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Detection and Selective Smoothing of Transmission Errors in Linear PCM

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We consider detection of transmission errors in PCM by means of statistical hypothesis testing of the received quantized sequence. When errors are detected, a median filter is used to smooth waveform discontinuities. We describe two error detectors, one (CDC), based on correlation measurements, and the other (DDC), based on sample-to-sample difference measurements. While both offer s/n advantages over conventional PCM in the presence of errors, DDC is more promising both in terms of performance and simplicity of implementation.

I. INTRODUCTION

An acceptable decoded signal-to-noise ratio (s/n) can be maintained in a pulse code modulation (PCM) system in the presence of transmission errors if error detecting and correcting codes are added to the transmitted PCM signal. This approach[1,2] when combined with a Huffman coder in juxtaposition to the PCM and channel encoders offers the best theoretical solution, given that the properties of the transmission channel can be specified. By best solution we mean that for a specified channel bandwidth and error rate, the highest decoded s/n can be achieved. However, this approach is not usually justified economically, and partial solutions may be appropriate.

One solution is to retain a conventional transmitter and to modify the receiver of a PCM system to make provision for the possibility of transmission errors. Jayant[3] has observed that when the channel error rate is high, a linear or non-linear filter prior to the desampling filter reduces the noise due to transmission errors, but only at the expense of a degradation of speech quality when the channel error rate is low. This approach is analogous to reducing the bandwidth of a high-frequency receiver or introducing a noise filter in a high-fidelity system.

In this paper, we discuss a system which, like one of those described by Jayant, uses a median filter[4,5] to squelch channel error noise. How-

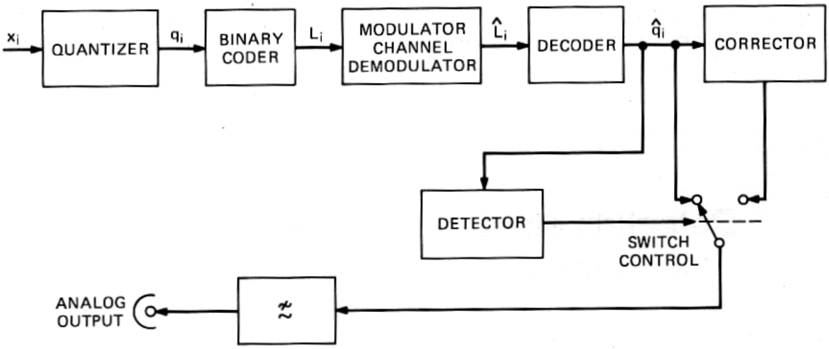

Fig. 1—PCM system with error protection.

ever, in our system this filter is introduced selectively under the control of an error detector, which measures certain statistics of the received PCM signal and makes inferences about whether individual samples or short blocks of samples have been contaminated by channel errors. Only when errors are detected is the median filter introduced. We use the results of computer simulations to show that this approach can lead to significant improvements in system signal-to-noise ratio. Another detection and correction system has been proposed for differential PCM systems operating substantially above the Nyquist rate[6].

II. ERROR DETECTION AND CORRECTION

We have investigated the system shown in Fig. 1, using a third-order median filter as a corrector. When the detector infers the presence of an error in either an individual sample or a block of samples, it causes the switch to be in the right-hand position, thus introducing the median filter, which, at time m, replaces the sample $\hat{q}_m$ with the median value of the samples $\hat{q}_{m+1}, \hat{q}_m, \hat{q}_{m-1}$. With no error detected, the switch is in the left-hand position and $\hat{q}_m$ goes directly to the low-pass filter, which transforms the sequence of quantized samples to a continuous waveform. Higher-order median filters,[5] though less satisfactory in the absence of an error detector,[3] may well improve the performance of a selective correction scheme. Linear smoothing is also likely to be effective.

The detector is essentially looking for an unexpected event in $\{\hat{q}_i\}$, and the more unexpected the event, the greater is the likelihood of detection. Correspondingly, large errors are more likely to be observed than small ones. Very small errors are unlikely to be detected due to their statistical similarity to the sequence $\{q_i\}$ at the transmitter. This characteristic is a limitation, although not too serious as it is the large errors that cause the greatest degradation of s/n.

