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Modal Dispersion in Optical Fibers With Arbitrary Numerical Aperture and Profile Dispersion

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Design criteria to minimize modal dispersion have been found for a broad class of practical, multimode, circular-symmetric, isotropic, optical fibers having any numerical aperture and any profile dispersion (which is a function of the derivative of the index with respect to the wavelength). The impulse-response width of these fibers, the rms width of the impulse response, the optimum profiles to minimize those widths, and the sensitivity to profile departures from ideal are found to be surprisingly simple closed-form generalizations of previous results that are mostly applicable to fibers with small numerical aperture and constant profile dispersion. The minimum impulse-response width of the optimized fiber is a function only of its numerical aperture and consequently is independent of the index profile and of the profile dispersion.

I. INTRODUCTION

Circular-symmetric, multimode, optical fibers intended for large communication capacity must have low modal dispersion and this is achievable by the quasi-complete equalization of the group velocities of all modes[1] (or rays). This equalization depends critically both on the refractive-index profile and on the profile dispersion of the fiber. The profile dispersion is defined in Section II, but here it is enough to know that it is related to the derivative of the index with respect to the wavelength.

To understand better the objectives of this paper, let us first review some recent evolution of thoughts linking the index profile and the profile dispersion of a fiber to the pulse broadening caused by modal dispersion.

Gloge and Marcatili[2] showed that if the numerical aperture (NA) of the fiber is arbitrary but the profile dispersion is negligible, there is a family of fibers—for which the dielectric constant profiles decrease radially according to power laws—that is important for two reasons. The first reason is that the family encompasses a wide variety of easy-to-make fibers (step-index, quasi-parabolic, etc.) possessing the unique property that the group velocity of each mode is a function only of its propagation constant; this drastically simplifies the analysis. The second and more important point is that for an almost parabolic power law of the dielectric profile, a fiber with small NA has the very narrow impulse response needed for high-speed communication.

Olshansky and Keck[3] extended these results in a very important way by showing that if the profile dispersion is constant across the core, narrow impulse response is achievable in small NA fibers by a simple modification of the exponent of the dielectric-constant profile's power law.

In many cases, though, the two requirements—smallness of NA and constancy of profile dispersion—are not satisfied. For example, to increase the coupling efficiency to incoherent sources and to decrease microbending losses,[4] fibers with large NAs[5] are being made. They are heavily doped and, particularly if the doping element is boron, the profile dispersion may not be a constant[6,7] as a function of the radius. Similar lack of constancy may occur in fibers that are doped with several materials for the purpose of improving optical or mechanical properties.[8] Arnaud and Flemming[9,10] have calculated the impulse response for these fibers, treating the variable portions of the profile dispersion as a perturbation. Using a numerical method Arnaud[11] has also calculated the pulse spreading in a multimode planar fiber with arbitrary index profile and profile dispersion.

In this paper, we extend the previous results by finding, within the WKB approximation, a surprisingly simple closed-form description of the modal dispersion in a broad class of circular-symmetric isotropic fibers which have arbitrarily large NA and arbitrary profile dispersion.

The gist of our paper is in Sections II and III. In Section II, the profiles of the fibers belonging to the group are defined and their impulse-response widths are calculated. In Section III, the optimum index profile required to minimize the impulse-response width is determined together with the sensitivity of this response to departures of the index from optimum. The rms of the impulse response is the subject of Section IV. In Section V, some approximate results about the influence of index profiles on the minimization of the rms width of the impulse response are derived and conclusions are drawn in Section VI.

II. FIBER PROFILE AND WIDTH OF ITS IMPULSE RESPONSE

We start by looking for the dielectric-constant profile of a circular symmetric isotropic fiber such that, as in Ref. 2, the group velocity of each mode is only a function of its propagation constant. In the process, we will also find the width of the impulse response of that fiber.

