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Engineering Traffic Networks for More Than One Busy Hour

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A procedure is described that is used to engineer traffic networks for more than one hour of point-to-point load data. The procedure differs significantly from existing methods, which are based upon the concept of "economic load on the last trunk" (ECCS). (When the peak-load hours on most routes coincide, however, the procedure reduces to the ECCS method.) This "multihour" procedure has been implemented in a computer program used in design studies of three end offices in the Los Angeles local network. For the cases examined, the multihour technique produced networks whose costs averaged approximately 7 percent below those achieved with the presently used single-hour methods. Thus, the multihour technique appears to promise considerable cost benefits in future network designs.

I. INTRODUCTION

In this paper, we describe a procedure used to engineer networks for more than one hour of point-to-point traffic data. Specifically, for a given routing structure, set of switching and transmission costs, and point-to-point offered load between each pair of offices for each of several hours, this method produces a (nearly) least-cost network that satisfies the constraint that the blocking probabilities on all final groups be below a predetermined value (the "grade of service") for all hours. This multihour procedure is a major departure from currently used single-hour methods based upon the concept of "economic load on the last trunk"[1] (ECCS). The new technique reduces to the ECCS method, however, when the peak-load hours on most routes coincide. After computer programs are written and operating practices are developed, this new procedure should become suitable for routine field use.

The underlying theoretical basis for multihour engineering was developed by Rapp.[2] Rather than attempting to construct an optimal solution, however, Rapp proposed an alternative approximate technique. Our aim is to get an exact solution. Although we do not fully achieve this aim, we obtain significant improvement in network performance relative to a single-hour approach.

A computer program that implements the multihour procedure was used to study three end offices located in the Los Angeles area. For the cases examined, where a significant amount of noncoincidence of peak-load hours existed, the multihour method produced network cost savings averaging 7 percent over the single-hour methods currently employed. In addition, in each case, a very sizable reduction of tandem switching load was achieved.

II. SINGLE-BUSY-HOUR ENGINEERING

Before discussing the multihour technique, let us first review the considerations involved in engineering for a single hour. Figure 1 depicts a single high-usage group, the direct route, overflowing to an alternate route. (For now, we make the simplifying assumption that the alternate route consists only of a single trunk group. We later consider more realistic alternate-route configurations.) The cost per trunk of the direct route is C_D, and the cost per trunk of the alternate route is C_A (which is assumed to include the cost of tandem switching). The offered load in the hour being considered is a. The problem is to determine the value of n, the number of trunks in the high-usage group, so that the total cost is minimized; however, the minimization of cost is subject to the con-

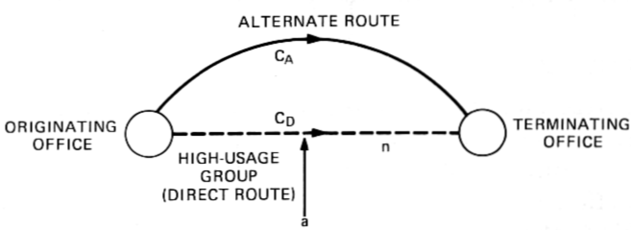


Fig. 1—High-usage trunk group overflowing to alternate route.

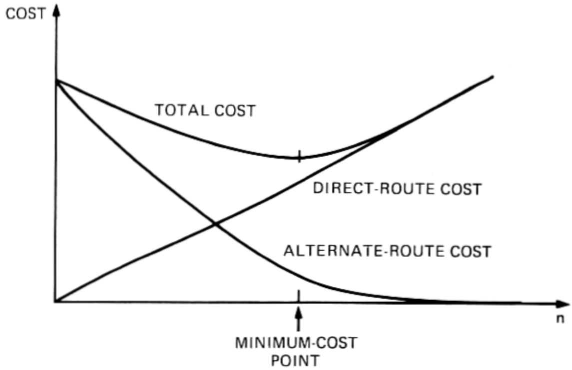


Fig. 2—Single-hour trunk-group sizing.

straint that the blocking probability on the alternate route is below a predetermined value.

The total cost is equal to the cost of trunks on the direct route plus the cost of trunks on the alternate route. To simplify matters, it can be assumed that the alternate-route cost is composed of a fixed component (the cost of carrying the "background" alternate-route load) plus a variable component (the cost of carrying the overflow) whose magnitude is proportional to the amount of overflow traffic.* Since the cost required to carry the background alternate-route load is independent of n, we may neglect this component and write the cost to be minimized as

$$\text{COST} = \frac{C_A}{\gamma} aB(n,a) + C_D n. \tag{1}$$

Here, γ is the marginal capacity of the alternate route, and $B(n,a)$ is the Erlang-B blocking probability. The marginal capacity is the amount of additional traffic that it is assumed can be offered to the alternate route, at fixed blocking, for the addition of one trunk. Thus, $aB(n,a)$ is the load overflowing to the alternate route, $1/\gamma\, aB(n,a)$ is the assumed number of additional alternate-route trunks required to carry this overflow, and $C_A/\gamma\, aB(n,a)$ is the cost of these additional trunks. $C_D n$ is, of course, the cost of trunks on the direct route. These two components of cost and their sum are shown as a function of n in Fig. 2. The total cost is seen to be a U-shaped curve having a minimum at the point indicated. This point is determined by the condition that the rate of change of COST with respect to n be equal to zero:

* This assumption is not quite true, particularly when the peakedness of the overflow traffic is taken into account. Nevertheless, in most cases of interest, the assumption yields a configuration whose cost differs negligibly from the optimal plan.

$$\frac{d}{dn}\,\text{COST} = 0. \tag{2}$$

From eq. (1), this implies

$$-\frac{d}{dn}\,[aB(n,a)] = \frac{\gamma}{C_R}, \tag{3}$$

where $C_R = C_A/C_D$ is the cost ratio, as discussed in Ref. 1. The quantity on the left-hand side of eq. (3), the rate of change of overflow with respect to the number of trunks on the high-usage group, is very nearly equal to the "load on the last trunk." * The quantity on the right-hand side of eq. (3) is the "economic" load on the last trunk, or ECCS (CCS is 100 call seconds per hour). Thus, the minimum cost is achieved by sizing the high-usage group such that its load on the last trunk is equal to its "economic" value, γ/C_R.

In this discussion, it is assumed that the network is designed to carry only a single hour's load. In practice, of course, the load on the high-usage group, as well as the background load on the alternate route, varies from hour to hour. The question arises as to which of the hours of loads should be used to engineer the group.

It is clear that it would be uneconomical to engineer a high-usage group for its individual group busy hour if this hour does not coincide with the busy hour of the alternate route; the alternate route has spare capacity in off-hours. A moment's reflection reveals that the appropriate hour for which to size the group is the alternate-route's busy hour; only in this hour does the cost of carrying the overflow traffic from the high-usage group need to be considered.

This fact has long been recognized by traffic engineers.[1] The method of choosing the engineering hour which was adopted, consequently, involved the concept of the "cluster busy hour." A "cluster" is defined as a set of high-usage trunk groups originating at the same office and overflowing to a common alternate-route leg, together with the alternate-route leg itself. The cluster busy hour is defined as that time-consistent hour for which the total load offered to the cluster (specifically, the sum of the carried loads on all high-usage groups in the cluster, plus the offered load on the alternate-route leg) is maximum. It was assumed that the alternate-route busy hour would be the same as the cluster busy hour, and thus the adopted engineering practice was to size every high-usage group for its cluster-busy-hour load.

The difficulties that can arise with this method, however, are illustrated by the example in Fig. 3. In the figure, we show a simple network

* "Load on the last trunk" is defined to be $a[B(n\text{-}1,a)\text{-}B(n,a)]$ if the group has n trunks.

cluster consisting of two one-way outgoing high-usage groups, A and B, overflowing to a common alternate route, F. The cost ratio is assumed to be 2, and the marginal capacity of the alternate route is assumed to be 28 CCS. Since a total of 1100 CCS is offered to the cluster in Hour 1 and 1000 CCS is offered in Hour 2, the cluster busy hour is Hour 1. For the Hour-1 loads the ECCS method yields a network consisting of 10 and 26 trunks on high-usage groups A and B, respectively. It is seen, however, that this network has a high overflow from group A during Hour 2. This high overflow occurs because the group is engineered for only 300 CCS, while in Hour 2 the offered load is 600 CCS. As a consequence, the total load on the alternate route is greater during Hour 2, contradicting the original assumption that the alternate route was busier in Hour 1. To guarantee a given grade of service in both hours, it is necessary to add trunks to the alternate route for its load in Hour 2.* Under the assumption that the number of extra alternate-route trunks required to carry this load is $n_F = 276/\gamma = 9.8$, we find the total cost of the network to be \$55.60.†

Figure 3 also shows the network derived on the basis of the Hour-2 loads. For this network, due to high overflow from group B during Hour 1, the final-group busy hour is Hour 1, again contradicting the initial assumption. The total cost of this network is \$59.40.

The third network in Fig. 3 was derived using the multihour technique. As can be seen, this network nearly equalizes the load on the alternate route in the two hours. (In general, the multihour technique tends to equalize the hourly loads on the alternate route or routes.) The cost of this network is \$46.80, substantially less than either single-hour network.

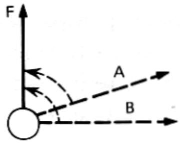
This example illustrates some of the problems inherent in single-hour engineering methods and the potential improvement obtainable with the multihour technique. We describe this technique in detail in Section III.

III. MULTIHOUR ENGINEERING

Figure 4 again shows a single high-usage group overflowing to an alternate route, where the trunk costs C_D and C_A are defined as before. The loads a_1 and a_2 are offered to the high-usage group in Hours 1 and 2, respectively. Also shown in the figure are the background loads in

* In practice, the servicing up of the alternate route might take place after the engineered network was in operation, when the service degradation in the side hour was actually observed.

† The use of an assumed marginal capacity for the alternate route, while reasonable for the purpose of sizing high-usage groups, is actually inappropriate for determining trunk requirements on the alternate route. Our aim here, however, is merely to obtain a rough indication of alternate-route cost for comparative purposes.



