THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 56, No. 6, July–August 1977
Printed in U.S.A.

Cancellation of Polarization Rotation in an Offset Paraboloid by a Polarization Grid

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(Manuscript received July 20, 1976)

The polarization rotation properties of the field radiated from a polarization grid have been found to be similar to those in the aperture of an offset parabolic antenna. This observation suggests broadband cancellation of the polarization rotation in a large offset reflector by the opposite prerotation of the incident feed radiation via a polarization grid. Reduction of cross polarization from −24 dB without the grid to −39 dB with the grid wires is predicted in a numerical example. A previously unexplained polarization rotation measured using an offset parabolic grid is shown to be in good agreement with calculation.

I. INTRODUCTION

Current interest in frequency reuse through orthogonal polarizations creates a strong incentive for improving polarization properties of antennas. Cross-polarized radiation from an offset reflector[1] is often regarded as a blemish on an otherwise excellent antenna, which offers both low sidelobe level and good impedance matching.[2] Although the cross polarization can be minimized using a large effective f/D ratio, the corresponding requirements of small offset angle and large feed aperture are not always convenient in applications. Recently a trimode feed horn[3] has been discussed as a means of reducing the cross-polarized radiation of offset reflectors; however, a multimode arrangement is inherently narrow in bandwidth. Our purpose here is to propose a broadband method for reducing the cross-polarized radiation of the offset reflector antenna. This scheme is based upon cancellation of polarization rotation due to reflector curvature[1] by opposite rotation generated by a polarization grid.[4]

In Section II we examine the similarity between cross polarization in the aperture of an offset reflector and that of a polarization grid. The condition of cancelling the first-order cross-polarization terms is then

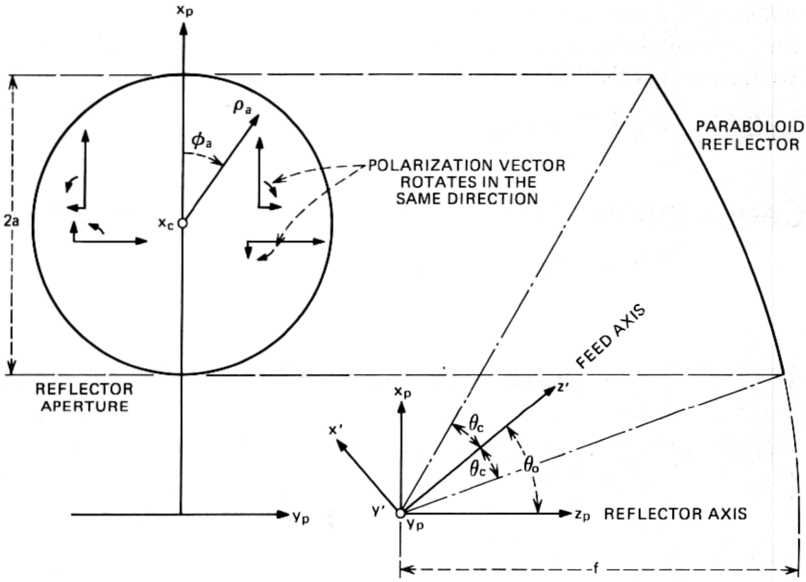

Fig. 1—Geometry of offset reflector.

deduced. In Section III numerical calculations of practical examples are given.

In the appendix the previously unexplained measured polarization rotation* of a parabolic grid[5] is calculated. This example is given here as experimental evidence of agreement with the predicted rotation of radiation from a polarization grid.

