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# A Model Relating Measurement and Forecast Errors to the Provisioning of Direct Final Trunk Groups*

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*This paper describes a mathematical model of the provisioning of direct final trunk groups with forecasting and measurement errors. This model can be used to study the effects of applying standard trunking formulas to possibly inaccurate load forecasts. An important consideration in the process is the degree to which the trunk forecast is actually followed. This so-called provisioning policy is modeled parametrically to allow consideration of a range of strategies, from following the forecast precisely to complete reluctance to remove trunks when indicated by the trunk forecast. When load forecast errors are combined with a reluctance to remove trunks, there will be a net reserve capacity on the average, i.e., more trunks than would be needed if the loads were known exactly. Using the mathematical model, a set of curves known as Trunk Provisioning Operating Characteristics is calculated. These relate percentage of reserve capacity to service (as measured by the fraction of trunk groups with blocking exceeding 0.03). The accuracy of the estimate of the traffic load defines the curve on which one is constrained to operate. The degree of reluctance to remove trunks together with the traffic growth rate determines the operating point. Improved estimation accuracy corresponds to a more desirable operating characteristic. The accuracy of the forecast load estimate is influenced by many factors, such as data base errors (e.g., measuring the wrong quantity due to wiring or other problems), recording errors (e.g., key punch errors), and projection ratio errors. This type of modeling may be useful both in evaluating the potential effects of proposed improvements in measurement or forecasting accuracy, and in studying the effects of changes in provisioning policy. A discussion is given of trunk provisioning process issues based on the viewpoint presented in this paper.*

* A version of part of this paper was presented at the 8th International Teletraffic Congress, Melbourne, November 1976.

## I. INTRODUCTION

Traffic measurements in the Bell System are used as the basis of those efforts aimed at planning an efficient network by providing appropriate quantities of trunking and switching equipment. They also form the basis of many efforts aimed at efficiently administering the network as well as serving as primary inputs for purposes of evaluating network performance.

New traffic measurement systems typically bring with them a variety of benefits such as more accurate and detailed data and more automated and convenient collection and processing of the raw data, together with possible new uses for the data which these improvements allow. Of course, to prove in economically, these advantages must offset the costs associated with installing and operating the system. Clerical savings associated with data collection and processing represent a good example of a relatively easily quantifiable advantage. Other advantages are not so simply equated to dollar savings.

To quantify the traffic-related benefits of improved measurements, models of the trunk provisioning process are required. We describe here a mathematical model of the provisioning of direct final trunk groups* with measurement and forecasting errors. An important consideration which ultimately determines the number of installed trunks is the degree to which the trunk forecast is actually followed. This so-called provisioning policy is modeled parametrically to allow consideration of a range of possibilities, from following the forecast precisely to complete reluctance to remove trunks when indicated by the trunk forecast. Errors in the load forecast will result in some trunk groups having more trunks than required while others will not meet the service criterion. When these errors are combined with a reluctance to remove trunks, there will be a net reserve capacity on the average, i.e., more trunks than would be needed if the loads were known exactly.

Using the mathematical model as a building block, a set of curves known as the Trunk Provisioning Operating Characteristics (TPOCS) is developed. These relate percent reserve capacity (i.e., the amount of trunks in service in excess of what would be required if the load were known perfectly) to service (as measured by the fraction of trunk groups with blocking >0.03). Figure 1 illustrates a typical set of TPOC curves. The accuracy of the estimate of the traffic load defines the curve on which one is constrained to operate. The reluctance to remove trunks together with the traffic growth rate determines the operating point.† Improved estimation accuracy corresponds to a more desirable operating characteristic.

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* Also called full direct or nonalternate route groups.

† Traffic growth and reluctance may be related (e.g., high growth may cause high reluctance).

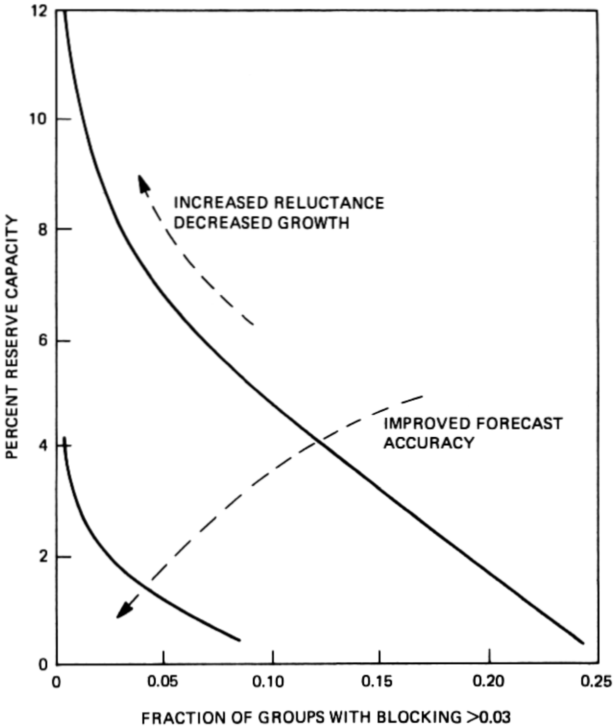


Fig. 1—Trunk provisioning operating characteristics.

This type of modeling can be useful in evaluating the potential effects of proposed changes in measurement or forecasting accuracy as well as studying the effects of changes in provisioning policy. However, as the results which are presented in this paper are based on a simplified model of the provisioning process for the case of direct final trunk groups only, they are intended to serve primarily as a way of viewing the problem with focus on some of the important trade-offs which are present. Much work remains to be done to develop and more fully exploit these ideas.

The paper is broken down as follows: Section II describes the mathematical models which we use to characterize the trunk provisioning process and generate the TPOC curves. Basically, they consist of a procedure for estimating the traffic load for the next busy season coupled with an engineering rule describing the provisioning policy for converting this estimate into the number of trunks provided. The mathematical details associated with these models are presented in the appendix.