	OFFERED LOAD		COST RATIO = 2, γ = 28CCS, ECCS = 14
	HOUR 1	HOUR 2	
A	300	600	
B	800	400	
	1100	1000	

HOUR 1 ENGINEERING

$n_A = 10$

$n_B = 26$

	OVERFLOW		TOTAL COST = 10 + 26 + 2 x 9.8 = 55.6
	HOUR 1	HOUR 2	
A	41	276	
B	56	0	
	97	276	$\Longrightarrow n_F = 9.8$

HOUR 2 ENGINEERING

$n_A = 20$

$n_B = 13$

	OVERFLOW		TOTAL COST = 20 + 13 + 2 x 13.2 = 59.4
	HOUR 1	HOUR 2	
A	0	47	
B	369	49	
	369	96	$\Longrightarrow n_F = 13.2$

MULTIHOUR ENGINEERING

$n_A = 14$

$n_B = 20$

	OVERFLOW		TOTAL COST = 14 + 20 + 2 x 6.4 = 46.8
	HOUR 1	HOUR 2	
A	6	165	
B	172	0	
	178	165	$\Longrightarrow n_F = 6.4$

Fig. 3—Comparison of engineering methods in the presence of noncoincidence.

Hours 1 and 2, A_1 and A_2, offered to the alternate route. The background loads are the total loads offered to the alternate route, not including the overflow from the high-usage group under consideration.

In sizing the high-usage group, we attempt to minimize total cost. We

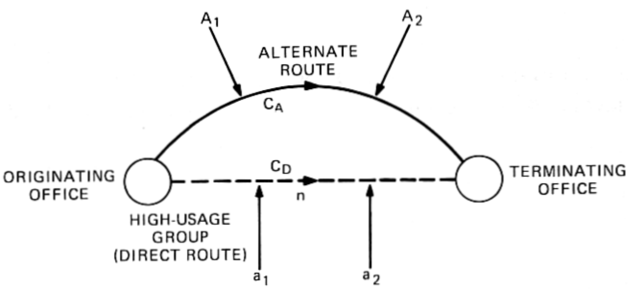


Fig. 4—High-usage group with offered load for two hours.

must recognize, however, that since the grade of service must be guaranteed for both hours, the cost of the alternate route depends upon the *greater* of its total offered loads in Hours 1 and 2. The cost is thus given by the formula*

$$\mathrm{COST} = \frac{C_A}{\gamma} \operatorname{Max} \begin{Bmatrix} A_1 + a_1 B(n,a_1) \\ A_2 + a_2 B(n,a_2) \end{Bmatrix} + C_D n. \tag{4}$$

The total load on the alternate route during Hour 1 is equal to the background load A_1 plus the overflow from the high-usage group, $a_1 B(n,a_1)$. The total load during Hour 2, similarly, is $A_2 + a_2 B(n,a_2)$. The controlling load for the alternate route is the greater of these. The total cost equals the maximum alternate-route load times C_A/γ plus the cost of the high-usage group $C_D n$.†

The term "multihour engineering" denotes the process of designing a network by searching along the cost curve of eq. (4) for each group—or actually, the more general cost curve of eq. (5) discussed below—to determine the minimum-cost point. The optimal number of high-usage trunks in each group determined by the use of this technique varies depending upon the loads and costs. Figure 5 shows two different cases that can arise.

In Case I, the plot of cost curves shows that the Hour-1 load on the alternate route dominates the Hour-2 load for all n. In this case, therefore, the maximization operator of eq. (4) always selects Hour 1, as suggested by the shading of this cost curve in the figure. Thus, in Case I, the multihour method reduces exactly to the single-hour method by using the Hour-1 load. This example illustrates the case discussed above, where the use of the cluster-busy-hour concept yields the correct solution. Note that the correct solution is confirmed if the actual alternate-route busy hour, determined by examining the load offered *after* engineering, is the same as that originally assumed.

Case II in Fig. 5 illustrates a different type of behavior. In this example, the background load on the alternate route is greater in Hour 2, and the offered load on the high-usage group is greater in Hour 1. Thus, for n small there is heavy overflow during Hour 1, and this causes the total alternate-route load to be greater in Hour 1. For n large, however, the overflow is small, so that the alternate-route load is greater in Hour 2 due to the background component. The costs of carrying the alternate-

* This formula, in the more general form of eq. (5), was first given by Rapp (Ref. 2).

† The equation again is not strictly correct since the cost of the alternate route is not proportional to its offered load. However, we shall not use eq. (4) to evaluate the absolute cost, but only to determine its relative minimum with respect to n. For this purpose, the equation yields accurate results.