The initial point is a WKB approximation[2] that relates the propagation constant $\beta(\lambda)$ of a mode characterized by the radial and azimuthal wave numbers μ and ν to the free-space propagation constant $k = 2\pi/\lambda$, the refractive index $n(r, \lambda)$ of the fiber and the radial coordinate r, via the integral

$$\mu = \frac{1}{\pi} \int_{r_1}^{r_2} \rho \frac{dr}{r}, \tag{1}$$

where

$$\rho(r, \lambda) = \sqrt{(k^2 n^2 - \beta^2) r^2 - \nu^2} \tag{2}$$

and r_1 and r_2 are two neighboring turning points that make the radical zero and between which most of the field of the mode is concentrated. It is useful to redefine

$$n^2 = n_1^2 (1 - F) \tag{3}$$

$$\beta^2 = k^2 n_1^2 (1 - B), \tag{4}$$

where n_1 is the index on axis and the profile function $F(r, \lambda)$ is zero on axis, is an arbitrary function of r and λ within the core ($r \leqq a$), and is $2\Delta(\lambda)$, a function only of λ in the cladding ($r \geqq a$). Similarly, the mode parameter B varies between zero for the lowest-order mode and $2\Delta(\lambda)$ for the modes whose phase velocities coincide with that of a plane wave in the cladding.* With these definitions, the radical (2) becomes

$$\rho = \sqrt{(k n_1 r)^2 (B - F) - \nu^2}. \tag{5}$$

The group velocity of a mode (or ray) is introduced by taking the derivative of both sides of (1) with respect to the free-space wavelength λ,

$$\int_{r_1}^{r_2} \left[B \left(1 - \frac{n_1}{2N_1} \frac{\lambda}{B} \frac{dB}{d\lambda} \right) - F \left(1 - \frac{p}{2} \right) \right] \frac{r}{\rho} \, dr = 0, \tag{6}$$

where

$$N_1 = n_1 \left(1 - \frac{\lambda}{n_1} \frac{dn_1}{d\lambda} \right) \tag{7}$$

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* Similar but not identical profile function and mode parameter have been introduced previously in the literature.[12]

is the group index on axis and

$$p(r, \lambda) = \frac{n_1}{N_1} \frac{\lambda}{F} \frac{\partial F}{\partial \lambda} \tag{8}$$

is a generalized version of the profile dispersion parameter introduced in Ref. 12.

The derivative $dB/d\lambda$ in (6) can be expressed in terms of the group delay t of the mode by taking square roots and derivatives on both sides of (4). The result is

$$\frac{d\beta}{dkn_1} = \frac{t}{T} = \sqrt{1-B} + \frac{n_1}{2N_1} \frac{\lambda}{\sqrt{1-B}} \frac{dB}{d\lambda} \tag{9}$$

in which T, the flight time on axis, that is, the delay of a plane wave in a medium of group index N_1 and length L, is related to the velocity of the light in free space c via

$$T = \frac{LN_1}{c}. \tag{10}$$

The substitution of $dB/d\lambda$ from (9) into (6) yields the integral

$$\int_{r_1}^{r_2} \left[1 - \sqrt{1-B}\,\frac{t}{T} - F\left(1 - \frac{p}{2}\right)\right] \frac{r}{\rho}\, dr = 0. \tag{11}$$

This integral was solved in Refs. (2) and (3) by assuming p constant and $F = 2\Delta(r/a)^\alpha$, a power law, with α an arbitrary constant. To lift these restrictions and still solve (11) exactly, the following self-evident expression is introduced:

$$\int_{r_1}^{r_2} \frac{\partial \rho}{\partial r}\, dr = \rho(r_2, \lambda) - \rho(r_1, \lambda) = 0. \tag{12}$$

This integral becomes less obvious and very useful when the derivative $\partial\rho/\partial r$ is performed with the help of (5), yielding

$$\int_{r_1}^{r_2} \left[B - F - \frac{r}{2}\frac{\partial F}{\partial r}\right] r \frac{dr}{\rho} = 0. \tag{13}$$

Combining (13) with (11), we arrive at a general expression

$$\frac{1 - \sqrt{1-B}\,\dfrac{t}{T}}{B} = \frac{\displaystyle\int_{r_1}^{r_2} \left(1 - \frac{p}{2}\right) \frac{Fr}{\rho}\, dr}{\displaystyle\int_{r_1}^{r_2} \left(1 + \frac{r}{2F}\frac{\partial F}{\partial r}\right) \frac{Fr}{\rho}\, dr}, \tag{14}$$

that like (11), relates the group delay t of a mode characterized by its mode parameter B (or propagation constant β), and its azimuthal mode number ν (hidden in ρ) to the profile function F and profile dispersion p. This expression is valid for any circular-symmetric fiber with isotropic dielectric and, in general, still cannot be solved exactly. However, if a particular family of fibers is considered that satisfies the condition

$$\frac{1 + \dfrac{r}{2F}\dfrac{\partial F}{\partial r}}{1 - \dfrac{p}{2}} = D(\lambda), \tag{15}$$