Section III considers the sensitivity of the results to a number of the important submodels and assumptions which are used in the development. The TPOC curves are shown to be quite insensitive to the particular model of provisioning policy used as well as to the traffic growth factor and the distributions which characterize the various measurement and forecasting errors (though both the reluctance and growth affect the operating point). The curves do depend somewhat on trunk group size (corresponding to true offered load), however. While specific results are not presented in this paper, the curves also depend upon the mean values of measurement or forecasting errors.

Section IV is a discussion of perspectives on the trunk provisioning process based on the viewpoint presented in this paper. Section V summarizes and discusses further work which is needed.

## II. A PROVISIONING MODEL FOR DIRECT FINAL GROUPS

### 2.1 Basic model concepts

The trunk provisioning process actually used in practice is very complex based only on what is explicitly recommended and standardized.* Overlaid on this are decisions made on a judgment or discretion basis which take into account a multitude of practical considerations. The following steps summarize the highly simplified version of a portion of the trunk provisioning process for direct finals which we will analyze. In Section IV, we discuss the trunk provisioning process in more detail.

(*i*) Traffic measurements—typically consisting of usage, offered attempts (peg count), and overflow—are used to estimate the base offered load during the last busy season for each trunk group. Usage may include a component due to maintenance, as trunks removed from service for investigation and repair are often measured as being busy.

(*ii*) The estimates of offered load for the past busy season are projected ahead—using "projection ratios" that reflect anticipated growth—to yield estimates of base offered loads for subsequent busy seasons.

(*iii*) The estimated number of trunks required to meet a grade of service criterion for the upcoming busy season is determined from an appropriate engineering rule. Typically, trunk groups are sized to produce an average blocking of 0.01 during 20 consecutive business days of the busy season.

(*iv*) The trunk servicer uses the trunks forecast and the trunks in service as the primary information in deciding on the number of trunks to be available for the next busy season. If the forecast number of trunks is less then the number currently in service, there is a reluctance

* There is a large amount of literature dealing with aspects of the trunk provisioning process, generated both within and without the Bell System. Some recent introductory material is given in Refs. 1 to 3. References 4 and 5 also contain related material.

to take trunks out of service unless this frees up equipment needed for other purposes. A number of factors can justify reluctance as a prudent policy. In a growth environment, trunks not needed next year might be needed in the following year. The desire to avoid rearrangement costs also encourages leaving trunks as they are.

A mathematical model capturing the essence of these steps is discussed in the remainder of this section. Though simplified, this model is rich enough to provide valid insights into the trunk provisioning process.

### 2.2 A model of provisioning policy

It is useful to begin the model development by introducing a model for the provisioning policy. This policy relates the current number of trunks, $N_i$, and the estimated requirements for the next period, $\hat{N}_{i+1}$, to the number of trunks to be provided for the next period, $N_{i+1}$. This is modeled parametrically by,

$$N_{i+1} = \max\,[\hat{N}_{i+1},\, N_i - \beta\,(N_i - \hat{N}_{i+1})], \qquad 0 \leq \beta \leq 1, \qquad (1)^*$$

where $\beta$ is a parameter which we introduce to provide a measure of the reluctance to provision down. If $\beta = 0$, trunks will never be removed while, at the other extreme, $\beta = 1$ corresponds to the situation where exactly the estimated number of trunks is always provided, even if that requires removing a large number of the trunks currently installed. Intermediate values of $\beta$ correspond to intermediate provisioning policies.

It is convenient to rewrite (1) in normalized form by dividing through by $M_{i+1}$, the number of trunks required to handle the true traffic load in year $i + 1$. This yields

$$\frac{N_{i+1}}{M_{i+1}} = \max\left[\frac{\hat{N}_{i+1}}{M_{i+1}},\, \beta\frac{\hat{N}_{i+1}}{M_{i+1}} + (1-\beta)\frac{1}{\Delta_i}\frac{N_i}{M_i}\right], \qquad (2)$$

where $\Delta_i = M_{i+1}/M_i$ represents the growth in trunks required to handle the true traffic load in going from year $i$ to year $i + 1$.† The equilibrium solution to (2) has been determined both from (approximate) analytic techniques and from simulation. Specifically, important characteristics of the distribution of the quantity $N/M$ are found. The mean value of this variable is a direct measure of the reserve capacity of the trunk groups under consideration, while the grade of service provided by a collection of trunk groups is related to the distribution of $(N - m)/M$ where $m$ is the number of maintenance trunks (e.g., the fraction of

* Another model of reluctance was also used and was found to give similar results. This is discussed further in Section III.

† The results which follow are in terms of the traffic growth.

groups with blocking $\geq 0.03$ is given by prob$[(N-m)/M < \alpha]$, where $\alpha$ is a suitably chosen constant which depends upon group size). Figure 2 qualitatively shows this relationship. In fact, the TPOC is a plot of $E[N/M] - [1 + (Em/M)]$, the reserve capacity, vs. prob$[(N-m)/M < \alpha]$.

Inspection of eq. (1) or (2) indicates that the various forecasting uncertainties which enter into the provisioning process are all summarized in the distribution of $\hat{N}_{i+1}$, which represents our estimate of the number of trunks which will be required in year $i+1$. This is the reason that the provisioning policy model was introduced first.

### 2.3 A model of trunk forecasting

Having discussed the model which converts the estimated trunk requirement, $\hat{N}_{i+1}$, into the number of trunks provided, $N_{i+1}$, and a simplified discussion of the way it leads to the tradeoff curves, we now turn to the process of estimating the trunks required from the traffic measurements.

