the transistor will be modeled by the circuit given in Fig. 4b. The circuit given in Fig. 4a will now be converted to the form given in Fig. 2. Z_s and Z_L have their obvious counterparts. z_x is given by the impedance from base to ground with aI_0 equal to zero; this is obtained when $I_0 = 0$, which can be obtained by setting α_1 (first stage α) to zero. From the



(a)



(b)

Fig. 4—Design example N odd.

transistor model, this is evidently

$$z_x = r_b' + r_e + Z_E. \tag{37}$$

I_s is given by the collector current on the Nth transistor when the base current on the first is unity. With the cascade of N transistors, this yields

$$I_s = \beta_1, \cdots, \beta_N i_x; \qquad \beta_i = \frac{\alpha_i}{1 - \alpha_i}; \qquad k = \beta_1, \cdots, \beta_N. \tag{38}$$

The term a is given as the value of open circuit input voltage (Z_s and Z_F removed) that exists when I_0 equals unity. This is given by

$$a = Z_E + \frac{Z_E + r_e}{\beta_2, \cdots, \beta_N}, \qquad N > 1 \tag{39}$$

$$a = Z_E + r_e, \qquad N = 1. \tag{40}$$

For $|\beta_2, \cdots, \beta_N| \gg |Z_E + r_e|$, eq. (39) can be approximated by

$$a = Z_E, \qquad N > 1. \tag{41}$$

The last remaining parameter b can be obtained by evaluating the y_{12} parameter of the network Y, yielding

$$b = -(y_{12}) = -\left(\frac{-1}{Z_F}\right) = \frac{1}{Z_F}. \tag{42}$$

Loading effects of the Y networks, i.e., y_{11}, y_{22}, can be ignored if they are sufficiently small.

As a numerical example, the value of k, Z_F, and Z_E are calculated to yield an input and output reflection coefficient of 0.18 (return loss of 15 dB) and a gain to within a factor of 1.26 (1 dB) of $f^2/400$, f in MHz, in the band from 80 to 140 MHz. The remaining parameters of the transistors are $r_b' = 1$, and $r_e = 0.173$.

Following the four-step design procedure yields

(*i*) Let $Z_s = Z_L = 20$, $Z_s Z_L^* = 400$.

(*ii*) $g = \dfrac{f^2}{400} = \dfrac{1}{400|b|^2}$

$|b| = \dfrac{1}{f} = \left|\dfrac{1}{Z_f}\right|$

$|Z_f| = f$, f in MHz.

If Z_f is chosen as an inductor, then

$|Z_f| = 2\pi f L = f$; thus,

$L = 1/2\pi\ \mu\text{H}$, and

$b = \dfrac{1}{2\pi f L j} = \dfrac{1}{fj}.$

(*iii*) $a = Z_s Z_L^* b = 400b = 400 \dfrac{1}{Z_f} = \dfrac{400}{2\pi L f j}$

$a = \dfrac{400}{fj}$; thus, a can be realized when

$Z_E(\simeq a)$ is a capacitor of value

$C = \dfrac{1}{2\pi(400)}\,\mu\mathrm{F}.$

(*iv*) Using eq. (12) and $a \approx Z_E$, the following is obtained:

$$T = \frac{-2ab\,\mathrm{Re}\,(X_L)}{z_a b Z_L^* + a}\,k = \left(-2\,\frac{400}{fj}\,\frac{1}{fj}\,20\right)k \Bigg/ \left[\left(1.173 + \frac{400}{fj}\right)\frac{1}{fj}\,20 + \frac{400}{fj}\right]$$

$$T = \frac{-16000k}{8000 + fj423.4} = \begin{cases} -0.46e^{-j77°}k \text{ at } 80 \text{ MHz} \\ -0.27e^{-j82°}k \text{ at } 140 \text{ MHz.} \end{cases}$$

For $k = 20$, $|1 - T| = |F|$ at 140 MHz (the worst case point) is given by $|1 + 5.4e^{-j82°}| = |1.75 - j5.35| = 5.63$. This reduces the reflection coefficient by $1/5.03 = 0.18$. Thus, the input and output reflection coefficient specification is initially satisfied.

The gain deviation at 140 MHz is calculated from eq. (15) and is

$$\frac{|T|^2}{|1 - T|^2} = \frac{|5.4|^2}{|5.63|^2} = 0.919.$$

This implies a gain deviation from nominal of 0.37 dB, and initially satisfies the design requirements.