D being an arbitrary function of λ, the seemingly formidable right-hand side of (14) is reduced to $1/D$ and the group delay of the mode characterized by the mode parameter B is

$$t = T\frac{1 - \dfrac{B}{D}}{\sqrt{1 - B}}. \tag{16}$$

These last two equations are the basic results of the paper. Equation (16) says that t, the group delay of a mode (or ray) is only a function of the mode parameter B and the dispersion parameter D. More important, the group delay is independent of the mode number (which means that modes with the same propagation constant have the same delay), it is independent of the profile function F, and it is independent of the profile dispersion p. Equation (16) is used in the following sections to study the impulse response of the fiber.

On the other hand, eq. (15) is the recipe for the design of the fiber whose time response is given by (16). It can be solved in several ways depending on the control that the fiber designer has over F and its derivatives with respect to λ. The least demands on these functions occur if the fiber is designed to operate at one wavelength only. Then, in (15), D becomes a constant, p is only a function of r, and the partial derivative of F is reduced to an ordinary one. Without introducing new symbols for D, F, and p, the simplified design formula is

$$\frac{1 + \dfrac{r}{2F}\dfrac{dF}{dr}}{1 - \dfrac{p}{2}} = D. \tag{17}$$

This equation in turn can be solved in two ways. One way consists in prescribing the profile function F to satisfy, perhaps, requirements different from modal dispersion. Then, the profile dispersion p must be tailored to satisfy (17).

For example, assume the dielectric profile depicted in Fig. 1a. The profile function is

$$F = \begin{cases} 2\Delta \left(\dfrac{r}{a_1}\right)^{\alpha_1} & \text{for} \quad 0 < r < a_0 \\ 2\Delta \left(\dfrac{r}{a}\right)^{\alpha_2} & \text{for} \quad a_0 < r < a, \end{cases} \tag{18}$$

where

$$a_0 = a_1 \left(\frac{a_1}{a}\right)^{\alpha_2/(\alpha_1 - \alpha_2)} \tag{19}$$

and the inequalities

$$\begin{gathered} a_1 < a \\ \alpha_1 > \alpha_2 > 0 \end{gathered} \tag{20}$$

guarantee that the profile looks indeed like that in Fig. 1a.

Substituting (18) in (17) and assuming for the dispersion parameter D a value D_0 that optimizes in some sense the impulse response of the fiber, the required profile dispersion turns out to be

$$p = \begin{cases} 2 - \dfrac{2 + \alpha_1}{D_0} & \text{for} \quad r < a_0 \\ 2 - \dfrac{2 + \alpha_2}{D_0} & \text{for} \quad a_0 < r < a \end{cases} \tag{21}$$

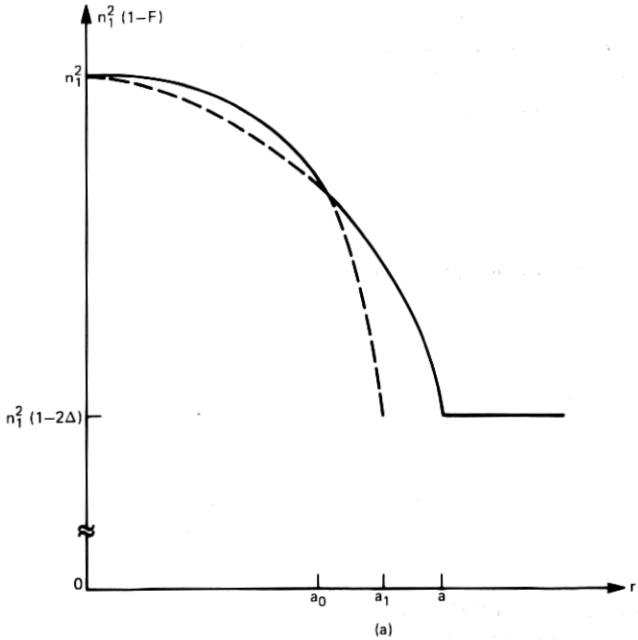
and is shown in Fig. 1b.