It is necessary to consider a specific estimation model. This model is defined by the following equations.

$$\hat{a}_i = \frac{\hat{U}_i}{1-\hat{B}_i}, \tag{3}$$

$$\tilde{a}_{i+1} = \hat{g}_i \hat{a}_i, \tag{4}$$

and

$$B(\hat{N}_{i+1}, \tilde{a}_{i+1}) = 0.01. \tag{5}$$

In these equations, $\hat{U}_i$ represents the measured carried load (which typically consists of the sum of the measured traffic load plus the measured maintenance usage), $\hat{B}_i$ is the blocking estimate (determined

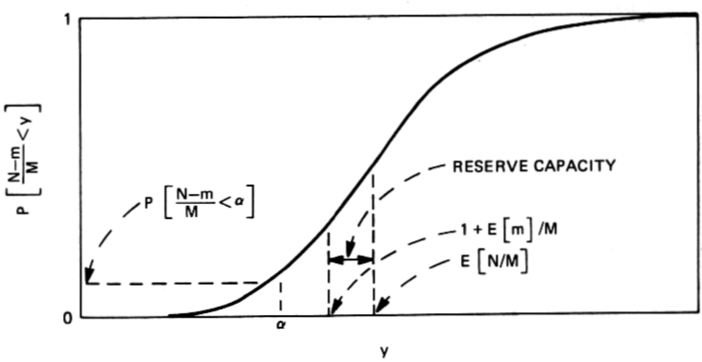


Fig. 2—A distribution of $N/M$ indicating reserve capacity and $P[(N-m)/M < \alpha]$, the fraction of groups with poor service.

from measured peg count and overflow), $\hat{a}_i$ is the estimate of offered load, $\hat{g}_i$ is the projection ratio (which is simply an estimate of the traffic growth on the trunk group), and $B(N, a)$ is the Erlang blocking formula.

As can be seen from (5), the distribution of the estimated number of required trunks is determined by the distribution of the estimated offered load, $\tilde{a}_{i+1}$. The (normalized) variance of the estimation error associated with $\tilde{a}_{i+1}$ turns out to play a major role in results which we will soon describe. If we denote this normalized variance by $\sigma_x^2$, then the first order analysis of (3) and (4) given in the appendix (which assumes statistically independent, zero-mean error components) yields

$$\sigma_x^2 = \sigma_{U_i}^2 + \sigma_{B_i}^2 + \sigma_{g_i}^2, \tag{6}$$

where*

$\sigma_x^2 = \text{var}\left(\dfrac{\tilde{a}_{i+1} - a_{i+1}}{a_{i+1}}\right)$ = normalized variance of error in forecast load,

$\sigma_{U_i}^2 = \text{var}\left(\dfrac{\hat{U}_i - U_i}{u_i}\right)$ = normalized† variance of total carried usage error,

$\sigma_{B_i}^2 = \text{var}\left(\dfrac{\hat{B}_i - B_i}{1 - B_i}\right)$ = normalized variance of error in blocking estimate, and

$\sigma_{g_i}^2 = \text{var}\left(\dfrac{\hat{g}_i - g_i}{g_i}\right)$ = normalized variance of error in traffic growth factor.

To further facilitate the identification of the various error sources which contribute to our total estimation error, we may (approximately) decompose $\sigma_{U_i}^2$ into its component parts as follows:

$$\sigma_{U_i}^2 = \sigma_{d_i}^2 + \sigma_{s_i}^2 + \sigma_{m_i}^2. \tag{7}$$

In the above expression, $\sigma_{d_i}^2$ represents the variance of the data base and data handling induced errors in usage, e.g., due to measurement system deficiencies, $\sigma_{s_i}^2$ represents the statistical‡ errors in usage and $\sigma_{m_i}^2$ represents the variance of the fraction of trunks plugged busy for maintenance purposes (the trunking engineer, in general, does not

* These terms are defined more fully in the appendix.

† $u_i$ represents the mean traffic usage and thus does not have a maintenance component.

‡ The statistical error term represents the effects of basing an estimate on a fixed number of samples of the busy-idle state of each trunk over a finite time interval.

know how much of the total usage is attributable to trunks plugged busy for maintenance purposes* and how much is traffic usage). Each of these quantities is normalized to the group traffic usage.

To compute the reserve capacity and the fraction of groups with blocking greater than 0.03, it is necessary to know the expected traffic growth, the number of trunks actually required, and the statistical behavior of the fraction of trunks plugged busy for maintenance. The form of the model output is shown in Fig. 3. The model parameters used to generate the curve apply to trunk groups of nominal size 25, a traffic growth rate of 5 percent annually, $\sigma_m = 0.02$ and $\sigma_x = 0.2$.† Each point on the curve corresponds to choosing one value of the provisioning parameter $\beta$ and then computing the corresponding percent reserve capacity for the groups and the fraction of groups which have blocking greater than 0.03. Several values of $\beta$ are indicated on the curve. Thus this curve, which we shall refer to as the Trunk Provisioning Operating Characteristic (TPOC), indicates the tradeoff which exists between the percent reserve trunk capacity and the fraction of groups with blocking exceeding 0.03.

Figure 4 shows the same tradeoff curve as Fig. 3, but also includes the tradeoff curve for a system which has $\sigma_x = 0.1$. The operating point on the tradeoff curve depends on the amount of reluctance to service down, but the tradeoff curve depends on the $\sigma_x$ of the provisioning system. Improving the accuracy of the forecast (which depends in part on measurement accuracy) for the trunk provisioning process causes a decrease in $\sigma_x$. That is, it places the operating point on a more desirable tradeoff curve. The service improvement or trunk savings realized by such an improvement depends on the location of the operating point on the curve.