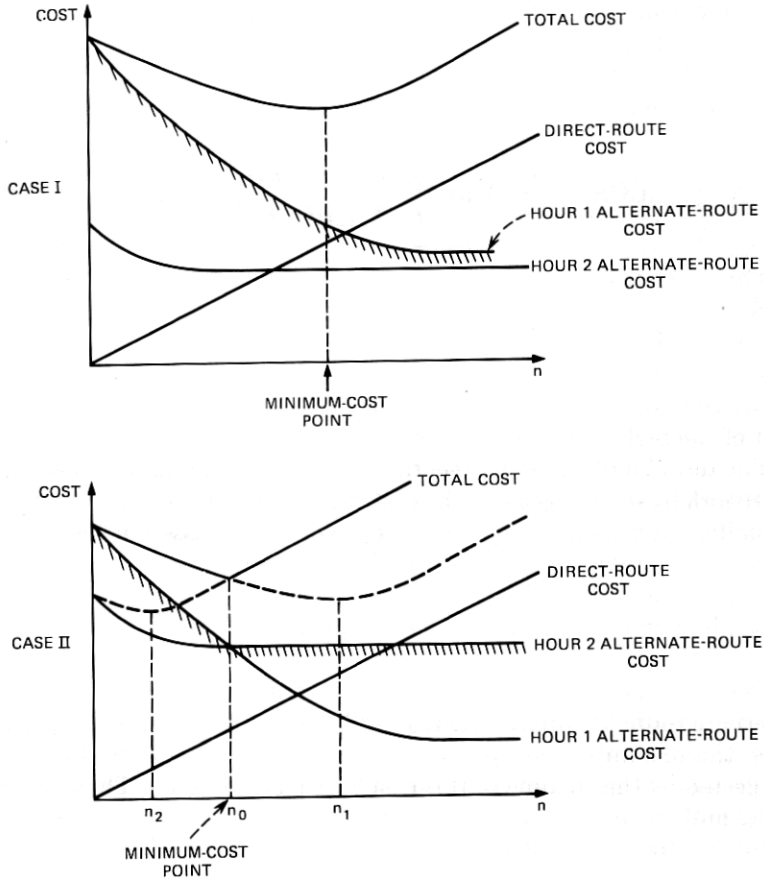


Fig. 5—Multihour trunk-group sizing for the network in Fig. 4.

route loads in Hours 1 and 2 are drawn in the figure and are seen to intersect. To the left of the intersection point, the cost of the alternate route is determined by the Hour-1 load, and to the right, the cost is determined by the Hour-2 load. The maximization operator selects these portions of the curves; this is suggested by the shading in the diagram. The total cost is the sum of the alternate-route cost and the straight-line direct-route cost and is represented by the solid-line curve on the top of the diagram. The curve shows a minimum at the point $n = n_0$ and has a discontinuous derivative at this point. The calculation of n_0 for this example, then, differs radically from the calculation in our previous examples. Whereas before, the minimum-cost design was determined by requiring the load on the last trunk to be equal to a prescribed value, here n_0 is determined to be that value of n which *equalizes* the loads on the alternate route in the two hours.

It is instructive to observe what would happen if a single-hour method were used in this example. If Hour 1 were chosen as the engineering base hour, the resulting high-usage group size would be n_1, shown in the diagram. Note that if n_1 trunks were installed, the alternate route would actually be busier in Hour 2 than in Hour 1, contrary to what was assumed. The total cost of the network after adding trunks to the alternate route to handle the Hour-2 load would be higher than at the optimum point n_0. Similarly, if Hour 2 were the selected engineering base hour, the resulting group size would be n_2. The alternate route would actually be busier in Hour 1, and again the total cost would be higher than optimum.

Up to this point we have, for simplification, been treating the case where the alternate route has consisted simply of a single trunk group. Actually, of course, the alternate-route configuration is more complicated. Figure 6 shows one possible arrangement where the alternate route consists of a final group, a tandem switch, and a tandem-completing group. The background loads for the two hours are F_1 and F_2 for the final, S_1 and S_2 for the tandem switch, and T_1 and T_2 for the tandem-completing group. The cost per trunk of the final group is C_F and the cost per trunk of the tandem-completing group is C_T. We assume that the switching cost is proportional to the load and is equal to C_S per CCS switched. The cost of each component of the alternate route again is determined by the maximum traffic offered to it. Thus, the engineering of the high-usage group requires three separate busy-hour comparisons.

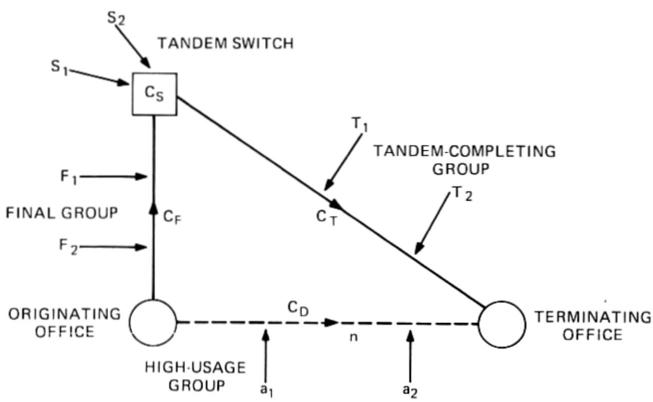


Fig. 6—Typical alternate-route configuration.

The total cost is given by the formula

$$\text{COST} = \frac{C_F}{\gamma} \text{Max} \begin{Bmatrix} F_1 + a_1 B(n,a_1) \\ F_2 + a_2 B(n,a_2) \end{Bmatrix} + C_S \text{ Max} \begin{Bmatrix} S_1 + a_1 B(n,a_1) \\ S_2 + a_2 B(n,a_2) \end{Bmatrix} + \frac{C_T}{\gamma} \text{Max} \begin{Bmatrix} T_1 + a_1 B(n,a_1) \\ T_2 + a_2 B(n,a_2) \end{Bmatrix} + C_D n, \quad (5)$$

which is explained exactly like eq. (4).