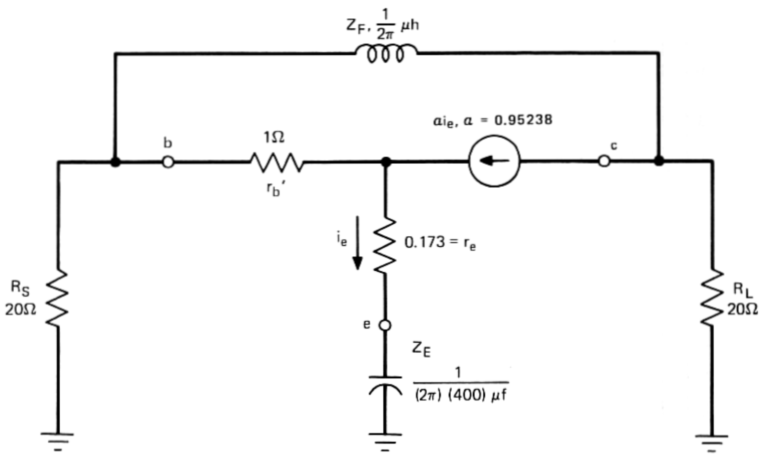


Fig. 5—Numerical example N odd.

The resulting circuit for N odd is given in Fig. 5. The actual transducer gain, input impedance, and output impedance values for this circuit were obtained by a computer-aided design program.[15] A summary of the results is given in Fig. 6. It can be seen from this figure that this procedure yields a practical first iteration in the design procedure.

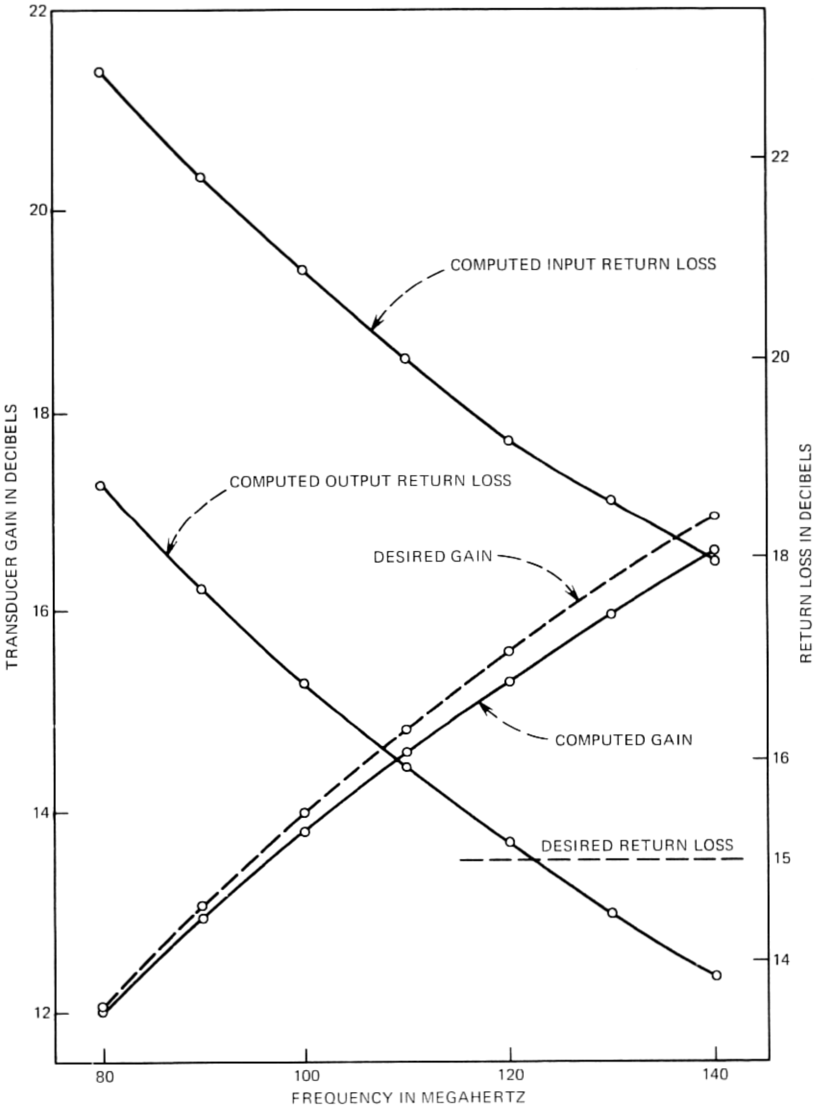


Fig. 6—N odd results.