One additional point: a perfect measuring system corresponds to a nonzero $\sigma_x$ due to nonmeasurement errors, such as statistical errors. This is indicated in Fig. 4 by the shaded area which contains TPOCS which cannot be reached by only decreasing measurement errors.‡

A number of studies were conducted to assess the sensitivities of the tradeoff curves to the model assumptions. These are discussed in the next section.

## III. SENSITIVITY RESULTS FOR TRADEOFF CURVES

The provisioning policy model used is an approximation to the actual provisioning decisions which occur in the provisioning process.

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* Maintenance usage may or may not be directly measured; even if measured, it varies from year to year.

† Normality is assumed; sensitivities to distribution (and to other factors) are considered in the next section.

‡ In the forecasting process, other factors, such as day-to-day variations, also contribute to a minimum $\sigma_x$. These factors are discussed, for example, in Refs. 6 to 8.

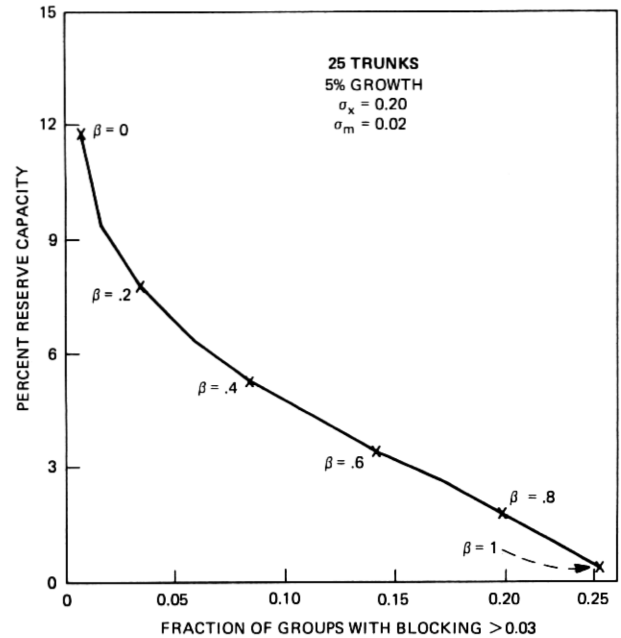


Fig. 3—Typical model output, TPOC tradeoff curve.

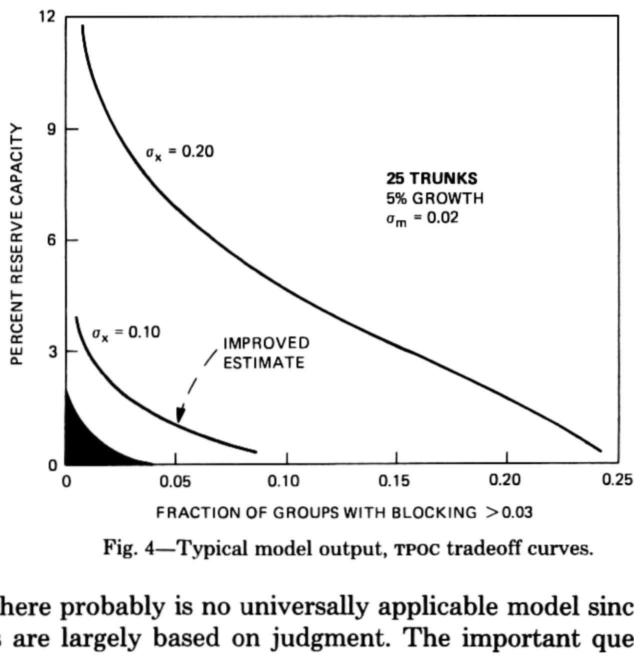


Fig. 4—Typical model output, TPOC tradeoff curves.

In fact, there probably is no universally applicable model since many decisions are largely based on judgment. The important question is whether a different model for provisioning policy would significantly change the tradeoff curves. To investigate the sensitivity of the curves to the assumed model of provisioning policy, we considered a different model for the reluctance to remove trunks.

This model (the $\omega$-model) is given by

$$N_{i+1} = \max \{\hat{N}_{i+1}, \omega N_i\}, \tag{8}$$

where $\omega$ is between zero and one. When $\omega$ is zero, the $\omega$-model always provides the estimate of the trunks required, just as the $\beta$-model, eq. (1), does when $\beta = 1$. Also, when $\omega = 1$ the $\omega$-model never removes trunks, just as with the $\beta$-model when $\beta = 0$. Thus, the two models are identical when there is no reluctance to remove trunks, $\omega = 0$ and $\beta = 1$, and when there is complete reluctance, $\omega = 1$ and $\beta = 0$. However, the models are quite different in the case of an intermediate amount of reluctance. The $\omega$-model will freely remove trunks down to $\omega N_i$, but

