THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 57, No. 4, April 1978
Printed in U.S.A.

Loop Plant Modeling:

The Feeder Allocation Process

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(Manuscript received August 20, 1977)

Allocation is an engineering process by which economical use of spare feeder capacity in the loop network may be planned. The basic concept is to apportion the available spare feeder pairs along an entire feeder route such that placement of the relief cable is deferred for the section of the route with the shortest time to relief, thus reducing capital costs. Since rearrangements are planned on a route basis, rather than made expediently, operating costs are also reduced. This paper first illustrates the concept of allocation by applying a simple manual technique to a small example. Then a more sophisticated method, which has been computerized, is described. Finally, a generalized mathematical description is given.

I. INTRODUCTION

An important part of an operating company outside plant engineer's job is to decide how best to use the spare capacity in the feeder network.[1] This activity is called *feeder administration.* A major part of feeder administration is the *allocation* of spare feeder pairs.

1.1 The basic concept of allocation

Allocation is the process of planning the use of spare feeder capacity (i.e., spare feeder cable pairs) along an entire feeder route. The purpose of allocation is to make the best, or most economical, use of both existing and future feeder facilities, and to enable the engineer to plan splicing configurations which will avoid complexity and reduce as much as possible the need for costly future rearrangements.

The basic concept of allocation is to apportion the available spare pairs along the entire route, according to forecast growth rates. Doing so may economically defer feeder relief which leads to higher feeder utilization (the fraction of the feeder actually in use).

1.2 Dual view of the feeder

Consider the schematic of a feeder route in Fig. 1. The feeder route is like an expressway for cable pairs which extends out from the central office. Lateral cables are spliced to the feeder cable at various points along the route and these lateral cables are joined to distribution cables which are connected to subscribers.

The distribution plant along the route has been organized geographically into regions called *allocation areas.* Allocation areas are used for feeder planning and administration; they provide the basis for engineering manageable portions of the route, rather than trying to look at the entire route all at once. Conversely, the use of allocation areas encourages the feeder engineer, when solving local feeder/distribution problems, to consider the requirements of the rest of the route. Allocation areas are also used for monitoring facility problems, described elsewhere in this issue.[2]

The feeder itself has traditionally been broken up into segments called feeder *sections.* The feeder sections, which include all of the cable and conduit in a cross section between two points (usually important manholes) along the route, are used for relief planning and design.[3] The reason for this is that new relief cables are economically sized and placed in one or more sections at a time and then are spliced together with other cables to provide pairs from the central office to the various areas served by the network.

The other way of looking at the feeder facilities is the *pair group.* A pair group is a "bundle" of feeder pairs (not necessarily in the same cable sheath) which extend outward from the central office to a specific portion of the distribution plant, a single allocation area. Thus there is a one-

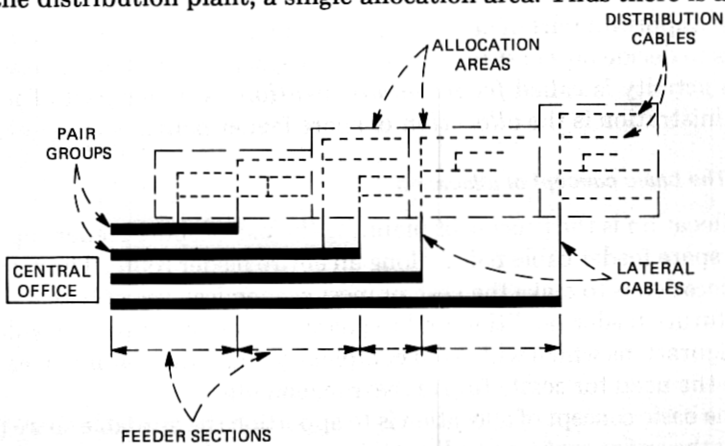

Fig. 1—Feeder route schematic.

to-one relationship between pair groups and allocation areas; the feeder *complements* (groups of pairs, usually in multiples of 25 pairs) in a particular pair group serve the allocation area associated with that pair group. Pair groups, then, are used primarily for feeder administration. Notice that a single section may have several pair groups passing through it and each pair group generally passes through several sections.

II. A SIMPLE APPROACH TO ALLOCATION

How should spare feeder complements be apportioned (or allocated) to the allocation areas along the route? This depends on the *lifetimes* of both the feeder sections and pair groups. "Lifetime" as used here does not mean the time until the feeder plant is replaced, but rather the time until additional plant must be provided. This is also known as fill time, time to exhaustion, or time to relief.

A section's lifetime is based on the number of spare pairs in the cross section and the growth into and through the section. A pair group's lifetime, however, is based on the number of spare pairs and the growth in that pair group only. Thus the feeder section lifetimes are usually different from the pair group lifetimes.

2.1 An illustrative example

Consider the example* shown schematically in Fig. 2. Here there are three pair groups feeding three allocation areas: AA1, AA2, and AA3. There are also three feeder sections: 2101, 2102, and 2103. Notice that

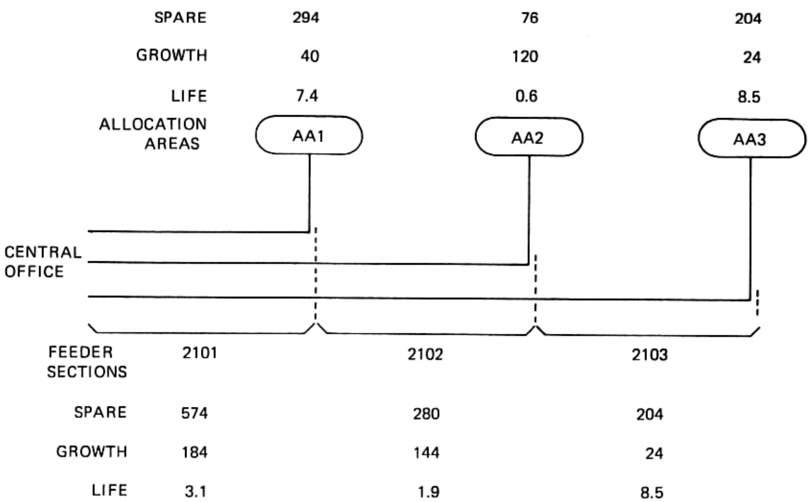

Fig. 2—An example route showing allocation area and feeder section lifetimes.