We have studied two detectors, both of which process blocks of M samples and compute a statistic characteristic of each block. They also

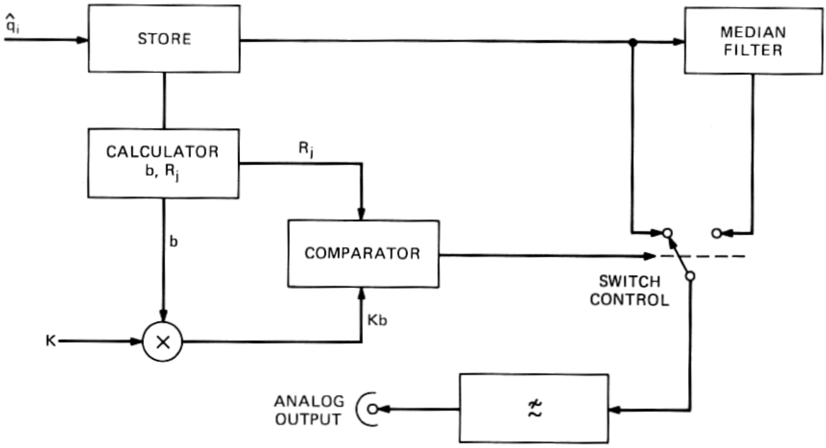

Fig. 2—Correlation detection and correction.

process shorter blocks (of length SB) imbedded in each long block, compute corresponding block statistics, and infer errors in a short block whenever its statistic is substantially different from that of the long block. In the correlation detection and correction system (CDC), the statistic is the first-order autocorrelation coefficient. In the difference detection and correction system (DDC), the statistic is the rms value of the sample-to-sample difference. In both cases, the long block length is $M = 64$ for speech sampled at 8 kHz. In CDC, the short block has $SB = 16$ samples while in DDC, $SB = 2$ and the block statistic is simply the magnitude of a single sample-to-sample difference.

III. TWO ERROR DETECTORS—DEFINITIONS

3.1 Correlation detection and correction

Figure 2 is a schematic representation of CDC. Correlation coefficients b and R_j are computed over blocks of two different sizes. The coefficient b, computed over a long block of samples of length M, provides an estimate of the local correlation of the transmitted sequence. It is compared with $\{R_j\}$, a sequence of correlation coefficients computed over short blocks of length SB, which lie within the long block. The presence of one or more errors in a short block can result in correlations substantially lower than b because channel errors are independent of the signal source. Thus, if R_j is substantially lower than b, the system infers the presence of an error in the block of length SB and replaces the samples in that block:

$$\hat{q}_j, \hat{q}_{j+1} \ldots, \hat{q}_{j+(SB-1)}$$

with the corresponding outputs of the median filter.

Specifically, for the first M received samples, we have

$$b = \frac{\frac{1}{M-1} \sum_{i=1}^{M-1} \hat{q}_i \, \hat{q}_{i+1}}{\frac{1}{M} \sum_{i=1}^{M} (\hat{q}_i)^2}. \tag{1}$$

Within this block, the detector computes R_1, the correlation coefficient of samples $\hat{q}_1$ to $\hat{q}_{SB}$; R_2 is based on $\hat{q}_2$ to $\hat{q}_{SB+1}$; etc. In general,

$$R_j = \frac{\hat{q}_j\hat{q}_{j+1} + \hat{q}_{j+1}\hat{q}_{j+2} + \ldots + \hat{q}_{j+SB-2}\hat{q}_{j+SB-1}}{(SB-1)\, S_j{}^2}, \tag{2}$$

where

$$S_j = \frac{1}{SB} \sum_{i=1}^{SB} (\hat{q}_{j+i-1})^2; \qquad j = 1,2, \ldots, M - SB + 1. \tag{3}$$

The second long block begins at $k = M - SB + 2$ and provides a value of b to be compared with $R_{M-SB+2}, R_{M-SB+3}, \ldots, R_{2(M-SB+1)}$. The third long block begins at $k = 2M - 2SB + 3$ etc. The time windows defining short blocks move one sample at a time; these defining long blocks slide over $M - SB + 1$ samples.

A particular block j of SB samples is deemed to contain errors if R_j is sufficiently smaller than b; i.e., if

$$R_j < Kb, \tag{4}$$

where $K < 1$ is a design parameter of the CDC system.