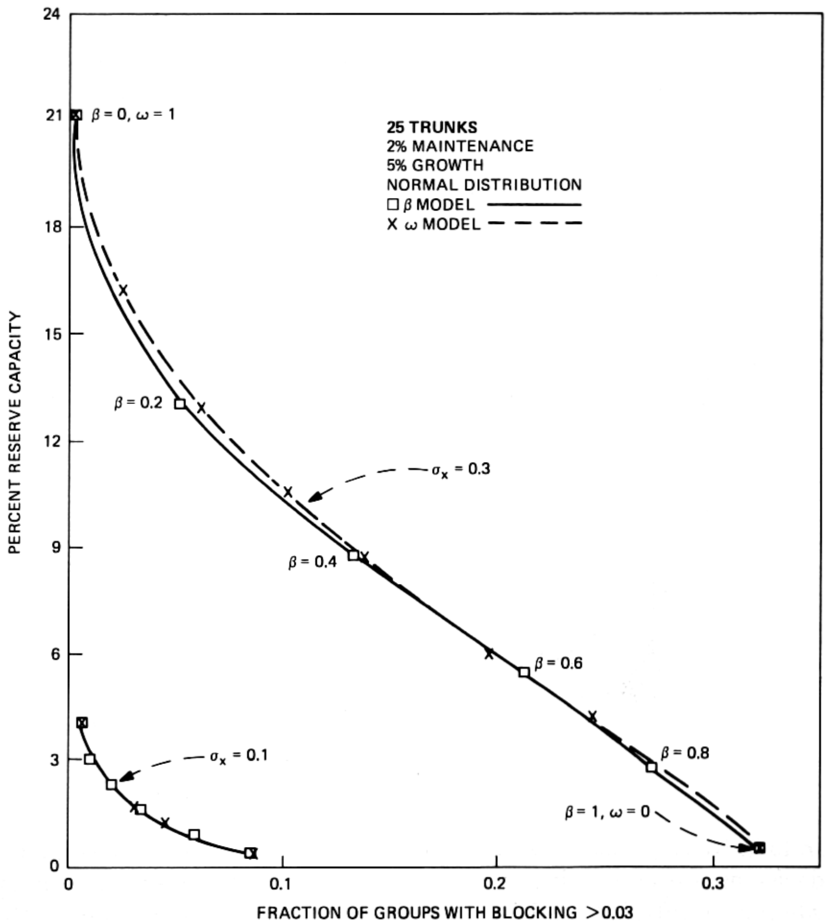


Fig. 5—Sensitivity to reluctance model.

won't remove any beyond that, while the $\beta$-model removes a fixed fraction of any excess of $\hat{N}_{i+1}$ over $N_i$.

TPOCS for each of these models are shown in Fig. 5 for nominal parameters of trunk group size 25, a 5-percent growth rate, fraction of maintenance busy trunks (0.02) and values of $\sigma_x$ of 0.1 and 0.3 (standard deviation of load forecast error). As expected, the end points of these plots are identical, since for $\omega = 0$, $\beta = 1$ and $\omega = 1$, $\beta = 0$ the models are identical. Furthermore, the curves are seen to be close throughout the entire range. We thus observe that the two reluctance functions provide different mechanisms for tracing out approximately the same tradeoffs.

Figure 6 is a plot of the tradeoff curves for several different growth

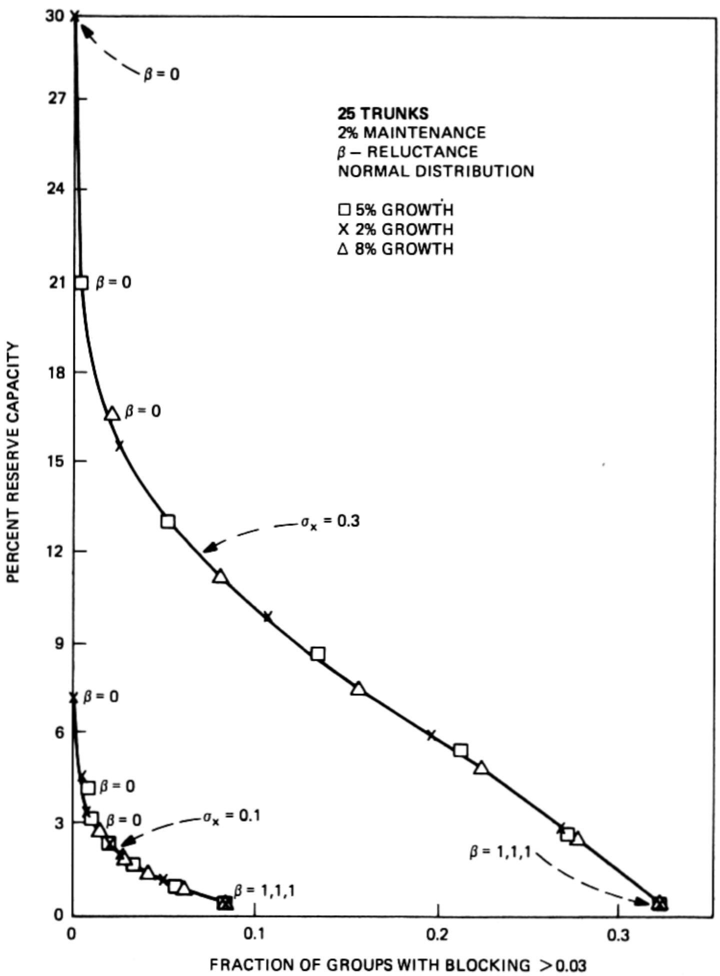


Fig. 6—Sensitivity to growth.

rates (2, 5, and 8 percent). We note that the curves are quite insensitive to the particular traffic growth rate, although the specific operating point, for a given reluctance, can be quite sensitive. Also, the extent of the curves are sensitive to growth. We noted earlier that, for a given growth, increasing reluctance caused one to ride up a tradeoff curve. Here we observe that, for a given reluctance, increasing the growth rate causes one to ride down a tradeoff curve. This is illustrated by the $\beta = 0$ (complete reluctance) points indicated on the figure. We are thus observing, as expected, higher growth rates tending to reduce reserve capacity. The results were also shown to be relatively insensitive to the distribution function of the forecast of trunk requirements and the level of maintenance busies; however, trunk group size* and mean forecasting error did influence the tradeoff curves.

## IV. APPLICATION OF THE MODEL TO THE PROVISIONING PROCESS

### 4.1 Provisioning process issues

Trunk provisioning process issues can be broken down into the general areas:

(*i*) Input data quality.
(*ii*) Improving the use of data.
(*iii*) Monitoring the performance of the provisioning process.

Figure 7 shows a simplified functional view of the trunk provisioning process which we shall use to discuss these issues.