* All of the feeder routes in this paper are simplified in order to make clearer examples. Actual feeder routes frequently are composed of 20 or more sections.

the *ideal lifetime* (the number of spare pairs divided by the growth rate) of allocation area AA2 (and its associated pair group) is only 0.6 year. The section 2102 ideal lifetime, however, is 1.9 years. Farther out in the route, both allocation area AA3 and section 2103 have large ideal lifetimes of 8.5 years. It would appear that allocation area AA3 might be able to share some of its spare pairs with allocation area AA2. The only alternative to a rearrangement such as this is to place a relief cable in section 2102 within 0.6 year.

Notice that the ideal lifetime of section 2102 (1.9 years) is the shortest section lifetime in the route. This section is referred to as the *critical section*. It is this section that will exhaust first if growth occurs as forecast and if all possible rearrangements are made to defer relief. That is, if all spare pairs can be used, then the time to exhaustion for section 2102 could be up to 1.9 years. But this section will exhaust sooner given the way the spare pairs are presently distributed to the allocation areas. Relief can be deferred in the critical section as much as possible by apportioning the spare pairs in the critical section to the various pair groups running through it such that they all have identical ideal lifetimes.

Since the ideal lifetime of AA3 is greater than the critical section's lifetime, then a *surplus* of spare pairs is associated with or allocated to AA3. In the same way, a *deficit* of spare pairs is allocated to AA2. *Load balancing* is the process of reallocating spare pairs from areas with a surplus to those with a deficit. Load balancing often helps to identify necessary physical rearrangements in the feeder network.

2.2 A procedure for load balancing

A load balance worksheet has been developed for use by outside plant engineers. Figure 3 illustrates its application to the example route of Fig. 2. The entries on lines A through H correspond to those in Fig. 2, discussed in the previous section.

On line J is the *ideal allocation lifetime*. This is the lifetime each allocation area (pair group) should have in order to theoretically balance the route and defer the next relief for as long as possible. The ideal allocation lifetime is the same as the most critical section's lifetime for those sections beyond and including the critical section. Thus, both areas AA2 and AA3 have ideal allocation lifetimes of 1.9 years. For the allocation areas fed from sections between the most critical section and the next most critical section, the ideal allocation lifetime is the lifetime of the second critical section.

The *theoretical spare pair allocation* (line K) for each allocation area is the allocation area's growth rate multiplied by its ideal allocation lifetime. Note the entries in Fig. 3.

Finally, each allocation area's spare pair surplus/deficit is the difference between the number of spare pairs it currently has and its theo-

LOAD BALANCE WORKSHEET

DATE ____________

WIRE CENTER ____________

ROUTE ____________

A. SECTION NUMBER	2101	2102	2103							
B. ALLOCATION AREA	AA1	AA2	AA3							
C. SPARE PAIRS IN ALLOCATION AREA	294	76	204							
D. ACCUMULATED SPARE PAIRS IN CROSS SEC. (ACCUM. C FROM FAR END)	574	280	204							
E. GROWTH IN ALLOCATION AREA	40	120	24							
F. ACCUMULATED GROWTH IN CROSS SEC (ACCUM. E FROM FAR END)	184	144	24							
G. IDEAL CROSS SECTIONAL LIFETIME (D÷F)	3.1	1.9	8.5							
H. CRITICAL SECTION(S)	#2	#1								
J. IDEAL ALLOCATION LIFETIME	3.1	1.9	1.9							
K. THEORETICAL SPARE PAIR ALLOCATION (E x J)	124	228	46							
L. SPARE PAIR SURPLUS/ DEFICIT (C–K)	170	−152	158							

Fig. 3—Load balance worksheet for example route in Fig. 2.

retical allocation. For this example the entries are 170 pairs for allocation area AA1, −152 for AA2, and 158 for AA3. Every deficit must be eliminated by using some surplus if the ideal allocation lifetimes of line J are to be achieved. Of course, it is not possible to use just *any* surplus; for example, here the 170 pairs in AA1 do not extend far enough out on the route to be used in AA2 (see Fig. 2).

Since the section and allocation area data are entered on the load balance worksheet from the central office outward, the table implicitly contains information on the configuration of the route. Therefore the load balance worksheet not only indicates spare pair surpluses and deficits, but it also shows how deficits can be resolved. Since AA3 is farther out in the route than AA2, then AA3's pair group passes through section 2102 and a transfer of surplus spare pairs could be made.

Why can't the 170 pair surplus in AA1 be used in AA2? Unless there are usable dead pairs in section 2102, there is no way to extend AA1's surplus spares without placing relief. How should the 170 pair surplus in AA1 be used? Ideally these pairs should be "held back" or reserved for use upon relief of the critical section. Unless section 2101 will be relieved along with section 2102, these pairs should not be allocated to allocation area AA1.

III. A MORE SOPHISTICATED APPROACH

An attractive feature of the load balance worksheet just described is its simplicity and consequential ease of preparation. Due to this simplicity, however, there are several important characteristics of the feeder which are not considered by the manual load balance worksheet: dead pairs, unallocated pairs, objective fills, and varying growth rates.

3.1 Improvements in the representation of the feeder route

First there are *dead pairs,* which are cable pairs that do not extend all the way back to the central office. Thus they cannot be used to provide service until they are spliced to cables which do reach the central office. Dead pairs are therefore a capital investment which is not making a return. Outside plant engineers need to know when and where it is practical and economical to use dead pairs, since this can defer feeder relief.