Figure 7 shows one possible example of the behavior of the cost curves as a function of n. In this example, the cost curves of the final group during Hours 1 and 2 intersect at a certain point. The switch cost curves also intersect, but at a different point. The tandem-completing cost is completely dominated by the Hour-1 load in this example. The sum of these three costs, plus the straight-line direct-route cost, yields the total cost curve shown at the bottom of the figure. In this example, the optimum design requires equalization of the switch loads in Hours 1 and 2. Of course, depending on the loads and costs, the minimum-cost point could instead have required equalizing of the final loads or of the tandem-completing loads. Alternatively, the minimum-cost network might not correspond to any of these "breakpoints" of the curve, but could lie on a smooth portion as in the single-hour case discussed previously.*

It is important to observe that even in the case where the optimum solution does not correspond to a breakpoint, the engineering does not necessarily reduce to the single-hour ECCS method. To use the single-hour ECCS method, we first compute the "effective alternate-route cost per trunk" as $C_A = C_F + \gamma C_S + C_T$. Then, using a single hour's load, we determine n such that the load on the last trunk during that hour is equal to γ divided by the cost ratio, C_A/C_D. To see how the multihour method differs from this, let us assume that the final group is dominated by its Hour-i load, the switch is dominated by its Hour-j load, and the tandem-completing group is dominated by its Hour-k load. (We make this assumption to avoid having to worry about breakpoints in the cost curve.) Then, it follows from the differentiation of eq. (5) that the optimum value of n satisfies the equation

$$\frac{C_F}{\gamma} L(a_i) + C_S L(a_j) + \frac{C_T}{\gamma} L(a_k) = C_D, \quad (6)$$

* Rather than attempting an exact minimization of eq. (5) in the manner we have described, Rapp adopted an approximate approach. He introduced the fictitious load, $\bar{a}$, as a function of the parameters a_1, a_2, A_1, A_2, and then used it in the single-hour formula of eq. (3) to produce a trunk-group size.[2]

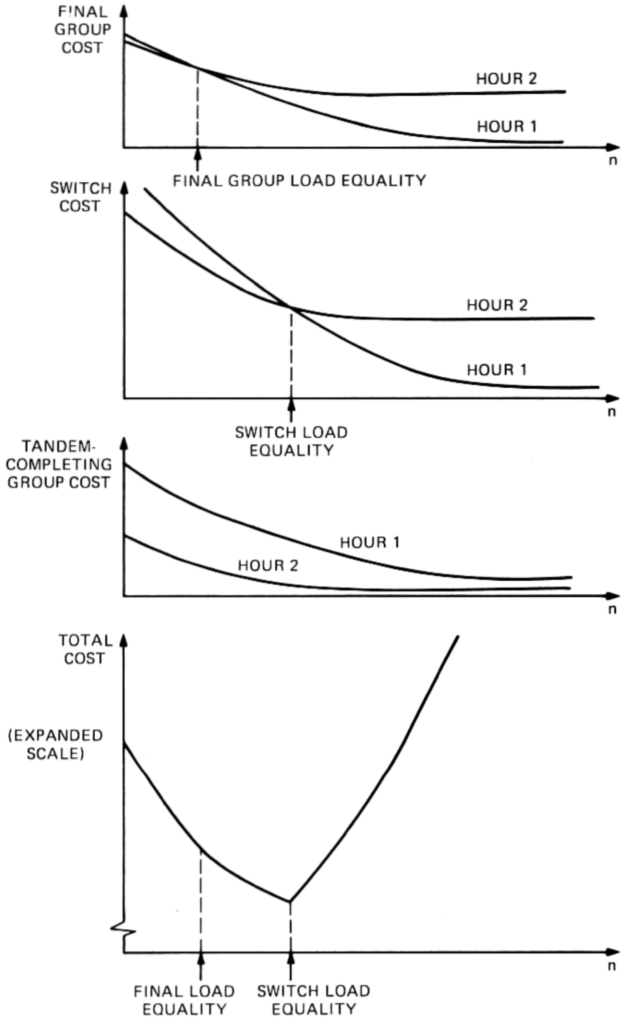


Fig. 7—Multihour trunk-group sizing for the network in Fig. 6.

where a_m ($m = i,j,k$) is the load offered to the high-usage group during Hour m and $L(a_m)$ is the load on the last trunk during that hour.* [This notation omits explicit indication that $L(a_m)$ is a function of n.] If $i = j = k$, so that all components of the alternate route are busy at the same time, then $L(a_i) = L(a_j) = L(a_k) \equiv L(a)$, and we can factor out this quantity. The equation becomes, in this case,

* More precisely, $L(a_m) = -(d/dn)[a_m B(n,a_m)]$.

$$L(a) = \frac{\gamma}{(C_A/C_D)}, \tag{7}$$

where $C_A = C_F + \gamma C_S + C_T$. This is precisely the ECCS formula. [See eq. (3)].

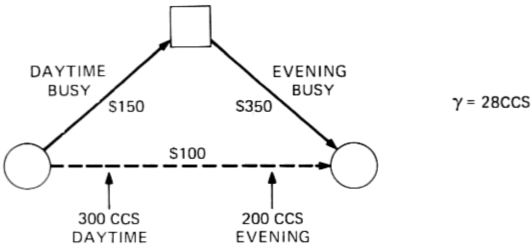
Thus we see, as expected, that the single-hour ECCS method is optimum only when all components of the alternate route have the *same* busy-hour (or when the high-usage load is the same in all hours). If this is not the case, then it is invalid to lump the various components into one alternate-route cost and to choose only a single hour's load for engineering the high-usage group. Equation (6) shows that, in general, the optimal sizing of the group involves the load on the last trunk in each of the three hours that are significant to the alternate route.