4.2 N even

For an even number of transistors, the circuit in Fig. 7 is used. The transistor model of Fig. 4b is again used. The evaluation of the parameters of Fig. 3 follows on the same basis as in the N odd case. Using Fig. 7, the calculations are summarized below.

$$z_x \approx r_b + r_e + Z_E, \tag{43}$$

$$I_s = \beta_1, \cdots, \beta_N i_x; \qquad k = -\beta_1, \cdots, \beta_N, \tag{44}$$

$$a = \frac{Z_E}{200 + Z_E} + \frac{r_e + (200Z_E)/(Z_E + 200)}{Z_L\beta_2, \cdots, \beta_N}$$

$$\approx \frac{Z_E}{200}; \qquad \beta_i \gg 1, \qquad i = 2, \cdots, N, \qquad |Z_E| \ll 200 \tag{45}$$

$$b = \frac{2}{Z_F + 2} \approx \frac{2}{Z_F}, \qquad |Z_F| \gg 2. \tag{46}$$

A numerical example is given to show the initial design steps for obtaining a maximum input and output reflection coefficient of 0.18 and a gain to within 1.26 of f^2 (f in MHz), from 80 to 140 MHz. The transistor parameters are again $r_b' = 1$ ohm and $r_e = 0.173$.

The four design steps yield

(*i*) Let $Z_s = Z_L = 20, \dfrac{Z_s}{Z_L^*} = 1.$

(*ii*) $g = f^2 = \dfrac{1}{|b|^2}$

$|b| = \dfrac{1}{f} = \dfrac{2}{|Z_f|}$

$|Z_F| = 2f.$

Let $Z_F = j2f = j2\pi fL$, L in μH

$L = \dfrac{1}{\pi}\,\mu\text{H}.$

(*iii*) $a = b = \dfrac{2}{j2\pi fL} = \dfrac{Z_E}{200} = \dfrac{1}{jf}.$

Thus, $Z_E = \dfrac{200}{jf}.$

This implies that Z_E is a capacitor of value

$C = \dfrac{1}{2\pi(200)}\,\mu\text{F}.$

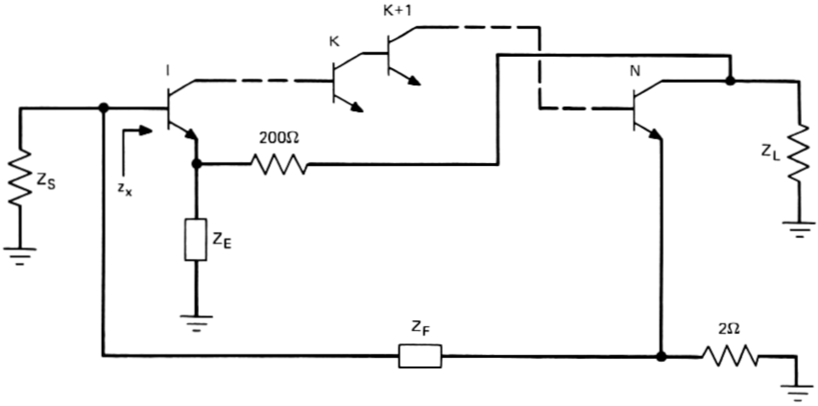


Fig. 7—Design example N even.

(*iv*) Using eq. (32),

$$T = \frac{2ab \text{ Re } (Z_L)}{bz_x + aZ_L^*} k = \frac{2(1/jf)(1/jf)20}{(1/jf)(r_b' + r_e + 200/jf) + 20/jf}$$

$$T = \frac{40}{200 + 21.173jf} k = \begin{cases} 0.023e^{-83^\circ j}k \text{ at } 80 \text{ MHz} \\ 0.015e^{-86^\circ j}k \text{ at } 140 \text{ MHz.} \end{cases}$$

For $k = -400$, $|1 - T| = |F|$ at 140 MHz is given by $|1 + 6e^{-86^\circ j}| = 6.1$.

The reflection coefficient is reduced by a factor of 1/6.1 (15.7 dB). Gain deviation can be calculated and is equal to 0.95 (0.22 dB); thus, the initial specifications are satisfied.

Figure 8 gives the resulting circuit. The results of the computer analysis of this circuit are given in Fig. 9. Again the data show that the approach yields good results.