Unlike dead pairs, *unallocated pairs* are connected all the way back to the central office. They are not, however, associated with any allocation area and frequently are not even spliced to any lateral cable. The engineer should be aware of the number and locations of unallocated complements. In some cases, too many pairs may remain unallocated; relief can be deferred if the unallocated complements are allocated and then committed to allocation areas approaching exhaustion. On the other hand, there may be too few unallocated pairs in a section adjacent to a critical section on the central office side. Unless this section is to be relieved simultaneously with the critical section, then a certain number of its pairs should be reserved for future use. Upon relief of the critical section, these unallocated pairs are spliced to the new cable, thus providing central office pairs through and beyond the former critical section.

Next consider spare lifetimes. The ideal lifetime calculated using the load balance worksheet is simply the number of spare pairs divided by the growth rate. A more realistic lifetime, or *time to relief,* may be determined if an *objective* or maximum *fill* is used in the calculation. This fill is the percentage of cable pairs at a particular point which are practical and economical to use at the time just prior to relief. A typical value is 85 percent. (See Ref. 4, for a discussion of optimal objective fills, i.e., the economic fill at relief.) The time to relief, then, is

$$\begin{matrix}\text{Time}\\ \text{to}\\ \text{relief}\end{matrix} = \frac{\begin{pmatrix}\text{objective}\\ \text{fill}\end{pmatrix} \times \begin{pmatrix}\text{number of}\\ \text{pairs available}\end{pmatrix} - \begin{pmatrix}\text{number of}\\ \text{pairs assigned}\end{pmatrix}}{\text{growth rate}}$$

where the number of pairs available includes all central office pairs, i.e., spare, defective, and assigned (working) pairs.

Finally, the load balance worksheet uses a single growth rate for each allocation area. This is frequently a reasonable assumption, but sometimes growth rates change considerably over time. For example, as a developing area saturates with subscribers, the growth slows. On the other hand, if the nature of an area is changing from single-family to multifamily residential, growth will increase. Or there may be a single "impact" growth due to a new office building. Thus a more realistic approach is to make use of some sort of growth function.

An experimental computer program has been developed which takes into account these characteristics of a route. In order to show how this technique is more flexible than the manual load balance worksheet, an example is presented next. A generalized mathematical description is given in the appendix.

3.2 Computerized load balancing

As an illustration of the computerized table, consider the example route in Fig. 4. Here there are five feeder sections and six allocation areas. In addition to the pair groups feeding the allocation areas, there is an unallocated 50 pair complement which extends out to section 2104. There are also 100 dead pairs in section 2105.

Table I summarizes both the fill count and growth forecast for the allocation areas. Note that in most cases the growth rate is declining. For example, the forecast for allocation area AA4 is 100 pairs growth for the first year (12/77 to 12/78), 70 pairs the next year, 50 pairs per year for the next two years, and then 40 pairs per year after 12/81.

Figure 5 shows output from the computer program for this route. By examining this output the reader should gain a further understanding of the allocation process.

The leftmost column lists the allocation area (AA) and feeder section (FS) designations, along with abbreviations for unallocated (UNAL) and dead (DD) pairs. Feeder sections along with their associated allocation area(s), etc., are separated by horizontal lines, with the most distant

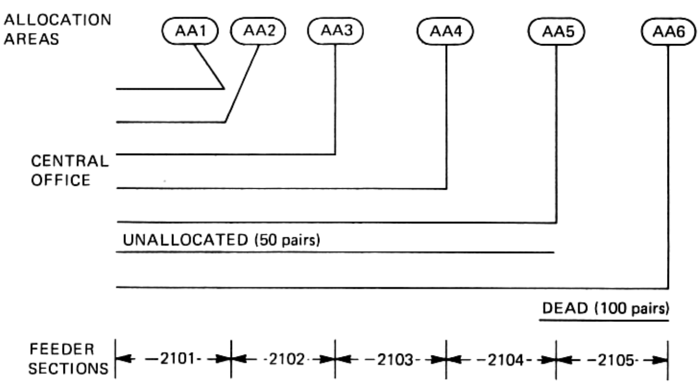

Fig. 4—Schematic of second example route.

Table I — Fill count and forecast for route in Fig. 4

Allocation area	Number of pairs available	Number of pairs assigned*	Objective fill	Growth forecast 12/77 to 12/78	Growth forecast 12/78 to 12/79	Growth forecast 12/79 to 12/81	Growth forecast Annual rate after 12/81
AA1	700	462	90	25	25	50	25
AA2	1500	1243	90	40	40	60	25
AA3	2450	1742	85	70	80	140	40
AA4	1850	1541	85	100	70	100	40
AA5	1000	771	85	55	40	80	30
AA6	400	228	90	30	20	40	10

* Fill count of 12/77.

section at the top of the table, and the section closest to the central office at the bottom.

Note, for example, section 2104, the second section from the top of the table. This section feeds allocation area AA5.

The second column is PAIRS AVAIL, the number of pairs available. For an allocation area this refers to the total number of central office pairs