An example is given in Fig. 8 which illustrates the difference between multihour and single-hour engineering in the case where the busy hours of the alternate-route legs do not depend upon the high-usage group size. In the example, a single high-usage group with offered loads of 300 CCS in the daytime and 200 CCS in the evening overflows to a final group with a known daytime busy hour and then to a tandem-completing group with a known evening busy hour. (The tandem is neglected for simplicity.) Since the busy hours of the alternate-route legs are fixed, eq. (6) gives the multihour solution for this example. The equation yields a trunk requirement of 11. Single-hour engineering produces a trunk requirement of 13 for the daytime load and 9 for the evening load. It can be seen in the figure that the multihour network cost is significantly lower than both single-hour networks in this example. Note that daytime engineering over-sizes the high-usage group due to an overestimate of the tandem-completing cost, while evening engineering undersizes the group due to an underestimate of the final cost.

IV. THE SIGNIFICANT-HOURS ALGORITHM

An alternative procedure for engineering networks for more than one hour of traffic data, called the "significant-hours" method, has recently come into use in the Bell System. In this section, we describe this algorithm as applied to a two-level local network, and compare it to the multihour method discussed above.

The significant-hours algorithm was devised to overcome shortcomings of the cluster-busy-hour ECCS approach caused by the fact that the various legs of the alternate-route path have busy hours different from that of the originating cluster. These shortcomings can be explained by considering the network in Fig. 9. In the cluster-busy-hour approach, the originating cluster-busy hour of each office is used for sizing all the



MULTIHOUR ENGINEERING

$150\ L(300) + 350\ L(200) = 28 \times 100 \quad \Longrightarrow \quad n = 11$

	OVERFLOW	INCREMENTAL COST	TOTAL COST = 11 x 100 + 150 + 38 = 1288
DAYTIME	28	28 x 150/28 = 150	
EVENING	3	3 x 350/28 = 38	

DAYTIME ENGINEERING

$(150 + 350)\ L(300) = 28 \times 100 \quad \Longrightarrow \quad n = 13$

	OVERFLOW	INCREMENTAL COST	TOTAL COST = 13 x 100 + 61 + 12 = 1373
DAYTIME	11	11 x 150/28 = 61	
EVENING	1	1 x 350/28 = 12	

EVENING ENGINEERING

$(150 + 350)\ L(200) = 28 \times 100 \quad \Longrightarrow \quad n = 9$

	OVERFLOW	INCREMENTAL COST	TOTAL COST = 9 x 100 + 306 + 142 = 1348
DAYTIME	57	57 x 150/28 = 306	
EVENING	11	11 x 350/28 = 142	

Fig. 8—Comparison of engineering methods for fixed alternate-route busy hours.

originating groups in each office. Thus, group A-Z is sized for its A-office cluster-busy-hour load, group B-Z for its B-office cluster-busy-hour load, and group C-Z for its C-office cluster-busy-hour load. If offices A,B, and C are business-dominated offices, the groups A-Z, B-Z, and C-Z would be sized for their daytime business loads. If office Z is a residence-dominated office, however, the loads on these groups may peak in the evening. Since the groups are sized for their smaller daytime loads, they would overflow heavily in the evening, and all this overflow would be offered to the tandem-completing group T-Z. This effect has been observed in actual networks; in some cases, extremely high loads occur on certain tandem-completing groups, requiring great quantities of trunks and switching termination equipment. The problem is clearly caused by the exclusive attention to the originating portion of the alternate-

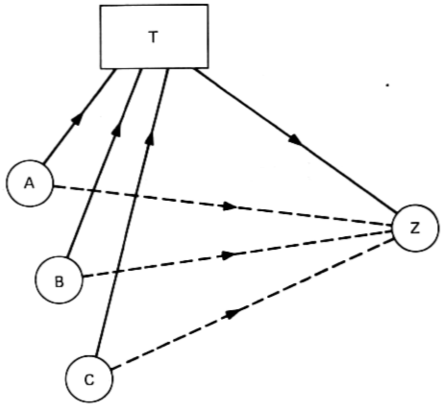


Fig. 9—Example of network.

route path and the total neglect of the terminating portion.

The significant-hours method solves this problem by giving equal treatment to both parts of the alternate-route path. For group A-Z, two significant hours are considered: the A-office originating cluster busy hour (defined above) and the Z-office terminating cluster busy hour (defined as that hour for which the total traffic terminating at office Z is maximum). Of these, the one for which the A-Z load is *larger* is chosen as the "control hour" for group A-Z. The group is then engineered for this load. By engineering the high-usage group for the larger of its significant loads, enough trunks are installed to eliminate the possibility of extremely heavy overflow in the busy hour of the tandem-completing leg.

There still remain two problems with this method, however. The first is the fact that the actual busy hours of the final and tandem-completing groups of the engineered network are not necessarily the same as those that the significant-hour calculation predicts.* The second problem is that even if the significant hours are the right ones, in the sense that the alternate-route busy hours after engineering agree with the original assumptions, this method will over-engineer the group unless either (*i*) the significant hours are all the same, or (*ii*) the offered loads on the high-usage group are the same in these hours.

* It should be emphasized that using *observed* final-group and tandem-completing-group busy hours, instead of the originating and terminating *cluster* busy hours, does not get around this problem. The observed busy hours depend on the *previous* network configuration. The point is that if a significant amount of noncoincidence of traffic loads exists, the busy hours of the newly engineered network will not agree with those assumed in the engineering, no matter how the hours are selected.