It can be seen in Fig. 9 that the difference in gain is greater than the computed 0.22 dB. This is due to the fact that a was taken as $Z_E/200$, rather than the term given in eq. (45). A more accurate evaluation (denoted by the hatted variables) of a is given as

$$\hat{a} = \frac{Z_E}{200 + Z_E} + \frac{r_e + (200Z_E)/(Z_E + 200)}{Z_L\beta_2} \approx \frac{Z_E}{200} + \frac{Z_E}{Z_L\beta_2}$$

$$\hat{a} \approx \frac{Z_E}{200} + \frac{Z_E}{(20)(80)} = \frac{Z_E}{200}(1 + 0.125) = 1.125a,$$

where a was the numerical value previously obtained. Using eq. (33) yields

$$|\hat{S}_{21\infty}|^2 = \frac{1}{|\hat{a}\hat{b}|} = \frac{1}{|1.125ab|} = \frac{1}{1.125}\frac{1}{|ab|} = \frac{1}{1.125}|S_{21\infty}|^2.$$

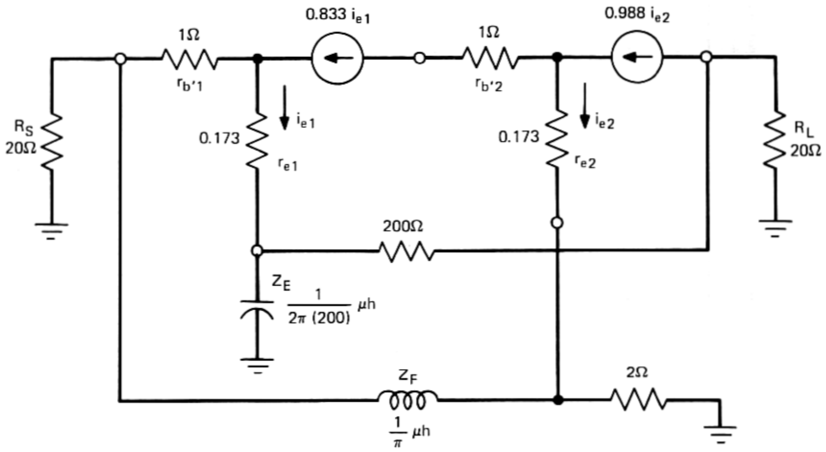


Fig. 8—Numerical example N even.

Again the unhatted quantities were the ones used in the five design steps. The factor of 1.125 accounts for an additional 0.51 dB of the gain difference.

In this example, the gain difference can easily be reduced by increasing the β of the second-stage transistor. This was not done since it was desired to keep the $\beta_1\beta_2$ product at 400. Since β_2 is 80, this forces β_1 to be 5; any high value of β_2 results in unrealistic values of β_1. Nonetheless it is evident that a high $\beta_2, \cdots, \beta_n$ product is needed for an even number of cascade stages.

V. CONCLUSIONS

In this paper, the basic characteristics of two forms of major multiloop feedback have been investigated. The design characteristics treated have been input and output impedance and frequency-dependent power gain. It has been shown that, with sufficient open loop gain, the equations that describe the gain and impedance quantities are very simple in nature. An initial circuit-design iteration can easily be performed since many complicating variables are eliminated.

This initial circuit-design concept would be extremely useful in a computer circuit analysis-optimization program. Well known is the major practical limitation of optimization programs: the obtaining of a convergent starting point. For dual-loop amplifiers, this paper offers the designer a method of easily finding a good starting point.

Although not reported here, several frequency-shaped amplifiers were actually built using multiloop feedback. The excellent perform-