Actual traffic in the network is sensed by a measurement, data collection, and processing system. The data from this system enter a trunk servicing system which estimates current requirements and service and a trunk forecasting system which, with other information, forecasts trunk requirements for 1 to 5 years out. The processed data from the trunk servicing and forecasting systems, together with the source data for these systems, are the focus for assessing input data quality. Typical questions which often arise in this context include:

(*i*) How big are the errors in these data, and what are the main contributors?
(*ii*) To what extent can these errors be controlled by better technology or maintenance?
(*iii*) What are the benefits of such improvements?

The outputs of the trunk forecasting system are used in facility and equipment planning and for trunk servicing, i.e., for deciding on the actual trunks to be provided for the next busy season. Trunk servicers base these decisions primarily on the forecast, but they factor in

* This sensitivity to trunk group size occurs if $\sigma_x$ is the relative error in the load, as it has been defined in this paper. However, it may be shown that, if the relative error in the number of trunks is used, this sensitivity becomes small.

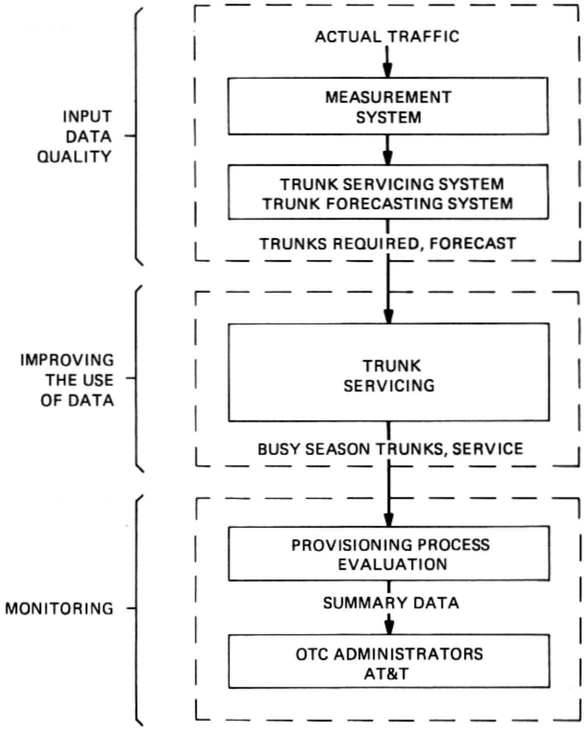


Fig. 7—Simplified view of the trunk provisioning process.

current trunks installed, recent trunk servicing data, facility and equipment availability, etc. This decision-making process is the focus for improving the use of data. Typical questions include:

(*i*) How can all available data be used so that the final judgment on required trunks in the decision-making process (i.e., the "effective" forecast) is an improvement on the quality of the nominal forecast (i.e., the "published" forecast)?

(*ii*) What provisioning policies (e.g., disconnect policies) should be followed, given that all uncertainties in forecasting cannot be removed?

(*iii*) What are the benefits from such improvements in the use of data?

The quality of the input data together with the actions based on the data determine the overall performance of the provisioning process. Data on service, trunks provided, network activity, etc., is gathered by the operating telephone companies, with summary results provided to AT&T for use in assessing and establishing system policies. This evaluation process is the focus of the monitoring issue. Typical questions here include:

(*i*) What data should be used to monitor the provisioning process at the local level; that is, to aid servicers and forecasters directly involved in provisioning, and at the global (or network) level, that is, to aid operating telephone company administrators and AT&T in overseeing the process?

(*ii*) How should the data be displayed and interpreted so that the causes underlying observed results become evident?

(*iii*) How can the monitoring results be used to improve the process?

The major issues which we have outlined can be directly related to the provisioning model developed in Section II. Input data quality is concerned primarily with the "nominal" forecast accuracy, i.e., the accuracy of the published forecast. The key parameters to quantify forecast accuracy are the forecast bias and the forecast standard deviation. Improving the use of data is concerned with decisions which determine operating points along a TPOC, and, perhaps more important, with methods to move to preferred TPOCs by improving on the nominal forecast accuracy (by using all available information). For example, generating a short-term forecast based on the most recent servicing information (traffic data) and combining it with the nominal forecast can result in an improved "effective" forecast. Monitoring is concerned primarily with assessing trunks and service results. The TPOC model clearly shows that one-dimensional measures of performance, such as service, do not provide sufficient information. From the TPOC point of view, methods which exploit knowledge of the reserve capacity and service tradeoff are desirable.

Relating provisioning process issues to the TPOC framework is important for several reasons. First, this framework helps to identify potential improvements to the provisioning process. Perhaps more important, this framework provides a basis for quantifying the benefits associated with such improvement. The ultimate dollar value of any of the improvements considered requires detailed specifications and application studies and consideration of the coupling between trunk servicing and facility and equipment provisioning, which are beyond the scope of this paper.

### 4.2 Input data quality

The key parameters for assessing input data quality are the nominal forecast bias, a measure of average error, and the nominal forecast standard deviation, a measure of the spread of errors. When analyzing a collection of trunk groups at an office, an error in the office growth factor would tend to show up as a bias in the forecast error on those groups. In practice, this can occur in any given year due, for example, to errors in commercial forecasting of main stations and loads per main station, which are inputs to the trunk forecasting process. For example, in the case of an unexpected economic downturn, which normally results in reduced calling, trunking requirements would tend to be overforecast *on average*. To the extent that this bias is influenced by unexpected exogenous economic factors, it may not be controllable by better measurement technology.