II. CANCELLATION OF POLARIZATION ROTATION

Let us first briefly review the salient properties of the cross-polarized field in the aperture of an offset paraboloid as shown in Fig. 1. For a balanced feed radiation,

$$\mathbf{E}_f = F(\theta', \phi') \begin{bmatrix} \cos\phi' & & \sin\phi' \\ \hat{\theta}' & \mp & \hat{\phi}' \\ \sin\phi' & & \cos\phi' \end{bmatrix} \frac{\exp(-j\rho)}{\rho}, \tag{1}$$

where (ρ, θ', ϕ') are spherical coordinates with respect to the z' axis and $(\hat{\theta}', \hat{\phi}')$ are the corresponding unit vectors. Since the reflected field from the paraboloid is

$$\mathbf{E}_r = -\mathbf{E}_f + 2\hat{n}(\mathbf{E}_f \cdot \hat{n}),$$

* The design was initially proposed by Comsat for the Comstar satellite.

where $\hat{n}$ is a unit vector normal to the reflector surface. The principal and cross-polarized field components in the reflector aperture can be written respectively:[1]

$$M = \mathbf{E}_r \cdot \begin{matrix}\hat{x}_p \\ \hat{y}_p\end{matrix} = \frac{F(\theta', \phi')}{t\rho} [\sin \theta' \sin \theta_0 \cos \phi' - \sin^2 \phi' \cdot (\cos \theta_0 + \cos \theta') - \cos^2 \phi'(1 + \cos \theta_0 \cos \theta')] \quad (2)$$

$$N = \mathbf{E}_r \cdot \begin{matrix}\hat{y}_p \\ \hat{x}_p\end{matrix} = \mp \frac{F(\theta',\phi')}{t\rho} [\sin \theta' \sin \theta_0 \sin \phi' - \sin \phi' \cos \phi'(1 - \cos \theta')(1 - \cos \theta_0)], \quad (3)$$

where $t = 1 + \cos \theta' \cos \theta_0 - \sin \theta' \sin \theta_0 \cos \phi'$, $M^2 + N^2 = F^2/\rho^2$, and N vanishes when $\theta_0 = 0$. The offset angle θ_0 is between the feed axis and the reflector axis. The sign combination in eqs. (2) and (3) indicates that the rotation of the polarization vector due to offset in a paraboloidal aperture has the same magnitude and is in the same direction as illustrated in Fig. 1 for any orientation of the incident linear polarization. The projection of the intersection of a circular cone (with vertex at the focus) and the offset paraboloid onto the $x_p y_p$ plane is a circular aperture with center

$$x_c = \frac{2f \sin \theta_0}{\cos \theta_0 + \cos \theta_c} \quad (4)$$

and radius

$$a = \frac{2f \sin \theta_c}{\cos \theta_0 + \cos \theta_c}, \quad (5)$$

where θ_c is the half angle of the cone. Equations (4) and (5) will be used later to obtain the relations in eqs. (12) and (13).

Radiation from transmitting and reflecting wire grids can be obtained by magnetic and electric current sheet models, respectively. The principal and cross-polarized components are[4]

$$P = -C[1 - \cos^2 \phi' (1 - \cos \theta') - \sin \theta' \cos \phi' \tan \delta] \quad (6)$$

$$X = \pm C[\sin \phi' \cos \phi'(1 - \cos \theta') + \sin \theta' \sin \phi' \tan \delta], \quad (7)$$

where C is a proportionality constant, θ' and ϕ' are the spherical coordinates of the feed (z') axis, and δ is the angle between the conducting wires and the $x'y'$ plane as shown in Fig. 2; the expressions inside the brackets are identical to those of eqs. (9) and (10) in Ref. 4, provided one makes the following substitutions: $\phi' = \phi - 90°$ and $\delta = 90° - \gamma$. The changes of notation are made for the purpose of comparison with eqs. (2) and (3). The upper and lower signs in eq. (7) correspond to the transmitting and reflecting cases. The orientations for the transmitting