LOOP FEEDER ANALYSIS TABLE

C.O. – WHIPPANY
ROUTE – 2A
DATE – 12/21/77

FACILITY COUNT (BASE) DATE: 12/77

AA FAS	PAIRS AVAIL	PAIRS ASGND	PRES FILL	OBJ FILL @RLF	GROW MARGN	—GROWTH FORECAST— PER1 1278	PER2 1279	PER3 1281	ANN RATE	YRS TO RLF	EQUALIZED YRS	PAIRS	SUR DEF
AA6	400	228	57.0	90	132	30	20	40	10	8.2	1.8	304	96
UNAL	0			90	0						0.0	0	0
DD/BT	100	'		90	90								
2105	500	228	45.6	90	222	30	20	40	10	17.2			
AA5	1000	771	77.1	85	79	55	40	80	30	1.6	1.8	1008	−8
UNAL	50			86	43						0.0	0	50
2104	1450	999	68.9	86	254	85	60	120	40	3.8			
AA4	1850	1541	83.3	85	31	100	70	100	40	0.3	1.8	1994	−144
UNAL	0			86	0						0.0	0	0
2103	3300	2540	77.0	86	285	185	130	220	80	1.8			
*** CRITICAL SECTION #1 – EXHAUST DATE 979 ***													
AA3	2450	1742	71.1	85	340	70	80	140	40	5.2	2.9	2299	151
UNAL	0			85	0						1.1	149	−149
2102	5750	4282	74.5	85	625	255	210	360	120	2.9			
*** CRITICAL SECTION #2 – EXHAUST DATE 1080 ***													
AA1	700	462	66.0	90	168	25	25	50	25	6.7	3.3	605	95
AA2	1500	1243	82.9	90	107	40	40	60	25	2.9	3.3	1513	−13
UNAL	0			87	0						0.4	84	−84
2101	7950	5987	75.3	87	900	320	275	470	170	3.3			
*** CRITICAL SECTION #3 – EXHAUST DATE 381 ***													

C.O.

Fig. 5—Computer output for example route in Fig. 4.

associated with the allocation area. This is, of course, the number of central office pairs in the allocation area's feeder pair group. (There are 1000 pairs available in AA5.) For unallocated pairs, the entry indicates the number of unallocated pairs ending in the section. Unallocated pairs passing through the section but ending farther out in the route are not included in this value. The number of dead pairs are included in each section in which they appear. There is an unallocated 50 pair complement ending in section 2104, but there are no dead pairs. There are 100 dead pairs in section 2105.

The number of pairs available in the feeder section is the total number of pairs in the cross section. (There are 1450 in section 2104.) This includes all of the pairs in pair groups feeding the section's allocation areas and the allocation areas beyond. All unallocated pairs in the section and beyond it, as well as dead pairs within the section, are accumulated also.

PAIRS ASGND refers to the number of pairs assigned. These are mostly working pairs but also include some pairs associated with specific subscriber locations but which are temporarily idle. These data are entered by allocation area and are accumulated by feeder section. (Neither dead nor unallocated pairs can be assigned.)

The next column (PRES FILL) provides the present fill for allocation areas and feeder sections. This is simply

$$\begin{matrix}\text{Present}\\\text{fill}\end{matrix} = \frac{\begin{pmatrix}\text{current number of}\\\text{assigned pairs}\end{pmatrix}}{\begin{pmatrix}\text{number of pairs}\\\text{available}\end{pmatrix}}$$

When calculated for an allocation area, it refers only to the pairs in the allocation area's pair group. For feeder sections, however, the entire cross section is considered: all pair groups and unallocated pairs passing through, as well as dead pairs in the section.

The objective fill at relief (OBJ FILL@RLF) was described earlier. This fill level is determined by the engineer for the allocation area pair groups.[4] Then, using these values, the program computes the objective fills for feeder sections and unallocated and dead complements.

The growth margin (GROW MARGN) is the number of spare pairs which can be used to provide service and bring the fill to the objective fill. It is

$$\begin{matrix}\text{Growth}\\\text{margin}\end{matrix} = \begin{pmatrix}\text{objective}\\\text{fill}\end{pmatrix} \times \begin{pmatrix}\text{number of}\\\text{pairs available}\end{pmatrix} - \begin{pmatrix}\text{number of}\\\text{pairs assigned}\end{pmatrix}$$

The growth forecasts are next and for this program there may be from one to four columns. The simplest form is a single annual growth rate for each allocation area. Whenever the growth forecast reflects changes in the growth rate over time, as in this case, the more flexible form of input may be used. The engineer then specifies up to three periods of time (PER1, etc.) and the growth that is expected to occur during each period within each allocation area. An annual growth rate (ANN RATE) is required for all time after the growth periods.

For example, consider again allocation area AA4. The first growth period extends from the facility count date of 12/77 to 12/78, one year in length. (Any time period is acceptable.) During this first period, the forecast growth is 100 pairs. During the next period, one year in length, the forecast growth decreases to 70 pairs. Then over the next two years the forecast growth is 80 pairs. (This is a smaller annual rate, of course, than the previous period.) Finally, after 12/81, the forecast annual growth rate for allocation area AA4 is 40 pairs per year.

The growth forecasts for a feeder section are an accumulation of the forecasts for all allocation areas fed by and beyond the section.

Using the growth margins and growth forecasts, the program computes the time (in years) to relief (YRS TO RLF), which is actually the time to reach the objective fill. This is determined for both allocation areas and feeder sections. As discussed earlier, the times to relief for a particular section and the allocation areas it feeds are frequently different. This may indicate that there is a better way to allocate existing spare pairs.

It is in the next three columns, however, that the ideal allocation becomes evident. EQUALIZED YRS and PAIRS represent the load balanced times to relief and number of pairs available for each allocation area and for the unallocated pairs in each section. The load balanced times to relief are based on the critical sections' times to exhaustion, as discussed earlier. Using these values, the equalized number of pairs available, i.e. the ideal allocation for theoretical load balance, may be determined. It is

$$\begin{matrix}\text{Equalized number} \\ \text{of pairs available}\end{matrix} = \frac{\begin{pmatrix}\text{number of} \\ \text{pairs assigned}\end{pmatrix} + \begin{pmatrix}\text{growth over equalized} \\ \text{time to relief}\end{pmatrix}}{\text{objective fill at relief}}$$

In the last column (SUR, DEF), the allocation surplus or deficit is printed. It is

$$\begin{matrix}\text{Allocation} \\ \text{surplus/deficit}\end{matrix} = \begin{pmatrix}\text{number of} \\ \text{pairs available}\end{pmatrix} - \begin{pmatrix}\text{equalized number} \\ \text{of pairs available}\end{pmatrix}$$

A negative value (deficit) for an allocation area indicates that there are too few pairs allocated to it. This allocation area will require relief sooner than if the ideal allocation were made to it. A positive value (surplus) for an allocation area indicates that too many spare pairs are available for the area. Its time to relief will be greater than its critical section's, and therefore, some other allocation area will have a deficit.