This is an interesting example not only because it clearly illustrates the power of the theory even to design optimized fibers in which the profile function and dispersion are discontinuous, but also because it may be of practical interest. For example, by using an index-increasing dopant for $r < a_0$ and an index-decreasing dopant for $r > a_0$ the NA of the fiber can be increased, keeping at the same time its optimum modal dispersion.

In the other way of solving (17), the profile dispersion p as a function F is assumed to be known, from experiment, and the index profile must be found from the integration of (17) that yields

$$r = a \exp \int_F^{2\Delta} \frac{dF}{[2 - D(2 - p)]F}. \tag{22}$$

This result will be used in a more general way later, but if for the time being we prescribe p to be a constant P_0, the profile function results:



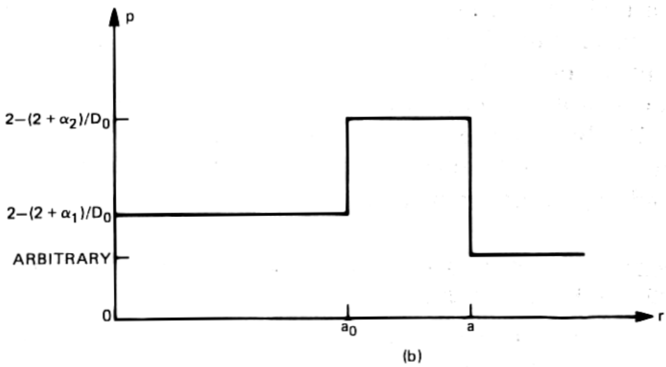


Fig. 1—(a) Dielectric profile (solid line). (b) Profile dispersion.

$$F = 2\Delta \left(\frac{r}{a}\right)^{\alpha}, \tag{23}$$

where

$$\alpha = D(2 - P_0) - 2. \tag{24}$$

This last equation establishes the relation between the dispersion parameter D of the fiber introduced in this paper and the α value so widely used in the literature[2,3] for fibers with constant profile dispersion P_0. It follows from (17) that only if p is a constant, is the profile function F a power law (23).

The two solutions described require only the control over the profile function F and its first derivative with respect to λ. But suppose that the fiber designer has also control over the second derivative. Then, to increase the range of wavelengths over which the fiber operates with low modal dispersion, he could demand, for example, that at a wavelength not only

$$D = D_0$$

but also, as proposed by Kaminow and Presby,[13]

$$\frac{dD}{d\lambda} = 0. \tag{25}$$

This requires the simultaneous satisfaction of (17) and

$$\frac{\frac{\partial}{\partial\lambda}\left(\frac{r}{F}\frac{\partial F}{\partial r}\right)}{\frac{\partial p}{\partial\lambda}} = -D_0 \tag{26}$$

derived from (15) and (25). It is this last equation that implies the control over the second derivative of F with respect to λ.

It can easily be extrapolated that control over higher derivatives permits even further demands on D. In fact, if all the higher derivatives were controllable, $D(\lambda)$ could be chosen arbitrarily and the profile F would be the solution of the linear partial differential equation of first order (15) subject to the conditions of being zero at $r = 0$ and $2\Delta(\lambda)$ at $r = a$; the result is well known[14] from a mathematical point of view, but of limited importance from a practical point of view.

III. MINIMIZATION OF THE IMPULSE-RESPONSE WIDTH AND ITS SENSITIVITY TO ERRORS IN THE PROFILE

The impulse-response width is determined from (16) by finding the difference between flight times of the slowest and the fastest modes (or rays) for any given value of the dispersion parameter D. It is simple to find that the minimum time spread, τ_{min}, between those modes occurs if D is chosen

$$D_0 = 1 + \sqrt{1 - 2\Delta}. \tag{27}$$

In fact, the modes characterized by $B = 0$ and $B = 2\Delta$ are the slowest and arrive at the end of the fiber at

$$t_{max} = T, \tag{28}$$

while the modes characterized by $B = 1 - \sqrt{1 - 2\Delta}$ are the fastest and arrive at

$$t_{\min} = T\frac{2(1-2\Delta)^{1/4}}{1+\sqrt{1-2\Delta}}. \tag{29}$$

The time spread between them is the minimum impulse-response width

$$\tau_{\min} = t_{\max} - t_{\min} = T\frac{[1-(1-2\Delta)^{1/4}]^2}{1+\sqrt{1-2\Delta}}. \tag{30}$$

Therefore, fibers with the same Δ have the same minimum impulse-response width $\tau_{\min}$, independently of their index profiles and profile dispersions, provided that they satisfy the design equation (17) with D substituted by the optimum value D_0 (27).