3.2 Difference detection and correction

This scheme, shown in Figure 3, is based on the notion that the differences between successive samples of a correlated input source tend to be relatively small. The detector infers that an unusually large sample difference is the result of a transmission error. Over the nth block of M samples the detector computes σ_b, the rms difference between successive samples, where

$$\sigma_b{}^2 = \frac{1}{M} \sum_{nM+1}^{(n+1)M} (\hat{q}_i - \hat{q}_{i-1})^2. \tag{5}$$

It then examines the magnitudes, $|\Delta_k|$, of individual sample-to-sample differences and if

$$|\Delta_k| \geqq L\sigma_b, \tag{6}$$

the system replaces $\hat{q}_k$ with the corresponding output of the median filter. We use for Δ_k

$$\Delta_k = \hat{q}_k - Q_k, \tag{7}$$

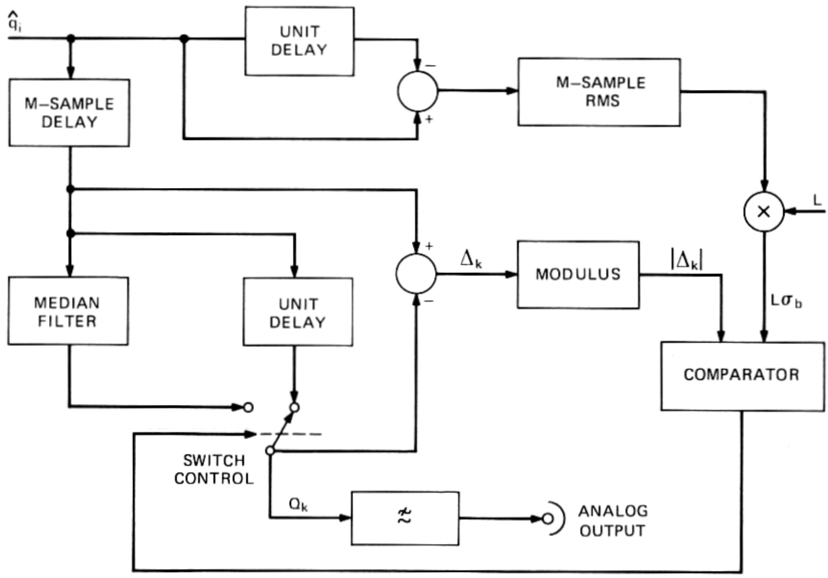

Fig. 3—Difference detection and correction.

where Q_k is the actual system output. It is $\hat{q}_{k-1}$, if $|\Delta_k| < L\sigma_b$; otherwise, it is a median filter output.

Notice that when an error is detected, the DDC system replaces an *individual* sample with a median filter output, while the CDC system uses the median filter to modify an *entire block* of samples. This greater selectivity of the DDC system results in the modification of fewer correctly received samples than with CDC. This property accounts for the fact that DDC provides better measured performance than CDC.

IV. PERFORMANCE EVALUATION

To gain insight into the detection and correction mechanisms, we implemented both detectors on a general-purpose computer and studied their operation on PCM samples derived from an artificial, statistically stationary source. The initial simulations were efficient computationally and demonstrated the effects of design parameters and signal characteristics on s/n. They also indicated that DDC performs better than CDC in the presence of isolated channel errors. These simulations were followed by investigations (using both software and special-purpose hardware) of DDC operating on PCM-coded speech transmitted over binary symmetric channels. With speech transmission, the error suppression provided by DDC is clearly audible and the s/n characteristics are similar to those observed with the artificial source.

In the next two sections, we present the results of the first simulations.

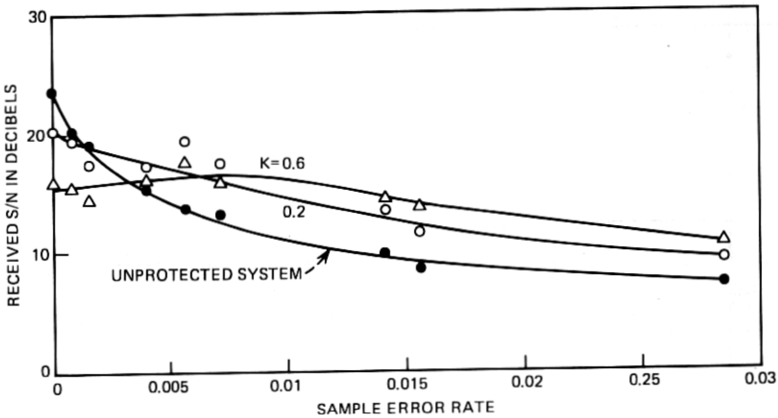

Fig. 4—CDC performance, 32-level quantizer.