For example, allocation area AA6 in section 2105 has an allocation surplus of 96 pairs. The time to relief for this allocation area is 8.2 years. Since it is beyond the most critical section (2103), its equalized time to relief is 1.8 years. Therefore, by allocating more than 1.8 years of spare pairs to this allocation area, other allocation areas are shortchanged.

Consider AA4 fed from section 2103. This section is the most critical section and its time to relief is 1.8 years. Allocation area AA4, however, has only 0.3 of a year to go. The deficit of −144 pairs indicates that 144 additional pairs should be allocated to this allocation area so that it might last as long as its critical section.

In each feeder section, the allocation surplus/deficit for unallocated pairs is also determined. The entire unallocated 50 pair complement ending in section 2104 is surplus. This complement should be allocated to an allocation area with a deficit.

Deficits of unallocated pairs may occur in the sections which are adjacent (on the central office side) to a critical section. Such is the case for section 2102 which precedes the most critical section 2103. Ideally there should be 149 unallocated pairs in section 2102. Upon relief of the critical section these unallocated pairs would be spliced to the new cable and then allocated to allocation areas farther out in the route.

Now, by taking a look at the route as a whole, what potential changes in the present allocation can the engineer identify?

The most critical deficit is in AA4. Not only is the deficit large (−144) but the time to relief for this allocation area is extremely short (0.3 year). It should be possible (though not necessarily economical) to eliminate this deficit using allocation surpluses in sections beyond 2103, i.e., those sections above it in the table. Since there is a surplus of 96 spare pairs in AA6 in section 2105, there may be a spare 100 pair complement which could be transferred from AA6 to AA4. Furthermore, the 50 unallocated pairs ending in section 2104 could be allocated to AA4 since they pass through section 2103.

Rather than using the 150 pairs just discussed, could 150 pairs be obtained from AA3 (151 pair surplus) in section 2102? Since this section is closer to the central office, the pairs in AA3's pair group do not extend far enough out into the route to be allocated to AA4. If, however, there were 150 dead pairs in section 2103, then the central office pairs might be extended by splicing them to the dead complements.

The surplus in AA3 should, however, be used to satisfy the deficit of unallocated pairs (−149) in section 2102. A 150 pair spare complement should be reserved for use upon the relief of section 2103.

Similarly, perhaps 100 pairs of AA1's allocation should be held back to satisfy the unallocated deficit in section 2101. This is less important, though, because the relief of critical section 2 is considerably later than for critical section 1. Furthermore the exhaust dates of critical sections 2 and 3 indicate that they may be relieved simultaneously.

3.3 Implementation of the ideal allocation

The determination of an ideal load balance does not complete the allocation process. Next the outside plant engineer must study practical and economical rearrangements suggested by the idealized feeder configuration.

Since it is probably not economical to perfectly balance an entire route, the engineer generally begins with the most serious shortage. This first involves a detailed examination of various cable records, such as underground schematics, to determine precisely what type of cable splicing work is required to move spare complements from an allocation area with a surplus to another with a deficit. The engineer determines how many pairs are to be rearranged, which particular spare complements are to be used, which splice closures must be opened, and whether a stub (short length of cable) will have to be placed in the manhole. The objective is to minimize cost, of course, but there is also a consideration of factors such as present manhole congestion, which is difficult to quantify. Several alternatives may be examined.

After practical rearrangements are identified, an economic evaluation should be made. This is a comparison of the rearrangement costs (cable splicing labor and materials) with the savings of deferred relief (due to delayed capital expenditure). Only if the rearrangement proves economical will the engineer prepare work orders for its implementation.

IV. A FEEDER ADMINISTRATION PACKAGE

The allocation process is part of a feeder administration package now being used by outside plant engineers in the operating telephone companies. This package includes a user-oriented manual containing guidelines, procedures, engineering tools, and documentation to aid these engineers in their jobs. In addition to these materials, there is also a comprehensive training course.

V. CONCLUSIONS

The allocation process may be applied to apportion existing spare pairs in a feeder route. This can help to identify rearrangements of the network which will defer relief, leading to higher utilization and reduced capital costs. Similarly, these techniques are useful in planning the allocation of an upcoming relief cable.

Whether existing or relief pairs are involved, allocation establishes a plan for the use of spare facilities over the entire route. Then fewer rearrangements will be made and those which are made will be cost effective. Expedient rearrangements, often involving working pairs, will be required less frequently. Thus, not only will operating costs be reduced, but further complication and congestion of the feeder plant will be avoided.

VI. ACKNOWLEDGMENTS

The recent developments in feeder administration reflect the work of a number of outside plant engineers throughout the Bell System. Many of their ideas were brought together by the Feeder Task Force of the Bell System Design and Utilization Committee.

Specific contributions to the concepts described in this paper were made by W. N. Bell (Bell Laboratories), G. J. Dean and D. W. Post (New Jersey Bell), R. A. Wallfred (Northwestern Bell), and J. G. Funk (Western Electric).

APPENDIX

A Mathematical Description of the Load Balancing Process

This appendix mathematically describes the load balancing process for a generalized feeder route. The computer program described in Section 3.2 is based on this model. The route is a single linear path with allocation areas, dead pairs and unallocated pairs. Allocation area growth forecasts may vary deterministically over time. Prespecified objective fills are permitted. Multigauge and multipath routes are not considered here.

A.1 Define route configuration and growth forecast

Let N_s be the number of feeder sections in the route. Let i be the index for the ith feeder section, with $i = 1$ representing that section most distant from the central office, and $i = N_s$ the section closest to the central office.