If

$$\Delta \ll 1,$$

(27) and (30) become

$$D_0 \cong 2 - \Delta \tag{31}$$

$$\tau_{\min} \cong \frac{\Delta^2}{8}T = 0.61\ \Delta^2\ \mu\text{s/km} \tag{32}$$

for $N_1 = 1.46$.

To find the sensitivity of the impulse-response width to departures of the index profile from optimum, we calculate the ratio $\tau/\tau_{\min}$ between the response width τ for

$$D = (1+\delta)D_0, \tag{33}$$

where

$$\delta \ll 1$$

and the minimum response width $\tau_{\min}$ occurring for $D = D_0$. After some straightforward calculations,

$$\frac{\tau}{\tau_{\min}} = \left\{1 + \frac{|\delta|}{8\Delta}(1+\sqrt{1-2\Delta})^2\,[1+(1-2\Delta)^{1/4}]^2\right\}^2 \tag{34}$$

and for

$$\frac{\tau}{\tau_{\min}} = \left(1 + \frac{2|\delta|}{\Delta}\right)^2. \tag{35}$$

It is known that the impulse-response width is indeed very sensitive to the choice of profile and more so for smaller Δ. If δ, the fractional departure of D from its optimum value, is equal to Δ, then the pulse width is nine times larger than $\tau_{\min}$.

The main results in the last two sections have been extended by Arnaud to optimize modal dispersion in fibers with noncircularly symmetric profiles.[15]

IV. THE RMS OF THE IMPULSE RESPONSE

From the point of view of the maximum information-carrying capacity of a fiber, more significant than the impulse response width is its rms width σ,[16] since $1/4\sigma$ is the repetition rate at which pulses can be transmitted with a reasonable loss penalty at the receiving end.[1]

Let us calculate first the impulse response $W(t)$ and then its rms width σ assuming that:

(*i*) The energy of the infinitely narrow impulse fed at one end of the fiber is equipartitioned among all modes.
(*ii*) All modes attenuate equally.
(*iii*) The number of modes is so large that the discrete pulses arriving at the receiving end can be replaced by a continuum.

The impulse response, then, is the rate of change of the number of modes reaching the end of the fiber,

$$W(t) = \frac{d}{dt} \int_0^{\nu_{\max}} \mu(\nu, t) d\nu, \tag{36}$$

and its rms width is, by definition,

$$\sigma = \left(\frac{1}{2} \frac{\int \int_0^\infty W(t_1) W(t_2)(t_1 - t_2)^2 dt_1 dt_2}{\int \int_0^\infty W(t_1) W(t_2) dt_1 dt_2} \right)^{1/2}. \tag{37}$$

To calculate $W(t)$, the value of μ given in (1) is substituted in (36) and the integration along ν is carried through yielding

$$W(t) = \frac{k^2 n_1^2}{4} \frac{d}{dt} \int_0^{r_B} (B - F) r dr. \tag{38}$$

The integral measures the energy arriving at the end of the fiber as a function of time and, since each contribution must be positive, the largest value that F can reach is B. Therefore, the upper limit r_B is the value of r that makes

$$F(r, \lambda) = B(\lambda). \tag{39}$$

The explicit value of r_B depends on the choice of fiber design. If the profile function F is prescribed, then r_B is obtained by solving (39). If, on the other hand, the profile dispersion is prescribed, then

$$r_B = a \exp \int_B^{2\Delta} \frac{dF}{[2 - D(2 - p)]F} \tag{40}$$

follows from (22).

Now the derivative with respect to time in (38) is carried out. The derivative of the integral is equal to the integral of the derivative since the terms that should contain the integrand times the derivatives of the limits are zero. Consequently,

$$W(t) = \frac{(kn_1r_B)^2}{8}\frac{dB}{dt}. \tag{41}$$

The reader interested in the explicit impulse response must substitute B in this expression with its time-dependent value obtained from (16).