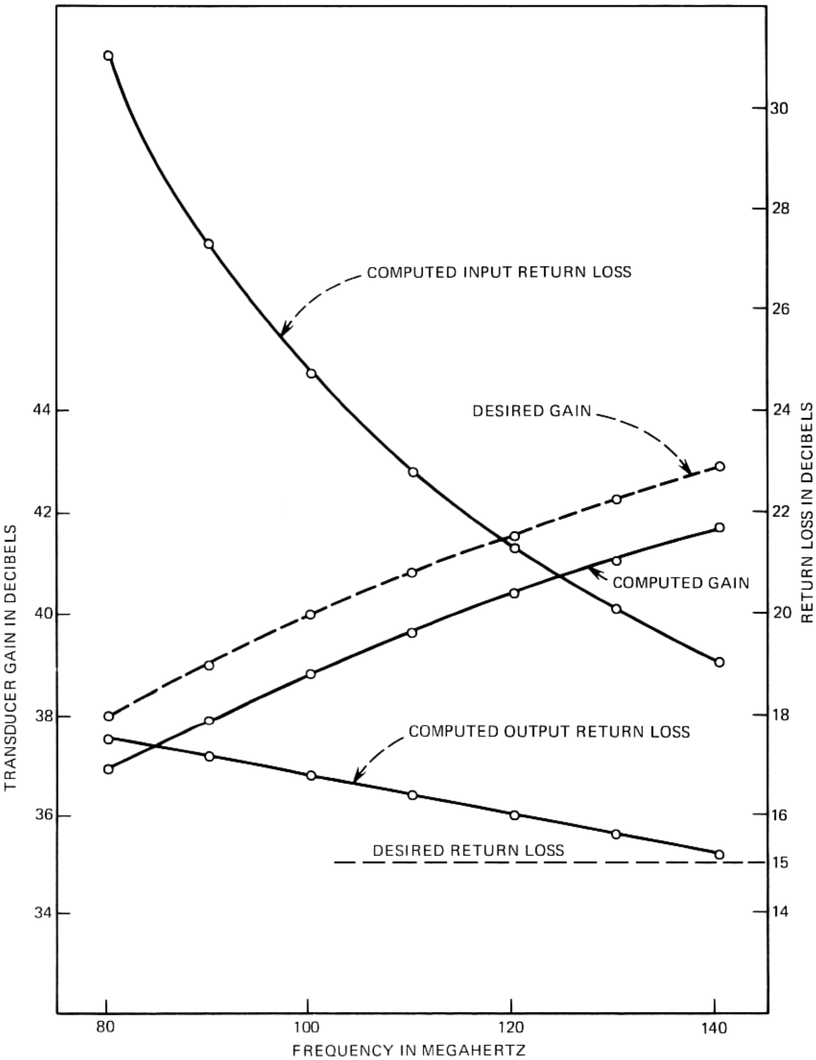


Fig. 9—N even results.

ance of these amplifiers, with respect to input and output matching and gain shaping, has precipitated the work reported in this paper. We anticipate that future papers will discuss more complicated active devices, feedback network loading effects, and feedback network synthesis.

APPENDIX A

Shunt Transadmittance: Series Transimpedance Feedback Calculations

A.1 Calculation of Z_{in}

$$i_x = \frac{V_{\text{in}} = aki_x}{z_x};$$
$$i_x(z_x + ak) = V_{\text{in}};$$
$$V_0 = -\,ki_xZ_L.$$

Thus,

$$I_{\text{in}} = i_x - bV_0 = i_x + kbi_xZ_L;$$
$$I_{\text{in}} = (1 + kbZ_L)i_x = \frac{(1 + kbZ_L)}{z_x + ak}V_{\text{in}};$$
$$Z_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{z_x + ka}{1 + kbZ_L}.$$

A.2 Calculation of Z_{out}

$$i_x = bV_0\frac{Z_s}{z_x + Z_s} - \frac{aI_0}{z_x + Z_s};$$
$$ki_x = I_0 = k\left(bV_0\frac{Z_s}{Z_s + z_x}\right) - k\left(\frac{aI_0}{z_x + Z_s}\right);$$
$$I_0(z_x + Z_s + ka) = kbZ_sZ_0;$$
$$Z_0 = \frac{V_0}{I_0} = \frac{(z_x + Z_s + ka)}{kbZ_s}.$$

A.3 Input reflection coefficient calculation

$$\rho_{\text{in}} = \frac{Z_{\text{in}} - Z_s^*}{Z_{\text{in}} + Z_s}, \qquad Z_{\text{in}} = \frac{z_x + ka}{1 + kbZ_L}, \qquad Z_s = \frac{a}{bZ_L^*},$$

$$\rho_{\text{in}} = \left(\frac{z_x + ka}{1 + bkZ_L} - \frac{a}{bZ_L}\right)\Big/\left(\frac{z_x + ka}{1 + kbZ_L} + \frac{a}{bZ_L^*}\right)$$
$$= \frac{z_xbZ_L + abkZ_L - a - abkZ_L}{z_xbZ_L^* + abkZ_L^* + a + abkZ_L},$$

$$\rho_{\text{in}} = \frac{z_xbZ_L - a}{z_xbZ_L^* + a + abk[2\,\text{Re}\,(Z_L)]},$$

$$\rho_{\text{in}_0} = \rho_{\text{in}|\,k=0} = \frac{z_xbZ_L - a}{z_xbZ_L^* + a}.$$

Therefore,

$$\rho_{\text{in}} = \frac{z_xbZ_L - a}{z_xbZ_L^* + a} \times \left(1 + \frac{2ab\,\text{Re}\,(Z_L)}{z_xbZ_L^* + a}k\right)^{-1}$$
$$= \rho_{\text{in}_0}\left(1 + \frac{2ab\,\text{Re}\,(Z_L)}{z_xbZ_L^* + a}k\right)^{-1}.$$