The source was a Gauss-Markov sample sequence with a correlation of 0.85, chosen to be representative of speech sampled at 8 kHz. Channel errors were introduced by replacing source samples with quantized samples of a white Gaussian process independent of the source.

4.1 CDC, Gauss-Markov source

With applications to speech communication in mind, we adopted, for the length of the long blocks $M = 64$, an interval (6–10 ms for typical sampling rates) over which correlation properties are expected to change slowly. The choice of the length, *SB*, of the short blocks reflects a compromise between the aims of obtaining: (*i*) reliable correlation measures (achieved with SB large), and (*ii*) accurate error localization (achieved with SB small). Unreliable correlation measures result in false alarms—spurious error detections—while imprecise error localization leads to the modification of a large number of correctly received samples. The other detector design parameter is K in eq. (4), which sets the threshold of error detection. A low value of K provides a stringent criterion, leading to fewer false alarms, but also fewer correctly detected errors than a high value of K.

After studying the influence of *SB* and K on the false alarm and correct detection probabilities, we arrived at $SB = 16$ as an appropriate compromise. Effective values of K range from 0.2 to 0.6, depending on the sample error rate (η).

The dependence of s/n on error rate is shown in Fig. 4 for a 32-level quantizer and $K = 0.2$ and 0.6. Observe that for $\eta > 0.002$, the CDC system having $K = 0.2$ is preferable to an unprotected PCM system. For $\eta > 0.006$, the improvement in the received s/n is approximately 3 dB. The CDC system offers an improvement of a further 1 to 2 dB when $\eta > 0.01$, K

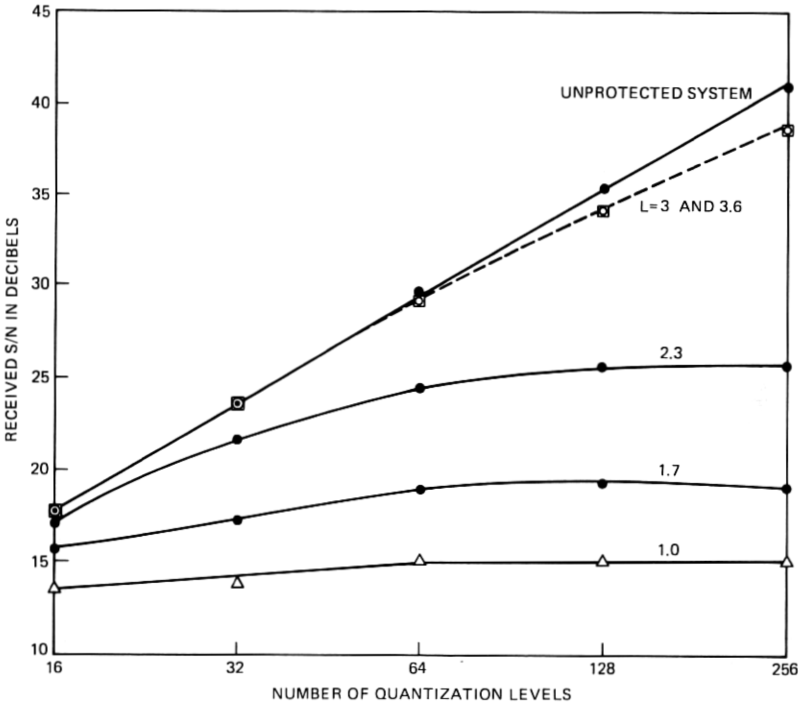

Fig. 5—DDC performance, error-free channel.

= 0.6. On the other hand, when the channel quality is reasonably high ($\eta < 0.002$), the CDC degrades s/n performance relative to an unprotected system. Even with $K = 0.2$, the deterioration is more than 3 dB in an error-free channel, and it is more substantial when the number of quantizer levels exceeds 32. The extra noise arises from the replacement of a block of 16 correctly received samples by median filter outputs whenever a false alarm occurs. This high cost of false alarms is a principal disadvantage of CDC. It does not exist in the DDC system, where an isolated false alarm introduces only one median filter output.

4.2 DDC, Gauss-Markov source

In this system, the detector design parameters are the block size M over which the rms difference signal, σ_b, is calculated and L in eq. (6), which determines the criterion of error detection. As in the CDC system, we used $M = 64$ to provide a syllabic measure of the rms sample-to-sample difference signal, and investigated the effects on s/n of several values of L under various transmitter and channel conditions.