For each section, let N_a^i = number of allocation areas fed from the ith section (as opposed to through it).

Let j be the index for the jth allocation area fed by the ith feeder section such that $j = 1$ to N_a^i, if $N_a^i \neq 0$. (If $N_a^i = 0$, then no allocation areas are fed by the ith section.)

For the jth allocation area fed from the ith feeder section (i.e., for each allocation area), the following are given:

$$P_{av}^{ij} = \text{number of pairs available}$$

$$P_{as}^{ij} = \text{number of assigned pairs}$$

A growth forecast for each allocation area is also given. As shown earlier, a simple annual growth rate may be used, with

$$G_r^{ij} = \text{annual growth rate}$$

This is frequently a reasonable assumption, but sometimes growth rates change considerably over time. A piecewise linear growth forecast is defined by

$$n = \text{number of growth time periods } (0,1,2,\ldots)$$

If $n > 0$, then the following are given:

$$t_k = \text{length of } k\text{th time period}$$

$$G_k^{ij} = \text{growth during } k\text{th time period, for } k = 1 \text{ to } n$$

At the end of the last time period, the annual growth rate G_r^{ij} is assumed to take effect. (If $n = 0$, then only the annual growth rate is given.) In order to determine a realistic lifetime, the objective fill at relief for each allocation area, F^{ij}, is also given.

The items above are given for each allocation area. The following items must also be given for each (the ith) feeder section,

$$P_d^i = \text{number of dead pairs}$$

$$P_u^i = \text{number of unallocated pairs ending in this section}$$

A.2 Accumulate section data from allocation area data

The growth forecasts for a feeder section are an accumulation of the forecasts for all allocation areas fed by and beyond the section.

$$G_r^i = \sum_{m=1}^{i} \left[\sum_{j=1}^{N_a^m} G_r^{mj} \right], \quad \text{for} \quad i = 1, \ldots, N_s$$

and, if $n > 0$,

$$G_k^i = \sum_{m=1}^{i} \left[\sum_{j=1}^{N_a^m} G_k^{mj} \right], \quad \begin{array}{l} \text{for} \quad i = 1, \ldots, N_s \\ \text{and } k = 1, \ldots, n \end{array}$$

The number of pairs available in the feeder section is the total number of pairs in the cross section. This includes all of the pairs in pair groups feeding the section's allocation areas and the allocation areas beyond. All unallocated pairs in the section and beyond it, as well as dead pairs within the section, are also accumulated.

$$P_{av}^i = \sum_{m=1}^{i} \left[\sum_{j=1}^{N_a^m} P_{av}^{mj} + P_u^m \right] + P_d^i, \quad \begin{array}{l} \text{for all } i \\ (i = 1, \ldots, N_s) \end{array}$$

Similarly, the number of assigned pairs in the section is

$$P_{as}^i = \sum_{m=1}^{i} \left[\sum_{j=1}^{N_a^m} P_{as}^{mj} \right], \quad \text{for all } i$$

A.3 Calculate time to relief

The *growth margin* is a generalization of spare pairs which includes the objective fill,

$$\begin{array}{c}\text{Growth} \\ \text{margin}\end{array} = \begin{pmatrix}\text{objective} \\ \text{fill}\end{pmatrix} \times \begin{pmatrix}\text{number of} \\ \text{pairs available}\end{pmatrix} - \begin{pmatrix}\text{number of} \\ \text{pairs assigned}\end{pmatrix}$$

For each allocation area the growth margin is

$$M^{ij} = F^{ij} P_{av}^{ij} - P_{as}^{ij} \quad \text{for all } i,j$$

Next, similar calculations are made on a section basis. The objective section fills are derived from the objective fills of all allocation areas fed by and beyond the section and the objective fills of all unallocated pairs beyond the section. For the section most distant from the central office ($i = 1$),

$$F^1 = \frac{\sum_{j=1}^{N_a^1} F^{1j} P_{av}^{1j}}{\sum_{j=1}^{N_a^1} P_{av}^{1j}}$$

and for $i > 1$,

$$F^i = \frac{\sum_{k=1}^{i} \sum_{j=1}^{N_a^k} F^{kj} P_{av}^{kj} + \sum_{k=1}^{i-1} F^k P_u^k}{\sum_{k=1}^{i} \sum_{j=1}^{N_a^k} P_{av}^{kj} + \sum_{k=1}^{i-1} P_u^k}$$

where objective fills for the unallocated pairs ending in the ith section and for the dead pairs in that section are set equal to that section's objective fill. The growth margins for unallocated and dead pairs, respectively, are

$$M_u^i = F^i P_u^i$$

$$M_d^i = F^i P_d^i, \quad \text{for all } i$$

Then, the grow margin for the ith section is

$$M^i = \sum_{k=1}^{i} \left[\sum_{j=1}^{N_a^k} M^{kj} + M_u^k \right] + M_d^i, \quad \text{for all } i$$

Now the *time to relief* for each feeder section and each allocation area may be calculated, using the growth margins and the growth forecasts, i.e.,

$$\text{Time to relief} = \frac{\text{growth margin}}{\text{growth rate}}$$

Since the growth forecast is a piecewise linear function, so too is the time to relief. For feeder sections when the growth margin is less than the forecast growth for the first time period, i.e.,

$$M^i < G_1^i$$

the time to section relief is

$$T^i = \frac{M^i}{G_1^i / t_1}$$

Similarly, for allocation areas when

$$M^{ij} < G_1^{ij}$$

then

$$T^{ij} = \frac{M^{ij}}{G_1^{ij}/t_1}$$

For feeder sections whenever

$$G_1^i \le M^i \le \sum_{k=1}^{n} G_k^i$$

then

$$T^i = \frac{\left(M^i - \sum_{k=1}^{m-1} G_k^i\right)}{G_m^i/t_m} + \sum_{k=1}^{m-1} t_k$$

where m satisfies

$$\sum_{k=1}^{m-1} G_k^i \le M^i \le \sum_{k=1}^{m} G_k^i, \quad (1 < m \le n)$$

Similarly, for allocation areas, whenever

$$G_1^{ij} \le M^{ij} \le \sum_{k=1}^{n} G_k^{ij}$$

then

$$T^{ij} = \frac{\left(M^{ij} - \sum_{k=1}^{m-1} G_k^{ij}\right)}{G_m^{ij}/t_m} + \sum_{k=1}^{m-1} t_k$$

where m satisfies

$$\sum_{k=1}^{m-1} G_k^{ij} \le M^{ij} \le \sum_{k=1}^{m} G_k^{ij}, \quad (1 < m \le n)$$