A.4 Return ratio calculation

Assuming the output current source I_s is disconnected and replaced by a 1-ampere current source, T is given by the current that flows through the disconnected current source.

$$i_x = bV_0 \frac{Z_s}{z_x + Z_s} - \frac{aI_0}{z_s + Z_s}; \qquad I_s = 1, \qquad V_0 = -Z_L,$$

$$i_x = -bZ_L \frac{Z_s}{z_x + Z_s} - \frac{a}{z_x + Z_s} = \frac{-bZ_L Z_s - a}{z_x + Z_s}; \qquad Z_s = \frac{a}{bZ_L^*};$$

$$T = ki_x = k \frac{-bZ_L(a)/(bZ_L^*) - a}{z_x + (a)/(bZ_L^*)} = -k \frac{abZ_L + abZ_L^*}{bz_x Z_L^* + a}$$

$$= \frac{-2ab \operatorname{Re}(Z_L)}{bz_x Z_L^* + a} k.$$

A.5 Output reflection coefficient

$$\rho_{\text{out}} = \frac{Z_{\text{out}} - Z_L^*}{Z_{\text{out}} + Z_L}; \qquad Z_{\text{out}} = \frac{z_x + Z_s + ka}{kbZ_s}; \qquad Z_L = \frac{a}{bZ_s^*};$$

$$\rho_{\text{out}} = \left(\frac{z_x + Z_s + ka}{kbZ_s} - \frac{a}{bZ_s}\right) \Big/ \left(\frac{z_x + Z_s + ka}{kbZ_s} + \frac{a}{bZ_s^*}\right),$$

$$\rho_{\text{out}} = \left(\frac{z_x + Z_s + ak - ak}{kbZ_s}\right) \Big/ \left(\frac{z_x Z_s^* + Z_s Z_s^* + akZ_s^* + akZ_s}{kbZ_s Z_s^*}\right),$$

$$\rho_{\text{out}} = \frac{z_x Z_s^* + Z_s Z_s^*}{z_x Z_s^* + Z_s Z_s^* + ak(Z_s + Z_s^*)},$$

$$\rho_{\text{out}_0} = \rho_{\text{out}}|_{k=0} = \frac{z_x Z_s^* + Z_s Z_s^*}{z_x Z_s^* + Z_s Z_s^*} = 1.$$

Therefore,

$$\rho_{\text{out}} = \rho_{\text{out}_0} \left(1 + \frac{ak(Z_s + Z_s^*)}{z_x Z_s^* + Z_s Z_s^*}\right)^{-1}; \qquad Z_s = \frac{a}{bZ_L^*};$$

$$\rho_{\text{out}} = \rho_{\text{out}_0} \left(1 + \frac{ak(a/bZ_L^* + a/bZ_L)}{z_x(a/bZ_L) + (a/bZ_L^*)(a/bZ_L)}\right)^{-1}$$

$$= \rho_{\text{out}_0} \left(1 + \frac{kabZ_L + kabZ_L^*}{z_x bZ_L^* + a}\right)^{-1},$$

$$\rho_{\text{out}} = \rho_{\text{out}_0} \left(1 + \frac{2ab \operatorname{Re}(Z_L)}{bz_x Z_L^* + a} k\right)^{-1}.$$

A.6 Transducer gain calculation

Assume a voltage source of value V_s is inserted in series with the source impedance Z_s in Fig. 2. Let $Z_s = a/bZ_L^*$. P_{AS} will denote the available power from the source and P_{out} the real power delivered to the load Z_L.

$$P_{AS} = \frac{|V_s|^2}{4\,\text{Re}\,(Z_s)} = \frac{|V_s|^2|Z_L|^2|b|}{4\,\text{Re}\,(Z_L)\,|a|};$$

$$V_{\text{in}} = \frac{V_s Z_{\text{in}}}{Z_s + Z_{\text{in}}} = \left(V_s \frac{z_x + ka}{1 + kbZ_L}\right) \Big/ \left(\frac{a}{bZ_L^*} + \frac{Z_x + ka}{1 + kbZ_L}\right)$$

$$= \frac{(z_x + ka)bZ_L^*}{a + kabZ_L + (z_x + ka)bZ_L^*} V_s,$$

$$V_{\text{in}} = \frac{(z_x + ka)bZ_L^* V_s}{a + z_x bZ_L^* + 2kab\,\text{Re}\,(Z_L)} = \frac{(z_x + ka)bZ_L^*}{a + z_x bZ_L^*} \frac{1}{1 - T} V_s$$