Replacing $W(t)$ in (37) and also substituting the explicit value of t from (16), the rms width of the impulse response results in

$$\sigma = \frac{T}{\sqrt{2}}\left(\frac{\int\int_0^{2\Delta}\left(\frac{1-x/D}{\sqrt{1-x}}-\frac{1-y/D}{\sqrt{1-y}}\right)^2 r_x^2r_y^2dxdy}{\int\int_0^{2\Delta} r_x^2r_y^2dxdy}\right)^{1/2}, \tag{42}$$

where x and y are dummy variables and r_x and r_y are given by (40) once B is substituted either by x or by y. It is easy to recognize in (42) that if $\Delta \ll 1$ and $D \cong 2$, the parenthesis is of the order of Δ^2 and σ is proportional to $\Delta^2 T$.

Unlike the simple impulse-response width, the rms width σ and the optimum value of D that minimizes it are dependent on the profile dispersion p and the profile function F. In general, the exact value of σ and its minimizations must be found numerically, but we push the analysis a little further in the next section where some simplifying assumptions are made.

V. APPROXIMATE RESULTS FOR RMS WIDTH OF THE IMPULSE RESPONSE, ITS MINIMIZATION, AND ITS SENSITIVITY TO PROFILE ERRORS

Within the family of fibers described in the previous sections there is a large group of particular importance that encompasses many of the available fibers today. This group has small NA and its profile dispersion is almost constant with respect to r. To introduce these properties, we assume

$$\Delta \ll 1, \tag{43}$$

then the profile dispersion is expanded in power series of the profile function F,

$$p = \sum_{s=0}^{\infty} P_s\left(\frac{F}{2\Delta}\right)^s, \tag{44}$$

and since F is a function of r, the near invariance of p with r implies

$$\sum_{s=1}^{\infty} P_s \left(\frac{F}{2\Delta}\right)^s \ll 1. \tag{45}$$

5.1 Profile function

Carrying this simplifying assumption to (22) by keeping only first powers of $P_s(s > 0)$, the profile function results:

$$F = 2\Delta \left(\frac{r}{a}\right)^{\alpha} \left\{1 + \frac{2}{\alpha} \sum_{s=1}^{\infty} \frac{P_s}{s} \left[1 - \left(\frac{r}{a}\right)^{s\alpha}\right]\right\}, \tag{46}$$

where

$$\alpha = D(2 - P_0) - 2 \tag{47}$$

and D is still an arbitrary number.

If the profile dispersion is constant, the summation in (46) disappears; then, and only then, will the profile function follow a pure power law.

5.2 Minimization of the rms width of the impulse response and its sensitivity to profile errors

We want to find $\sigma_{\min}$, the minimum rms width of the impulse response possible, and D_1, the optimum dispersion parameter for which $\sigma_{\min}$ is achieved. The optimum profile is obtained by substituting D with D_1 in (46). We are interested also in finding the sensitivity of σ to small errors in the profile.

To achieve these purposes σ^2 is expanded in a power series about D_1, and only the first three terms are kept,

$$\sigma^2 = \sigma_{\min}^2 + (D - D_1)\frac{d\sigma^2}{dD} + \frac{(D - D_1)^2}{2}\frac{d^2\sigma^2}{dD^2}. \tag{48}$$

The derivatives are to be taken at $D = D_1$. Since by definition σ^2 passes through a minimum of $D = D_1$, the equation

$$\frac{d\sigma^2}{dD} = 0 \quad \text{at} \quad D = D_1 \tag{49}$$

serves to determine the optimum dispersion parameter D_1.

From (42), (48), and (49), we obtain with the help of (43) and (45)

$$D_1 = 2\left[1 - \frac{\Delta}{2}\frac{1+2H}{1+4H}(1+\Sigma)\right] \tag{50}$$

$$\sigma_{\min} = T(\Delta H)^2 \frac{(1+H)^{1/2}}{(1+3H)(1+4H)(1+5H)^{1/2}} \tag{51}$$

$$\frac{\sigma}{\sigma_{\min}} = \sqrt{1 + \left(\frac{D-D_1}{D_1 \Delta H}\frac{1+4H}{1+2H}\right)^2 (1+3H)(1+5H)}, \tag{52}$$

where

$$\Sigma = 2\sum_{s=1}^{\infty} P_s \frac{(s^2+s+6)H^2+8H+2}{\{[(s+2)H+1]^2 - H^2\}\{[(s+3)H+1]^2 - H^2\}} \tag{53}$$

and

$$H = 1 - P_0. \tag{54}$$

The optimum value of the dispersion parameter D_1 is close to 2. The profile function that maximizes the information-carrying capacity of the fiber is obtained by substituting D with D_1 in (46). The dispersion-profile terms of order higher than zero appear in (50) only in the summation Σ and are multiplied by Δ. Therefore, their contribution is small indeed and is neglected in (51) and (52). It is kept in (50) because, as will be seen later, small errors in the profile affect substantially the value of σ. For $\Sigma = 0$, the substitution of (50) in (47) yields the same optimum α of Ref. 3.