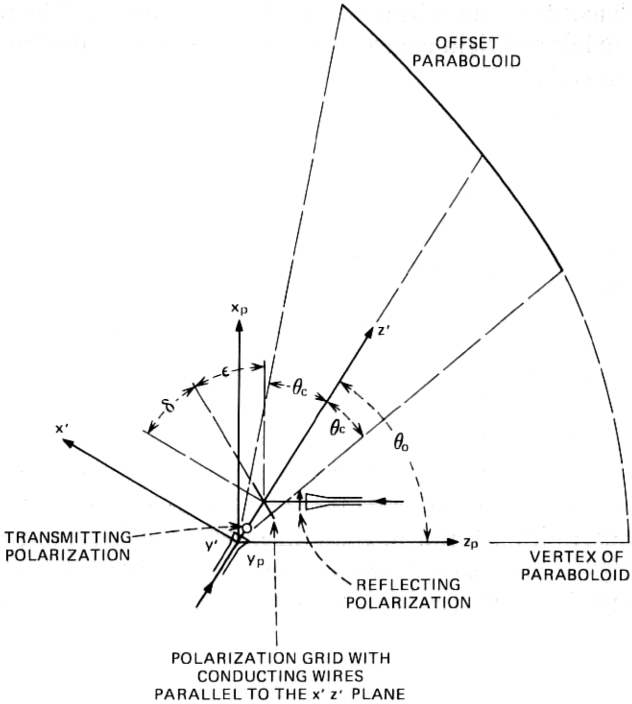

Fig. 2—Configuration for cancellation between polarization rotations of an offset paraboloid and a polarization grid.

and reflecting polarizations together with a given grid geometry are shown in Fig. 2 where the conducting wires are parallel to the plane of the figure.

One notes that the leading terms, which contain first power of θ', in eqs. (3) and (7) have the same sinusoidal dependence on θ' and ϕ'. Furthermore, the sign combination in eqs. (6) and (7) indicates that the transmitting and reflecting orthogonal polarizations rotate in the same direction, opposite to the rotation in the aperture of an offset paraboloid.

Let us take the first-order approximation—i.e., $\cos\theta' \approx 1$—in eqs. (2), (3), (6) and (7), but $\sin\theta'$ is retained. Then the cross polarization in eq. (3) normalized with respect to eq. (2) cancels that in eq. (7) normalized with respect to eq. (6):

$$\frac{X}{P} + \frac{N}{M} = 0 \text{ if } \delta = \frac{\theta_0}{2}. \tag{8}$$

Polarization rotation of the radiation from a wire grid, as predicted by eqs. (6) and (7), also explains data measured using a cylindrical reflector made of a curved wire grid. The report on this experiment[5] misinterpreted the polarization rotation as a consequence of diffraction and the offset geometry. A comparison between the calculated and measured polarization rotations of this case is given in the Appendix.

III. NUMERICAL EXAMPLES

Since the cancellation of polarization rotation discussed in the preceding section only eliminates the leading terms, it is of interest to determine the residual cross polarization. Assuming that the offset reflector is located in the far zone of the radiation from a wire grid, as shown in Fig. 2, the principal and cross-polarized components in the reflector aperture can be written

$$P = F(\theta')[1 - \cos^2 \phi_p (1 - \cos \theta_p) + \sin \theta_p \cos \phi_p \tan \epsilon] \quad (9)$$

$$X = \mp F(\theta') [\sin \phi_p \cos \phi_p (1 - \cos \theta_p) - \sin \theta_p \sin \phi_p \tan \epsilon], \quad (10)$$

where $F(\theta')$ is the feed-radiation pattern and

$$\theta' = \cos^{-1} [\cos \theta_p \cos \theta_0 + \sin \theta_p \sin \theta_0 \cos \phi_p]. \quad (11)$$

The above equations are simply a decomposition of the grid radiation into the two orthogonal components of a balanced feed whose axis coincides with the paraboloidal axis. The expressions inside the brackets of eqs. (9) and (10) are of the same form as those of eqs. (6) and (7); but θ_p and ϕ_p are the spherical coordinates with respect to the paraboloidal (z_p) axis instead of the feed axis, and $\epsilon = (\theta_0 - \delta)$ is the angle between the conducting wires and the $x_p y_p$ plane, as shown in Fig. 2.