V. MULTIHOUR ENGINEERING OF A NETWORK

In the discussion of the theory of multihour engineering surrounding eqs. (4) and (5), we considered only the sizing of a single high-usage trunk group in isolation. The background loads on the alternate route were given and assumed fixed. In a network, however, these background loads consist partly of overflows from other high-usage groups—groups which themselves have to be sized during the engineering process. For a network consisting of more than one group, therefore, the use of eq. (5) alone is insufficient, since it does not account for the interdependence between the high-usage groups that arises through their mutual effect on the background loads.

The optimal sizing of a network consisting of N high-usage trunk groups in fact requires the minimization of a cost function of N dimensions, instead of the one-dimensional cost function of eq. (5). An analysis of this optimal approach has been carried out by W. B. Elsner of Bell Laboratories[3]. All of our initial network results, however, including those described in the remainder of the present paper, were obtained using a simple iterative approach. These initial results allowed us to demonstrate the feasibility of multihour engineering and to quantify the order of magnitude of the associated cost savings. These preliminary findings justified the effort by Elsner to develop the exact algorithm.

We begin the iterative process by choosing initial sizes for every trunk group in the network.* This allows us to compute overflows from each high-usage group and thus to determine the total background loads which are offered to all alternate-route groups. We then size each high-usage group in turn by minimizing its one-dimensional cost function. The background loads used in each case consist of the first-routed loads plus the overflow from all *other* high-usage groups, that is, all high-usage groups except the overflow from the group being sized. After sizing every group once, the background loads that appear on the alternate-route groups differ from what they are at the beginning and, hence, the engineering procedure is iterated, each pass consisting of the resizing of every high-usage group. This process continues until the iteration converges.

An essential aspect of this procedure is the fact that the background loads are updated immediately after the sizing of each group and before the sizing of the next group in sequence. The background loads play a very important part in the process of multihour engineering and it is necessary that the computed background loads be accurate if the proper sizing is to take place. If the updating is not done promptly, the back-

* In obtaining the results that follow, we initialized each group to be numerically equal to the largest load on the group measured in erlangs.

ground loads used in sizing will differ from their true values, and misengineering of the network will result. If the updating is delayed to the end of each complete pass, for example (as was done in our first attempt), the iteration could even fail to converge.

When the iteration does converge, the resulting network has the property that, if all other groups are held fixed, each individual group is sized to minimize cost. It is possible, however, that at convergence network cost could be further reduced by changing the sizes of two or more trunk groups simultaneously. This simple iterative approach also has the property that the solution to which it converges is not unique; depending upon the initial trunk values assumed, and the order in which the groups are sized, the solution network can vary. Both of these undesirable properties of the iterative method are overcome with Elsner's approach.[3]

VI. COMPUTER PROGRAM AND RESULTS

A computer program incorporating the above iterative multihour procedure was written to design a network with the routing structure shown in Fig. 10. In this network, a single end office has a number of one-way outgoing high-usage trunk groups connected to other end offices. All high-usage groups overflow to a common final group and traffic reaches its terminating office via a one-way tandem-completing group.

The program was run using the load data from three California end offices: Gardena, Compton, and Melrose. In each case, two hours of loads were employed. Hour 1 was a morning busy hour dominated by business traffic, and Hour 2 was an evening busy hour dominated by residential traffic. In the absence of actual trunk and switching cost data, the trunk cost of every group was assumed identical, equal to $1000 per trunk, and

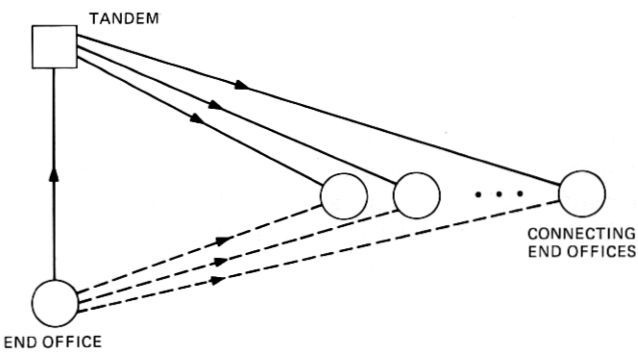


Fig. 10—Network configuration for multihour engineering program.

the switching cost was assumed to be $62 per CCS (which yields a realistic average switching-to-trunk-cost ratio). Both the switch and the tandem-completing groups were assumed to have zero background load. (This implies that the switch loads are identical to the final loads and that the tandem-completing busy hours are the same as their corresponding high-usage group's busy hours.)

Table I contains detailed results obtained for the Gardena office. The table shows the two hourly offered loads for each high-usage trunk group; it also shows the number of trunks and the resulting hourly overflows for the cluster-busy-hour network and the multihour network.

From the totals at the bottom of the table, it can be seen that the total cluster load in Hour 1 is 6712 CCS while the total cluster load in Hour 2 is 5154 CCS; clearly, Hour 1 is the cluster busy hour. On the other hand, for the network engineered for the cluster busy hour, the total overflow (i.e., the load offered to the final group) is 975 CCS in Hour 2 and only 502 CCS in Hour 1. Here we see again the phenomenon of the final busy hour differing from the cluster busy hour.