There is an important improvement in the error-free condition compared to CDC. Figure 5 shows that for the criteria $L = 3$ and 3.6, the

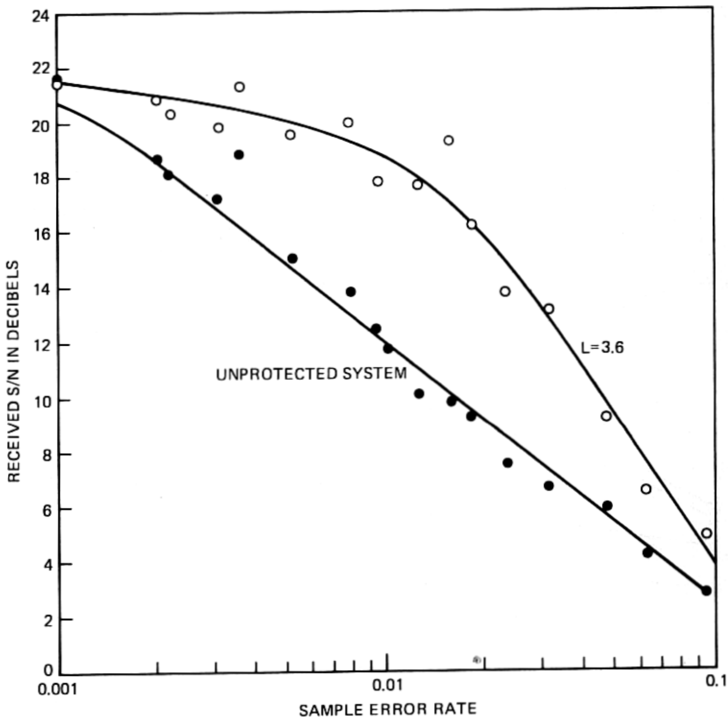

Fig. 6—DDC performance, 32-level quantizer.

degradation in received s/n due to false alarms is negligible for quantizers with 16 to 64 levels, and is only 2 dB for a quantizer with 256 levels.

The variation of s/n with η is displayed in Fig. 6 for a 32-level quantizer and $L = 3.6$. The DDC system is superior to the unprotected linear PCM system. With DDC, the s/n decreases by approximately 3 dB compared to the 9-dB reduction of the unprotected system when η increases from 0.001 to 0.01. At a 1 percent error-rate DDC provides a 7-dB improvement in s/n. With larger quantizers, the dependence of s/n on η follows the curves in Fig. 6 for $\eta > 0.003$. At these error rates the major part of the received noise is due to transmission errors rather than quantization.

Figure 7 shows s/n as a function of input power for a 32-level quantizer and $\eta = 0.016$. At low levels of input power, the quantized samples at the transmitter are generally quite small while transmission errors can cause very large samples to appear at the receiver. The resulting large sample-to-sample differences are reliably detected making DDC particularly effective at low input levels, which accounts for the fact that with DDC, s/n depends only to a small extent on input power. This property of DDC is in strong contrast to unprotected PCM in which the noise due to channel errors is essentially independent of signal level and s/n decreases