$$\frac{V_{\text{in}} - aI_0}{z_x} = i_x = \frac{ki_x}{k} = \frac{I_0}{k}.$$

Thus,

$$I_0 = \frac{k}{z_x + ka} V_{\text{in}};$$

$$P_{\text{out}} = I_0 I_0^* \,\text{Re}\,(Z_L) = \frac{|k|^2\,\text{Re}\,(Z_L)}{|z_x + ka|^2} |V_{\text{in}}|^2,$$

$$P_{\text{out}} = \frac{|k|^2\,\text{Re}\,(Z_L)}{|z_x + ka|^2} \frac{|z_x + ka|^2|b|^2|Z_L|^2}{|a + z_x bZ_L^*|^2} \frac{|V_s|^2}{|1 - T|^2}.$$

Therefore,

$$|S_{21}|^2 = \frac{P_{\text{out}}}{P_{AS}} = \frac{|k|^2\,\text{Re}\,(Z_L)|b|^2|Z_L|^2|V_s|^2 4\,\text{Re}\,(Z_L)|a|}{|a + z_x bZ_L^*|^2|1 - T|^2|V_s|^2|Z_L|^2|b|}$$

$$= \frac{|k|^2\,\text{Re}^2\,(R_L)|ab|4}{|a + z_x bZ_L^*|^2|1 - T|^2} = \frac{1}{|ab|} \frac{|T|^2}{|1 - T|^2}.$$

APPENDIX B

Current Transfer Shunt: Voltage Transfer Series Feedback Calculations

B.1 Calculation of Z_{in}

$$i_x = \frac{V_{\text{in}} - aV_0}{z_x} = \frac{V_{\text{in}} + aki_x Z_L}{z_x},$$

$$i_x = \frac{V_{\text{in}}}{z_x - akZ_L};$$

$$I_{\text{in}} = -bI_0 + i_x = (-bk + 1)i_x = \frac{V_{\text{in}}(1 - bk)}{z_x - akZ_L};$$

$$Z_{\text{in}} = \frac{V_{\text{in}}}{I_{\text{in}}} = \frac{z_x - akZ_L}{1 - bk}.$$

B.2 Calculation of $\mathbf{Z}_{out}$

$$i_x = \frac{bI_0 Z_s}{Z_s + z_x} - \frac{aV_0}{Z_s + z_x} = \frac{Z_s bI_0 - aV_0}{Z_s + z_x};$$

$$ki_x = I_0 = \frac{kZ_s bI_0 - kaV_0}{Z_s + z_x};$$

$$I_0(Z_s + z_x - kZ_s b) = -kaV_0;$$

$$Z_{\text{out}} = \frac{V_0}{I_0} = \frac{Z_s + z_x - kZ_s b}{-ka}.$$

B.3 Return ratio calculation

$$i_x = bI_0 \frac{Z_s}{z_x + Z_s} - aV_0 \frac{1}{z_x + Z_s}, \qquad I_0 = 1, \qquad V_0 = -Z_L;$$

$$i_x = \frac{bZ_s}{z_x + Z_s} + \frac{aZ_L}{z_x + Z_s}; \qquad Z_s = \frac{a}{b} Z_L^*;$$

$$T = ki = \frac{kb(a/b)Z_L^* + aZ_L k}{z_x + (a/b)Z_L^*} = \frac{2kab \operatorname{Re}(Z_L)}{bz_x + aZ_L^*}.$$

B.4 Input reflection coefficient calculation

$$\rho_{\text{in}} = \frac{Z_{\text{in}} - Z_s^*}{Z_{\text{in}} + Z_s}; \qquad Z_{\text{in}} = \frac{z_x - akZ_L}{1 - bk}, \qquad Z_s = (a/b)Z_L^*, \qquad (a/b) \text{ real};$$

$$\rho_{\text{in}} = \left(\frac{z_x - akZ_L}{1 - bk} - \frac{aZ_L}{b}\right) \Big/ \left(\frac{z_x - akZ_L}{1 - bk} + \frac{a}{b} Z_L^*\right)$$

$$= \frac{bz_x - abkZ_L - aZ_L + abkZ_L}{bz_x - abkZ_L + aZ_L^* - abkZ_L^*},$$

$$\rho_{\text{in}} = \frac{bz_x - aZ_L}{bz_x + aZ_L^* - abk \operatorname{Re}(Z_L)},$$

$$\rho_{\text{in}_0} = \rho_{\text{in}}|_{k=0} = \frac{bz_x - aZ_L}{bz_x + aZ_L^*};$$

therefore,

$$\rho_{\text{in}} = \rho_{\text{in}_0}\left(1 - \frac{2abk \operatorname{Re}(Z_L)}{bz_x + aZ_L^*}\right)^{-1}.$$