The reason for the high side-hour overflow can be seen by looking at the Hour-1 and Hour-2 overflow columns. The overflows in Hour 1 are quite uniform over all trunk groups; the trunk sizes are "matched" to the Hour-1 loads. In Hour 2, however, there is a great mismatch. A few trunk groups have very large overflows, the rest have virtually none. Three groups alone (14, 18, and 20), in fact, account for almost 60 percent of the total overflow in this hour. As can be seen, the pattern of overflow in the multihour network is more nearly balanced between the two hours; the total overflows in Hours 1 and 2 are nearly equal in this network.

Table II compares the main characteristics of the busy-hour and multihour networks for Gardena. The numbers of final trunks shown are sufficient to guarantee a blocking probability of 0.01 for both hours. For the busy-hour network, for example, the final must be sized for the Hour-2 load since that load is larger. The total cost shown is the sum of the costs of the high-usage groups, final group, tandem switching, and tandem-completing groups. (The tandem-completing cost is an approximation based upon the use of marginal capacity to determine the trunk requirement for each group.)

With this simple model, the total cost of the multihour network is approximately 7 percent less than the cost of the single-hour network. Also significant is the fact that the switching cost is reduced 26 percent by using the multihour technique. Table III shows cost comparisons of all three of the networks studied. As can be seen, total network costs for the three cases decrease in the range of 5 to 11 percent and tandem-switching costs decrease in the range of 17 to 26 percent with the use of the multihour technique. (Trunk costs of $1000 and switching costs of $62/CCS were assumed in each case.)

Table I — Gardena network results

Trunk Group	Offered Load (CCS)		Cluster-Busy-Hour Engineering			Multihour Engineering		
	Hour 1	Hour 2	Trunks	Hour-1 Overflow (CCS)	Hour-2 Overflow (CCS)	Trunks	Hour-1 Overflow (CCS)	Hour-2 Overflow (CCS)
1	60	140	3	10	62	4	4	42
2	119	9	6	8	0	3	45	0
3	82	20	4	10	0	4	10	0
4	305	76	12	20	0	6	126	1
5	30	0	2	5	0	0	30	0
6	59	7	3	9	0	1	37	1
7	102	56	5	10	1	4	19	3
8	256	161	11	13	1	8	47	8
9	366	230	15	15	0	12	46	4
10	469	310	18	20	1	18	20	1
11	115	115	5	15	15	5	15	15
12	144	34	7	9	0	7	9	0
13	206	335	9	13	80	10	7	61
14	310	650	13	13	233	16	3	154
15	284	319	12	13	25	12	14	25
16	93	152	4	15	50	5	7	33
17	17	24	1	5	10	1	5	10
18	74	325	4	8	200	6	1	143
19	102	158	5	10	37	5	10	37
20	137	322	6	14	141	8	3	92
21	222	247	9	18	28	10	11	18
22	252	390	11	12	78	12	7	59
23	445	194	17	21	0	17	21	0
24	176	86	8	11	0	8	11	0
25	83	29	4	11	0	4	11	0
26	98	21	5	9	0	5	9	0
27	158	74	7	13	0	7	13	0
28	124	36	6	10	0	6	10	0
29	54	25	3	7	1	3	7	1
30	38	1	2	8	0	2	8	0
31	31	17	2	5	1	2	5	1
32	140	46	6	15	0	6	15	0
33	96	30	5	8	0	5	8	0
34	122	62	6	9	0	6	9	0
35	163	57	7	15	0	7	15	0
36	163	72	7	15	0	7	15	0
37	296	238	12	17	5	12	17	5
38	33	28	2	6	4	2	6	4
39	240	3	10	16	0	10	16	0
40	136	7	6	14	0	6	14	0
41	54	4	3	7	0	3	7	0
42	52	35	3	7	2	3	7	2
43	206	9	9	13	0	9	13	0
Totals	6712	5154	295	502	975	287	713	720

Table II — Main characteristics of busy-hour and multihour networks for Gardena

Network Characteristics	Cluster Busy-Hour Engineering	Multihour Engineering
High-usage trunks	295	287
Final trunks	37	29
Switching cost	$60,450	$44,640
Total cost	$437,106	$405,315

Table III — Cost comparisons for three networks

	Network Total Costs ($1000)			Tandem Switching Costs ($1000)		
Office	Busy-Hour	Multihour	% Decrease	Busy-Hour	Multihour	% Decrease
Gardena	437	405	7	60.4	44.7	26
Compton	493	441	11	87.9	64.7	26
Melrose	303	288	5	45.2	37.6	17

VII. CONTINUING WORK

In this paper, we have described the basic theory of multihour engineering and have demonstrated, in a few simple networks, the potential savings it can bring about. A number of questions require answers for this technique to achieve acceptability for use in the field. These questions include the following: the "hours" of data used for engineering may occur in different seasons of the year as well as different times of the day. How many hours should be included in the engineering of a network and how should these hours be determined? How is multihour engineering to be accomplished in a large-scale network with more than two levels in the hierarchy? How is trunk administration to be carried out in a multihour environment? What changes are required for two-way trunk groups? These questions and others have been the subject of intensive study at Bell Laboratories, and the answers will be reported on in future papers.

VIII. CONCLUSIONS

Multihour engineering is a technique that can provide significant benefits in the design of alternate-route traffic networks. In a computerized engineering environment, especially since automated data-collection methods make it possible to collect larger amounts of (and more accurate) traffic data, this technique should prove to be a realistic and preferable alternative to the older single-hour techniques.

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