Consider now $\sigma_{\min}$, the minimum rms width of the impulse response. From (51), we might be tempted to conclude that $H = 1 - P_0$ should be made small to decrease $\sigma_{\min}$. However, the number of modes of the guide derived from (38) with the help of (46) is

$$N = \left(\frac{kn_1a}{2}\right)^2 \frac{\Delta H}{1+H}. \tag{55}$$

Therefore, if the number of modes of the fiber is to be kept constant, $\sigma_{\min}$ can be decreased by making

$$\frac{(1+H)^{5/2}}{(1+3H)(1+4H)(1+5H)^{1/2}}$$

small and this is achieved by choosing H large, not small.

The following table contains the minimum rms per unit length of fiber, $\sigma_{\min}/L$, and the concomitant pulse repetition rate $L/4\sigma_{\min}$ for different values of H as derived from (58), assuming $N_1 = 1.46$.

H	σ_{min}/L $\mu s/km$	$PRR = \frac{L}{4\sigma_{min}}$ Mb · km/s
1	$0.14\Delta^2$	$1.79/\Delta^2$
2	$0.16\Delta^2$	$1.56/\Delta^2$
∞	$0.18\Delta^2$	$1.38/\Delta^2$

For $H > 1$, the pulse repetition rate is fairly insensitive to the value of H. For $H = 1$ and $\Delta = 0.01$, the pulse repetition rate is ~18 Gb/km/s. This information-carrying capacity is only 33 percent smaller than that of the "ideal profile" reported by Cook.[17]

Let us turn now to the sensitivity of the rms width to errors in profile (52). Again, for $H > 1$, this result is insensitive to the value of H; indeed

$$\frac{\sigma}{\sigma_{min}} = \begin{cases} \sqrt{1 + 66.7\left(\dfrac{D - D_1}{D_1\Delta}\right)^2} & \text{for} \quad H = 1 \\ \sqrt{1 + 60\left(\dfrac{D - D_1}{D_1\Delta}\right)^2} & \text{for} \quad H = \infty. \end{cases} \tag{56}$$

For $H = 1$ and $(D - D_1)/D_1$, the fractional departure of D with respect to the optimum D_1 equal to Δ, the rms width σ is about 8.4 times wider than σ_{min}. As in the case of the pulse width, the rms width is very sensitive to profile errors.

A fiber designed to minimize the rms width ($D = D_1$) has only 30 percent more information-carrying capacity than a fiber with the same Δ designed to minimize the impulse-response width ($D = D_0$).

VI. CONCLUSIONS

For a vast class of circular-symmetric fibers made of isotropic dielectrics, simple and fundamental design criteria that minimize the impulse-response width due to modal dispersion at one wavelength have been found. This minimum width (30) is only a function of the NA and the time of flight along the axis. Therefore, if properly designed, a fiber with arbitrary profile dispersion has the same minimum impulse-response width as another fiber with the same NA and no profile dispersion. Their information-carrying capacity is about $1.4/\Delta^2$ Mb · km/s. The fiber engineer has a substantial freedom of choice to reach that optimum design: the profile dispersion may be arbitrarily chosen but then the index profile is uniquely determined by (22); or symmetrically, the index profile may be arbitrarily chosen and then the profile dispersion must satisfy (17). Only if the profile dispersion is a constant does the optimum dielectric profile that minimizes the impulse response follow a power law.

The profile dispersion entails the first derivative of the index with respect to the wavelength. If the second derivative can be controlled, then the minimization of the impulse-response width can be achieved at two neighboring wavelengths. This broadbanding of the fiber response can be expanded even further if higher derivatives are under control.

The width of the impulse response is very sensitive to errors in the fiber design. A fractional error of Δ in the dispersion parameter of the fiber makes the response about nine times wider than the minimum as seen from (35).

Only a marginal increase of about 25 percent in the information-carrying capacity of the fiber is achieved if the rms width of the impulse response is minimized instead of minimizing the impulse-response width.

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