To relate eqs. (9) and (10) to the normalized aperture coordinates $r = (\rho_a/a)$ and ϕ_a, the following expressions can be obtained with the aid of eqs. (4) and (5):

$$\theta_p = 2 \tan^{-1} \left[\frac{\sqrt{(r \sin \theta_c \sin \phi_a)^2 + (\sin \theta_0 + r \sin \theta_c \cos \phi_a)^2}}{\cos \theta_c + \cos \theta_0} \right] \quad (12)$$

$$\phi_p = \tan^{-1} \left[\frac{r \sin \theta_c \sin \phi_a}{\sin \theta_0 + r \sin \theta_c \cos \phi_a} \right]. \quad (13)$$

Numerical examples of several combinations of parameters (θ_0, θ_c and ϵ) have been calculated for the principal and residual cross-polarization components from eqs. (9) and (10). The feed pattern has a gaussian shape with 10-dB taper at the edge of the reflector. The principal polarization is close to unity (0 dB) around the center of the reflector aperture. The maximum, calculated, residual cross polarization is given in Table I for a number of examples. Fig. 3 shows a plot of both principal and cross polarizations for the case $\theta_0 = 50°$, $\theta_c = 20°$, and $\epsilon = 25°$. Only half of the

Table I — Cross Polarization in the aperture of an offset reflector

θ_0 (Deg)	θ_c (Deg)	ϵ (Deg)	Max. Residual Cross Pol. With Grid (dB)	Max. Cross Pol.[6] Balanced Feed Without Grid (dB)
50	20	25	−38.6	−24.0
50	20	23	−36.4	−24.0
50	20	27	−36.1	−24.0
60	20	30	−38.0	−22.5
60	30	30	−30.9	−18.0
90	20	45	−34.3	−17.5
90	14	45	−40.5	−20.0

aperture needs to be shown for each polarization because of the symmetry. The maximum residual cross polarization in this case is −38.6 dB, reduced from −24 dB for the same reflector aperture illuminated by a balanced feed without a polarizer. Keeping the same set of parameters, $\theta_0 = 50°$ and $\theta_c = 20°$, the calculated residual cross polarization

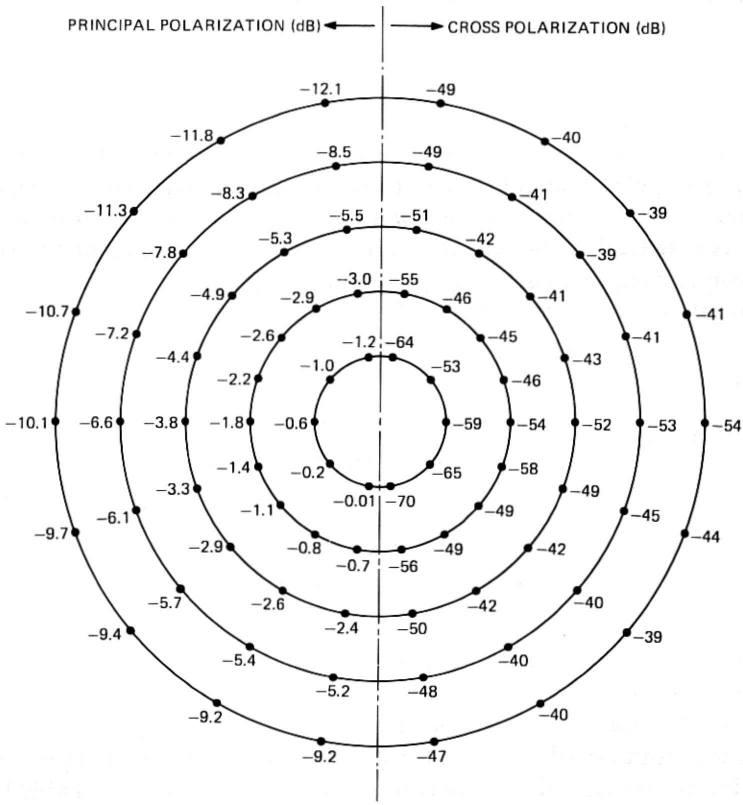

Fig. 3—Relative amplitude levels in the aperture of an offset paraboloid. $\theta_O = 50°$, $\theta_c = 20°$, $\epsilon = 25°$.

becomes −36.4 dB for $\epsilon = 23°$ and −36.1 dB for $\epsilon = 27°$. These results indicate that the residual cross polarization is not overly sensitive to a slight departure from the optimum orientation of $\epsilon = \theta_0/2$.