B.5 Output reflection coefficient calculation

$$\rho_{\text{out}} = \frac{Z_{\text{out}} - Z_L^*}{Z_{\text{out}} + Z_L}; \qquad Z_{\text{out}} = \frac{Z_s + z_x - kZ_s b}{-ka},$$

$$Z_s = (a/b)Z_L^*, \qquad (a/b) \text{ real};$$

$$\rho_{\text{out}} = \left(\frac{Z_s + z_x - kZ_s b}{-ka} - Z_L^*\right) \Big/ \left(\frac{Z_s + z_x - kZ_s b}{-ka} + Z_L\right)$$

$$= \frac{Z_s + z_x - kZ_s b + kaZ_L^*}{Z_s + z_x - kZ_s b - kaZ_L} = \frac{Z_s + z_x}{Z_s + z_x - ka2\,\text{Re}\,(Z_L)},$$

$$\rho_{\text{out}_0} = \rho_{\text{out}}|_{k=0} = 1,$$

$$\rho_{\text{out}} = \rho_{\text{out}_0}\left(1 - \frac{2ka\,\text{Re}\,(Z_L)}{(a/b)Z_L^* + z_x}\right)^{-1} = \rho_{\text{out}_0}\left(1 - \frac{2kab\,\text{Re}\,(Z_L)}{aZ_L^* + bz_x}\right)^{-1}.$$

B.6 Transducer gain calculation

Assume a voltage source of value V_s is inserted in series with the source impedance Z_s in Fig. 3. Let $Z_s = (a/b)Z_L^*$, a/b real. P_{AS} will denote the available power from the source and P_{out} the real power delivered to the load Z_L.

$$P_{as} = \frac{|V_s|^2}{4\,\text{Re}\,(Z_s)} = \frac{|V_s|^2|b|}{4\,\text{Re}\,(Z_L)|a|};$$

$$V_{\text{in}} = \frac{V_s Z_{\text{in}}}{Z_s + Z_{\text{in}}} = \left(V_s \frac{z_x - akZ_L}{1 - bk}\right) \Big/ \left(\frac{a}{b} Z_L^* + \frac{z_x - akZ_L}{1 - bk}\right)$$

$$= \frac{V_s(bz_x - abkZ_L)}{aZ_L^* - abkZ_L^* + bz_x - abkZ_L},$$

$$V_{\text{in}} = V_s \frac{bz_x - abkZ_L}{aZ_L^* + bz_x - 2abk\,\text{Re}\,(Z_L)} = V_s \frac{bz_x - abkZ_L}{aZ_L^* + bz_x} \times \frac{1}{1 - T},$$

$$\frac{V_{\text{in}} - aV_0}{z_x} = i_x = \frac{ki_x}{k} = \frac{I_0}{k} = \frac{V_{\text{in}} + aI_0 Z_L}{z_x};$$

$$I_0 = V_{\text{in}} \Big/ \left(\frac{z_x}{k} - aZ_L\right) = \frac{k}{z_x - akZ_L} V_{\text{in}};$$

$$P_{\text{out}} = I_0 I_0^*\,\text{Re}\,(Z_L) = \frac{|k|^2|V_{\text{in}}|^2}{|z_x - akZ_L|^2}\,\text{Re}\,(Z_L),$$

$$P_{\text{out}} = \frac{|k|^2\,\text{Re}\,(Z_L)}{|z_x - akZ_L|^2} \frac{|bz_x - abkZ_L|^2}{|aZ_L^* + bz_x|^2} \frac{1}{|1 - T|^2} |V_s|^2,$$

$$P_{\text{out}} = \frac{|k|^2|b|^2\,\text{Re}\,(Z_L)|V_s|^2}{|aZ_L^* + bz_x|^2|1 - T|^2};$$

therefore,

$$|S_{21}|^2 = \frac{P_{\text{out}}}{P_{AS}} = \frac{|k|^2|b|^2\,\text{Re}\,(Z_L)|a|4\,\text{Re}\,(Z_L)|V_s|^2}{|aZ_L^* + bz_x|^2|1 - T|^2|V_s|^2|b|}$$

$$= \frac{1}{|ab|} \frac{|T|^2}{|1 - T|^2}.$$

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