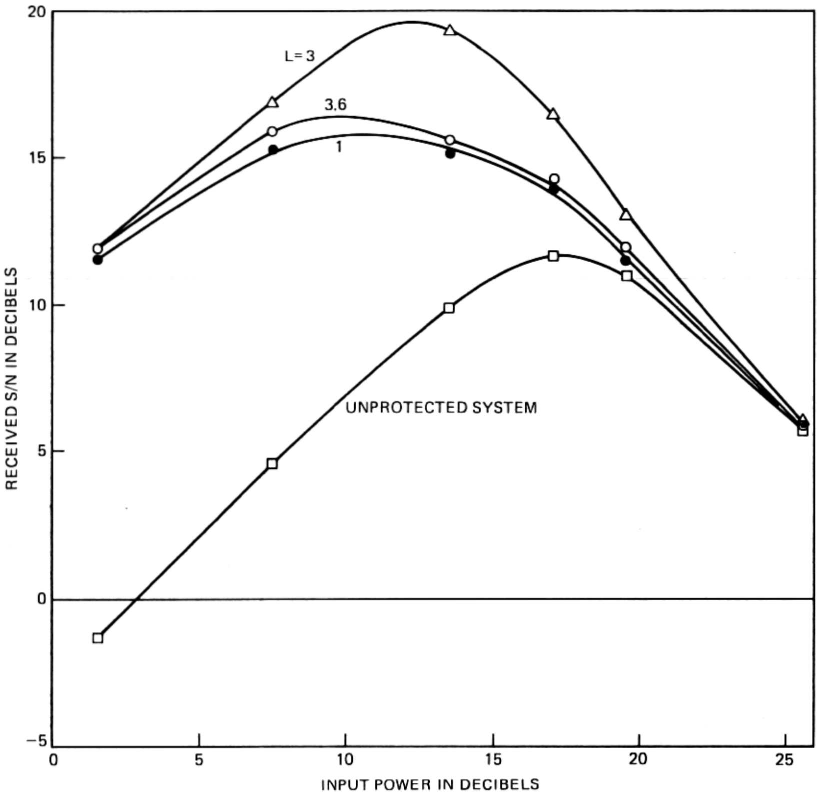

Fig. 7—DDC performance, 32-level quantizer, $\eta = 0.016$.

1 dB for each dB decrease in input power, as shown in Fig. 7. Because the s/n improvement at low signal levels does not depend on significant sample-to-sample correlations, the DDC system can be expected to perform well with speech signals which consist of essentially two waveform types: (*i*) high-level, highly correlated waveforms of voiced sounds, and (*ii*) low-level, uncorrelated waveforms of unvoiced sounds. Errors in both waveform types are detectable with DDC.

4.3 DDC, speech inputs

Encouraged by performance with Gauss-Markov inputs, we studied, by means of a computer simulation and a laboratory model, DDC systems operating on received PCM samples derived from a speech source. In the simulation, quantized samples were coded in a 5-bit sign-magnitude format and transmitted over a binary symmetric channel. The s/n performance, measured over an entire 2-second sentence, is shown in Fig.

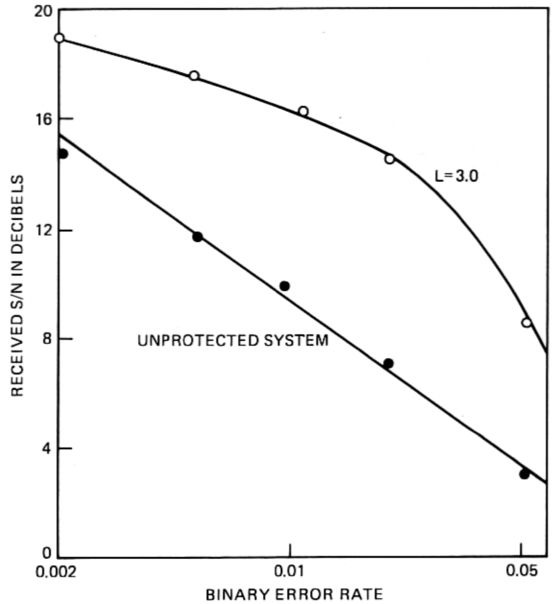

Fig. 8—DDC performance, speech transmission, 32-level quantizer.

8. The improvement introduced by DDC is immediately audible in the decoded outputs of DDC and the unprotected PCM receiver.

V. DISCUSSION

Two detection systems in connection with a smoothing filter corrector have been described and shown to give improvement in the s/n of a linear PCM codec in the presence of transmission errors. This improvement has been achieved without recourse to error detecting and correcting codes. The key element of both systems is a detector that allows the smoothing filter to be used selectively on the basis of inferences about errors in the received sequence. To date we have experimented with only one corrector, the third-order median filter. Although it increases s/n relative to an unprotected system, it is possible that other smoothing schemes offer even greater improvements.

Of the two detectors, DDC has better s/n performance and is easier to implement. It does not involve the calculation of correlation coefficients and the rms value of the quantized difference signal is easy to measure. It also is well-matched to the characteristics of speech waveforms.

There are existing and anticipated digital transmission systems in which performance is limited by channel quality. We hope that the approach taken here will be of value in upgrading performance at the expense of a tolerably small increase in receiver cost. Our method is also applicable to signal enhancement in systems other than PCM in which distortions are characterized by short, severe signal discontinuities. In such systems (FM signals exhibiting clicks is an example), the disturbed signal can be digitized, selectively smoothed, and desampled in the manner described here to produce an improved output.

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