The examples for $\theta_0 = 60°$ and $\epsilon = 30°$ show residual cross polarizations of −38.0 dB and −30.9 dB for $\theta_c = 20°$ and 30°, respectively. The second-order terms are not quite negligible at $\theta_c = 30°$; however, the cancellation of cross polarization appears to be significant.

IV. DISCUSSION

In view of the residual second-order $(1 - \cos\theta')$ terms, the half-cone angle of the reflector subtended at the focus should not exceed about 20° in order to take full advantage of the cancellation. If the reflectors, such as in an offset cassegrain configuration, do not cause significant cross polarization, the conducting direction of a polarization diplexing grid should be oriented to avoid introducing any polarization rotation, as discussed in Ref. 4.

When $\theta_0 - \theta_c$ is less than about 30° and $\epsilon = \delta = \theta_0/2$, the feed horn associated with the polarization reflected from the grid will introduce some blockage, as shown in Fig. 2. The blocking problem can be eased by using a smaller value of ϵ. This practical difficulty may prevent optimum orientation of the grid wires for reflectors of small offset angle, and hence reduce the effectiveness of the cancellation. The practical application of this scheme should indeed lie in reflectors with large offset angle.

The expression $\epsilon = \theta_0/2$ implies that the grid wires are approximately parallel to the tangent plane at the center of the offset reflector. One notes the similarity between this case and a symmetrical small-cone-angle paraboloid illuminated by a grid-covered feed.

ACKNOWLEDGMENT

The author is indebted to M. J. Gans and D. C. Hogg for valuable discussions, and to Mrs. Diane Vitello for assistance with the computation.

Appendix

Polarization Rotation of an Offset Parabolic Grid

An offset cylindrical-reflector system fed by two line sources of pillbox type (as shown in Fig. 4) was proposed[5] as a dual-polarized antenna with an elliptically shaped coverage pattern. The reflecting system consists of a vertically polarized grid attached to the surface of a parabolic cylinder, the front surface of the grid having the same curvature as the cylindrical reflector. The measured data[5] showed excellent orthogonality (within 1°) between vertical and horizontal polarizations over the whole

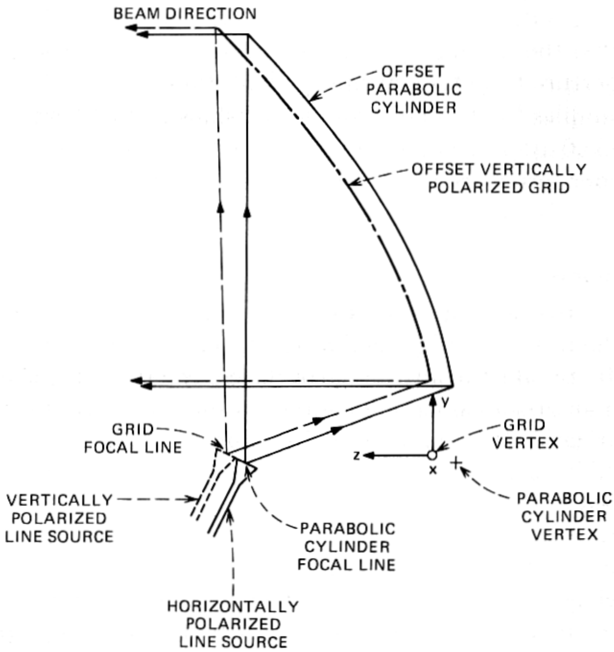

Fig. 4—Configuration of a dual-polarized cylindrical reflector antenna (Ref. 5).