Forecast standard deviation is primarily influenced by inherent variability of traffic (for example, day-to-day variability), measurement errors, and forecasting methodology (for example, the procedure by which aggregate traffic growth is allocated to individual trunk groups). Of these factors, measurement error can be an important, though technologically controllable, factor. For example, it has been generally accepted that wiring errors, which can cause individual trunk usage to be attributed to the wrong group, can occur in conventional Traffic Usage Recorders (TURS). Manual wiring changes in a TUR must be made when trunks are added to, or deleted from, a group, or if trunking rearrangements are made. The buildup of wiring errors is generally controlled by periodic audits. In contrast, an Electronic Switching System records group usage via the same trunk to group association used in the switching process. Thus, it is to be expected that measurement error and forecast standard deviation should be lower for ESS offices as compared to TUR offices. Limited data from a small number of offices supported this expectation, and suggested that the better accuracy of ESS technology may, in some cases, lead to a reduction of several percent in trunks in service, without a consequent degradation in customer service. This benefit can also be achieved by upgrading of conventional measurement technology in electromechanical systems. For example, the Engineering and Administrative Data System (EADAS), when equipped with Individual Circuit Usage Recording (ICUR), maintains the trunk-to-group mapping in software, with a variety of checks to guard against mapping errors.

### 4.3 Improving the use of data

The goal of improving the use of data, as suggested by the TPOC framework, is to control the effective (as opposed to the nominal) forecast accuracy driving the process, and to improve provisioning policies. As in any large scale process involving the acquisition, transfer, and processing of data to produce outputs—with perhaps both manual as well as automated segments—various irregularities can occur in trunk forecasting and servicing processes. Some simple data validation and screening techniques which can help trunk servicers control the effective forecast accuracy exploit: (*i*) incomplete data, e.g., a pattern of missing usage data which may suggest a faulty TUR, (*ii*) inconsistent data, e.g., apparent high blocking on a group with low usage, indicative of a possible data error, (*iii*) data outliers, e.g., unusually large changes in trunk forecasts, and (*iv*) redundant data, e.g., usage data from both ends of a group can be compared for consistency.

As a specific illustration of a method for improving the use of data suggested by the TPOC model, consider short-term forecasting. At the latest time a servicer must make a trunk group provisioning decision, i.e., just prior to an upcoming busy season, available data normally include the forecast trunks based on the last busy-season base load, and a recent traffic profile for the group. This latter data may be appropriately projected ahead to produce a "short-term forecast," essentially independent of the regular forecast. By linearly combining those two forecasts with appropriate weights (see Ref. 9), thus reducing variance, a significantly improved forecast can be constructed. As illustrated by Fig. 8, which assumes a regular and short-term forecast of comparable accuracy, a substantial savings in trunks turned up for service can potentially be achieved. The improvement obviously depends on the relative accuracies of these forecasts which require further studies to quantify.

The actual provisioning decisions by servicers, or more generally, provisioning policies, should reflect the fact that data are imperfect. The TPOC model shows that disconnect policy is a particularly important factor in the performance of the provisioning process and permits

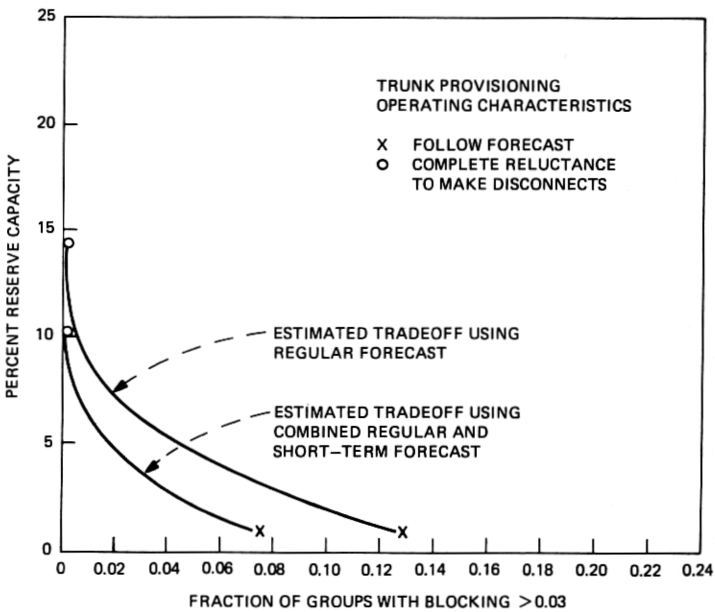


Fig. 8—Potential benefit of short-term forecast [regular and (assumed) short-term forecast standard deviation = 15 percent in trunks].

the study of various policies. In addition to reserve capacity and service, a prudent disconnect policy should factor in the cost of rearrangements and the possibility of unnecessarily disconnecting trunks which are needed in the next year or so. A modification of the TPOC model has been used by N. E. Kalb to examine various disconnect policies and define an appropriate one for Bell System use.

### 4.4 Monitoring

The goal of monitoring is to track the performance of the provisioning process, and to allow the underlying factors to be identified which contribute to that performance. For displaying performance results for particular administrative units, e.g., an operating division, the TPOC perspective indicates the importance of considering both reserve capacity and service, as illustrated with assumed data in Fig. 9. We note that comparing these assumed data points by service could be very misleading, i.e., the "best" service is obtained at the price of very high reserve capacity, while the "worst" service corresponds to a moderate level of reserve capacity. If one now further exploits the TPOC model and views each point as representing a particular operating point along a TPOC tradeoff curve, the data point that looks most like a weakspot (i.e., appears to lie on the worst tradeoff curve) does not have the worst service or reserve capacity. Similarly, the entity that might be considered to have the best performance does not correspond to either the