Finally, for feeder sections whenever

$$M^i > \sum_{k=1}^{n} G_k^i$$

then the time to relief is

$$T^i = \frac{\left(M^i - \sum_{k=1}^{n} G_k^i\right)}{G_r^i} + \sum_{k=1}^{n} t_k$$

Similarly for allocation areas whenever

$$M^{ij} > \sum_{k=1}^{n} G_k^{ij}$$

then

$$T^{ij} = \frac{\left(M^{ij} - \sum_{k=1}^{n} G_k^{ij}\right)}{G_r^{ij}} + \sum_{k=1}^{n} t_k$$

As an example, consider the case where

$$n = 3$$

and

$$G_1 + G_2 < M < G_1 + G_2 + G_3$$

(The feeder section and allocation area superscripts are deleted for simplicity in this example.) The time to relief, T, shown graphically in Fig. 6, is

$$T = \frac{M - (G_1 + G_2)}{G_3/t_3} + (t_1 + t_2)$$

A.4 Calculate the ideal allocation adjustment (surplus or deficit)

Next the feeder section times to relief, T^i, are used to identify the *critical sections.* The first (or most) critical section is that section with the shortest time to relief. Let I_i be the feeder section index of the ith critical section. Then

$$I_1 = j \text{ such that } T^j = \min_{1 \le k \le N_s} (T^k)$$

The second critical section is the section between the first critical section and the central office with the least time to relief. Thus

$$I_2 = j \text{ such that } T^j = \min_{I_1 < k \le N_s} (T^k)$$

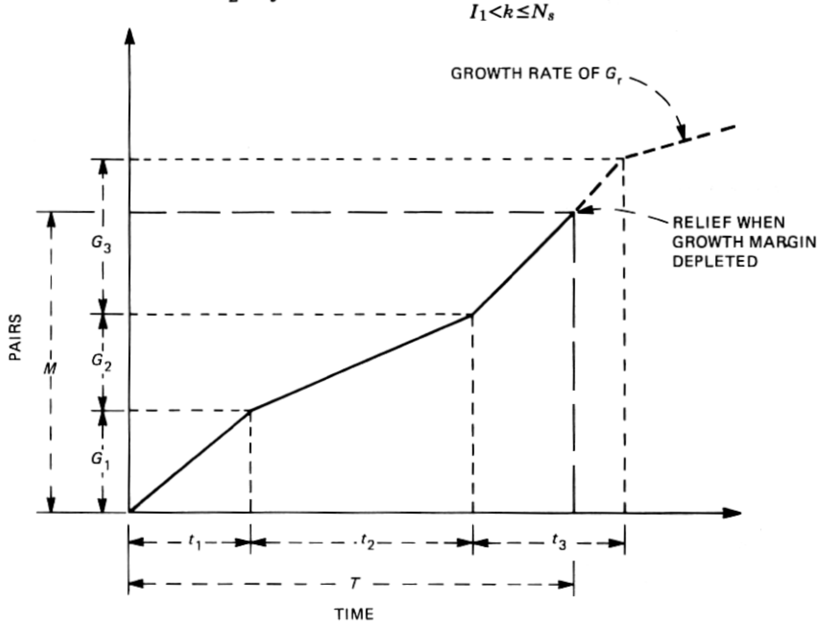

Fig. 6—Time to relief (T) for $n = 3$ and $G_1 + G_2 < M < G_1 + G_2 + G_3$.

The general form for this equation is

$$I_m = j \text{ such that } T^j = \min_{I_{m-1} < k \le N_s} (T^k)$$

for

$$m > 1 \text{ until } I_m = N_s$$

The number of critical sections, N_c, is the value of m when

$$I_m = N_s$$

Ideally, the relief of the route's most critical section can be deferred for the greatest time if all allocation areas fed by and beyond this section have the same time to relief. The *equalized time to relief* for the allocation areas (and the sections) fed by and beyond the first critical section is

$$E_T^{ij} = E_T^i = T^{I_1}, \quad \text{for } i = 1 \text{ to } I_1 \text{ and } j = 1 \text{ to } N_a^i$$

For the kth critical section ($k > 1$)

$$E_T^{ij} = E_T^i = T^{I_k}, \quad \text{for } k = 2 \text{ to } N_c,\ i = I_{k-1} + 1 \text{ to } I_k, \text{ and } j = 1 \text{ to } N_a^i$$

The *equalized growth margin* is the number of pairs each allocation area will grow over its equalized time to relief. It is used to determine how many pairs should be available to an allocation area so that it will not require relief until its equalized time to relief.