6.8° × 3.4° (3 dB) elliptical beam. However, significant polarization rotations, in the same direction for both polarizations, were observed. Maximum rotations of about 2° occur on the major axis (xz plane) of the half-power ellipse of the beam. No adequate explanation was given for this measured polarization rotation. Here we explain the rotation using electric and magnetic current sheet models for the grid.

The first-order approximation of polarization rotation by a wire grid is simply $\sin\theta \cos\phi \cot\gamma$ (eq. 7 in Section II or eq. (10) in Ref. 4), where θ is the angle off the beam axis, $\cos\phi$ is unity in the xz plane of maximum rotation, and γ the angle between the conducting wires and the beam axis. Since the wires of the curved grid have a variable direction, the rotation Δ in the plane of maximum rotation can be obtained by averaging $\cot\gamma$ over the parabolic curve, $y^2 = 4fz$.

$$\Delta = \frac{\sin\theta}{\int E\,ds} \int E \cot\gamma\, ds = \frac{\sin\theta \int E \dfrac{dz}{dy} \sqrt{(dy)^2 + (dz)^2}}{\int E \sqrt{(dy)^2 + (dz)^2}}, \tag{14}$$

where E is the aperture field distribution. Let $u = y/2f$, eq. (14) be transformed into

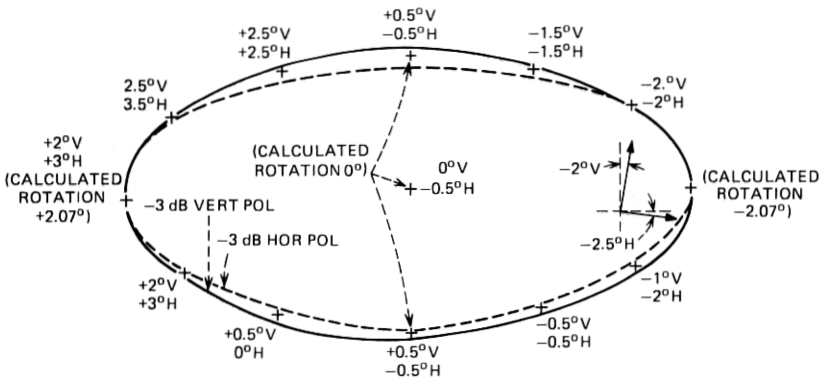

Fig. 5—Measured angles of nominal vertical and horizontal polarizations around −3-dB pattern contours of 6.8° × 3.4° for a dual-polarized cylindrical antenna with a parabolic grid; + indicates locations of measurement.

$$\Delta = \frac{\sin\theta \int_{0.1763}^{1} E\sqrt{1+u^2}\,u\,du}{\int_{0.1763}^{1} E\sqrt{1+u^2}\,du}, \tag{15}$$

where the upper and lower limits of integration are obtained from $y = 2f \tan \psi/2$ with $\psi = 90°$ and $20°$, respectively. Δ is not sensitive to the aperture distribution. Numerical calculation gives $\Delta = 0.61 \sin\theta$ for both cases when E is assumed to be uniform and when E has a 10-dB edge taper—i.e., when

$$E = -0.396 + 4.746\left(\frac{y}{2f}\right) - 4.034\left(\frac{y}{2f}\right)^2. \tag{16}$$

The quadratic form has been chosen to perform the integration in closed form. When $\theta = 3.4°$, $\Delta = 0.0362$ rad $= 2.07°$; agreement between this calculated value and Wilkinson's measured rotation shown in Fig. 5 is indeed very good.

Furthermore, eqs. (6) and (7) in Section II indicate that the transmitted and reflected orthogonally polarized fields from the grid rotate in the same direction, just as in the measured rotations. The planes of measured maximum and null rotation also agree with the predictions. Since the two polarizations have the same sense of rotation, orthogonality is preserved. However, if this rotation occurs in the feed radiation, nonorthogonal elliptically polarized radiations from the reflector will result.

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