The function for the growth over time is derived from the function for the time to relief for allocation areas. For

$$E_T^{ij} < t_1$$

the equalized growth margin over this time is

$$E_M^{ij} = \frac{G_1^{ij}}{t_1} E_T^{ij}$$

For the range

$$t_1 \le E_T^{ij} \le \sum_{k=1}^{n} t_k$$

then

$$E_M^{ij} = \frac{G_m^{ij}}{t_m} \left(E_T^{ij} - \sum_{k=1}^{m-1} t_k \right) + \sum_{k=1}^{m-1} G_k^{ij},$$

where m satisfies

$$\sum_{k=1}^{m-1} t_k \le E_T^{ij} \le \sum_{k=1}^{m} t_k, \quad (1 < m \le n)$$

Finally for

$$E_T^{ij} > \sum_{k=1}^{n} t_k$$

then

$$E_M^{ij} = G_r^{ij}\left(E_T^{ij} - \sum_{k=1}^{n} t_k\right) + \sum_{k=1}^{n} G_k^{ij}$$

The *equalized pairs available* for each allocation area is simply the number of available pairs each allocation area would have if it were perfectly load balanced. It is

$$\begin{matrix}\text{Equalized}\\ \text{pairs available}\end{matrix} = \frac{\begin{pmatrix}\text{number of}\\ \text{pairs assigned}\end{pmatrix} + \begin{pmatrix}\text{equalized}\\ \text{growth margin}\end{pmatrix}}{\text{objective fill at relief}}$$

Thus,

$$E_{av}^{ij} = \frac{P_{as}^{ij} + E_M^{ij}}{F^{ij}}$$

Finally, the *ideal allocation adjustment* (surplus or deficit) for allocation areas is the difference between actual and equalized pairs available,

$$\begin{matrix}\text{Ideal allocation}\\ \text{adjustment}\end{matrix} = \begin{pmatrix}\text{number of}\\ \text{pairs available}\end{pmatrix} - \begin{pmatrix}\text{equalized number}\\ \text{of pairs available}\end{pmatrix}$$

or

$$A^{ij} = P_{av}^{ij} - E_{av}^{ij}$$

Deficits of unallocated pairs may occur in the sections which are adjacent (on the central office side) to a critical section. Upon relief of the critical section these unallocated pairs would be spliced to the new cable and then allocated to allocation areas farther out in the route.

It is also possible for there to be too many unallocated pairs in a particular section. Thus the unallocated pair allocation adjustment must be determined for every feeder section.

First the equalized growth margin for unallocated pairs must be determined in each section. This is the growth that will occur between the equalized times to relief between adjacent critical sections. This time difference between any adjacent sections i and $i-1$ is

$$E_T^i - E_T^{i-1}, \quad \text{for} \quad i = 2 \text{ to } N_s$$

For

$$E_T^i - E_T^{i-1} > 0, \quad (i > 1, \text{implicitly})$$

the equalized growth margin for unallocated pairs is the growth occurring from time E_T^{i-1} to E_T^i.

Two evaluations must be made of the function for the equalized growth margin in a section, E_M^i, over time. (This function is similar to the function for allocation area equalized growth, above.) The function is given below with the time E_T^i (rather than E_T^{i-1}) as the independent variable. For $E_T^i < t_1$,

$$E_M^i = \frac{G_1^i}{t_1} E_T^i$$

For

$$t_1 \leq E_T^i \leq \sum_{k=1}^{n} t_k$$

then

$$E_M^i = \frac{G_m^i}{t_m} \left(E_T^i - \sum_{k=1}^{m-1} t_k \right) + \sum_{k=1}^{m-1} G_k^i$$

where m satisfies

$$\sum_{k=1}^{m-1} t_k \leq E_T^i \leq \sum_{k=1}^{m} t_k, \quad (1 < m \leq n)$$

Finally, for

$$E_T^i > \sum_{k=1}^{n} t_k$$

then

$$E_M^i = G_r^i \left(E_T^i - \sum_{k=1}^{n} t_k \right) + \sum_{k=1}^{n} G_k^i$$

Then the equalized growth margin for the unallocated pairs in a section is

$$E_{M_u}^i = E_M^i - E_M^{i-1}, \quad \text{for} \quad i = 2, \ldots, N_s$$

if the section farther from the central office has a shorter time to relief, i.e.,

$$E_T^i - E_T^{i-1} > 0$$

If, however, the section closer to the central office has the shorter lifetime, i.e.,

$$E_T^i - E_T^{i-1} \leq 0$$

or if the section is the most distant one from the central office ($i = 1$), then

$$E_{M_u}^i = 0$$

After the unallocated equalized growth margins are determined, the equalized available pairs for unallocated pairs may be calculated,

$$E_{av_u}^i = \frac{E_{M_u}^i}{F^i}$$

Finally the ideal allocation adjustment for unallocated pairs is

$$A_u^i = P_u^i - E_{avu}^i$$

A.5 Glossary of symbols

A.5.1 Route data

Given:

N_s = number of feeder sections
n = number of growth time periods
t_k = length of kth time period

Calculated:

I_m = section index of mth critical section
N_c = number of critical sections

A.5.2. Section data

Given:

N_a^i = number of allocation areas fed by each section
P_d^i = number of dead pairs in each section
P_u^i = number of unallocated pairs ending in each section

Calculated:

G_k^i = forecast growth during kth time period
G_r^i = forecast growth rate after all specified time periods
P_{av}^i = number of pairs available
P_{as}^i = number of pairs assigned
F^i = objective fill at relief
M^i = growth margin
M_u^i = growth margin for unallocated pairs
M_d^i = growth margin for dead pairs
T^i = time to relief
E_T^i = equalized time to relief
E_M^i = equalized growth margin
E_{Mu}^i = equalized growth margin for unallocated pairs
E_{avu}^i = equalized pairs available for unallocated pairs
A_u^i = ideal allocation adjustment (surplus or deficit) for unallocated pairs

A.5.3 Allocation area data

Given:

P_{av}^{ij} = number of pairs available
P_{as}^{ij} = number of pairs assigned
G_k^{ij} = forecast growth during kth time period
G_r^{ij} = forecast growth rate after all specified time periods

F^{ij} = objective fill at relief

Calculated:

M^{ij} = growth margin
T^{ij} = time to relief
E_T^{ij} = equalized time to relief
E_M^{ij} = equalized growth margin
E_{av}^{ij} = equalized pairs available
A^{ij} = ideal allocation adjustment (surplus or deficit)

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