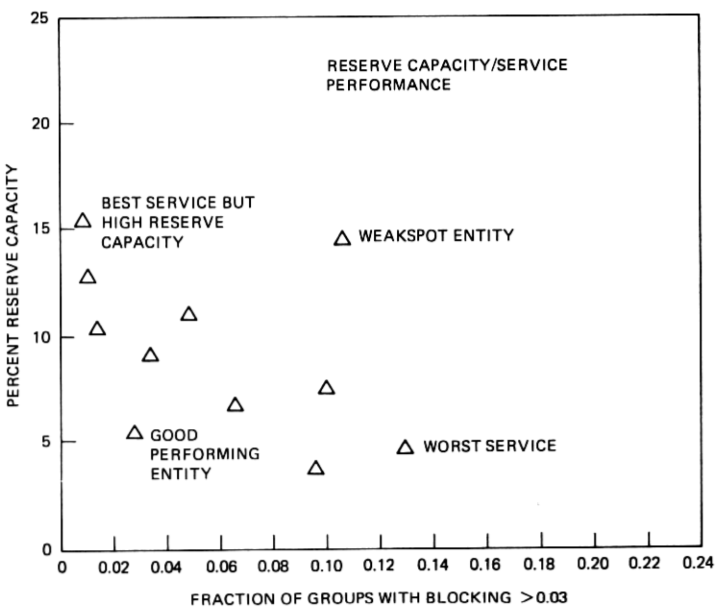


Fig. 9—Reserve capacity vs service display.

lowest reserve capacity or the best service. In both these cases, the actually operating points may have resulted from prudent disconnect policies that achieved a reasonable balance between reserve capacity and service, avoiding the extremes of no disconnects or of blindly following the forecast.

The preceding perceptions about performance, and the need to encourage an appropriate balance between reserve capacity and service, might be emphasized by subdividing the reserve capacity and service plane into appropriate regions, based on preferred operating points. This could result in a performance monitoring approach such as shown by Fig. 10, which clearly facilitates the comparison and ranking of overall performance of the various assumed data points. The shape of the performance regions shown in Fig. 10 reflect the undesirability of a disconnect policy which works at the tradeoff extremes. The performance partitions—which ideally would be designed to reflect equally desirable operating points, with cost factors considered—provide motivation both to improve forecast accuracy and to control disconnect policy. Further study would be required to determine an actual partitioning into performance regions.

In addition to allowing cross-comparisons of performance, monitoring should allow the underlying factors which contributed to the performance of a particular office to be identified. Again, the TPOC model suggests an approach. By supplementing the data needed to

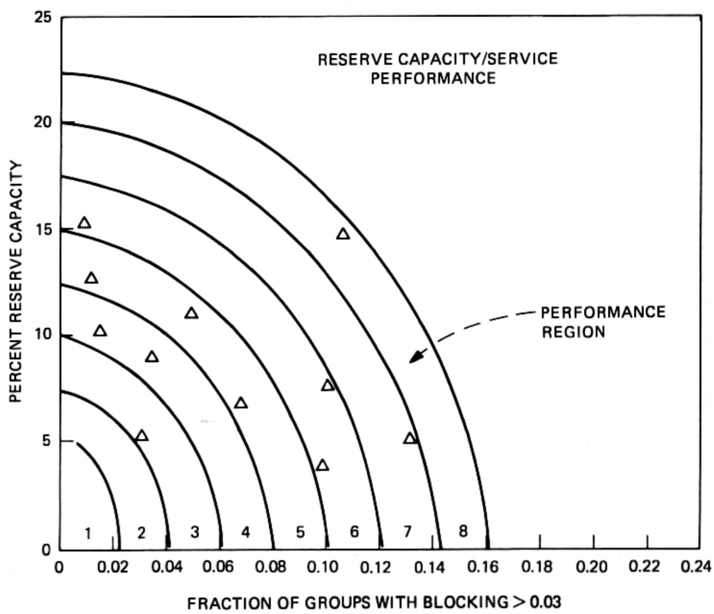


Fig. 10—Reserve capacity vs service display with (assumed) performance regions.

estimate service and reserve capacity with information on the previous busy season trunks, and the trunks forecast for the busy season, the operating points that would have corresponded to complete reluctance to disconnect and follow forecast can be computed.* By displaying these points with the actual operating point, as illustrated in Fig. 11, significant information is obtained. For example, it is immediately clear that reluctance to disconnect trunks could potentially have contributed about 12 percent in trunks to reserve capacity (labeled potential reluctance). However, the actual operating point indicates a prudent disconnect policy, with net reluctance adding only about 4 percent to reserve capacity. In contrast, the bias (perhaps due to a growth slowdown) has added about 5 percent to reserve capacity and is a dominant factor. The end result of this type of performance analysis is a decomposition of reserve capacity into basic components,† due to forecast bias plus actual reluctance, together with a relative measure of disconnect policy, i.e., actual disconnects/potential disconnects. One may also view the final service as being decomposed according to: follow forecast service minus service benefit of actual reluctance. Such graphical analysis, together with associated quantifications from the data (e.g., forecast standard deviation), could provide significant insight into provisioning performance and suggest where improvement is needed.

In addition to comparative evaluations and to evaluation of key factors on the global, or network, levels, it is important to provide information to servicers and forecasters responsible for decisions on individual trunk groups. Simple graphical displays using the same raw data needed for global evaluations can be used to provide servicers, forecasters, and trunk administrators with clear pictures of performance, and of the role of forecast accuracy, disconnect policy, and other aspects of the process. For example, Fig. 12 illustrates a graphical aid which can be used to display and quantify forecasting accuracy; a similar display using trunks in service and required can be used to display reserve capacity and service. Figure 13 illustrates a graphical aid which can be used to examine disconnect policy. Such displays also quickly point out unusual actions, such as the addition of trunks when the forecast calls for removals. This type of action may have been initiated if the ongoing servicing data from the group indicated higher traffic levels than those forecast.

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* It turns out that over a fairly wide range of group sizes, for example $\geq$10 trunks, the theoretical equivalent to P.03 service for direct final groups corresponds to an almost constant percent trunk shortage (about 13 percent).

† One may further extend this analysis (using the same data) to account for cases where all forecast adds may not have been made. This can occur when a growth slowdown has been recognized.

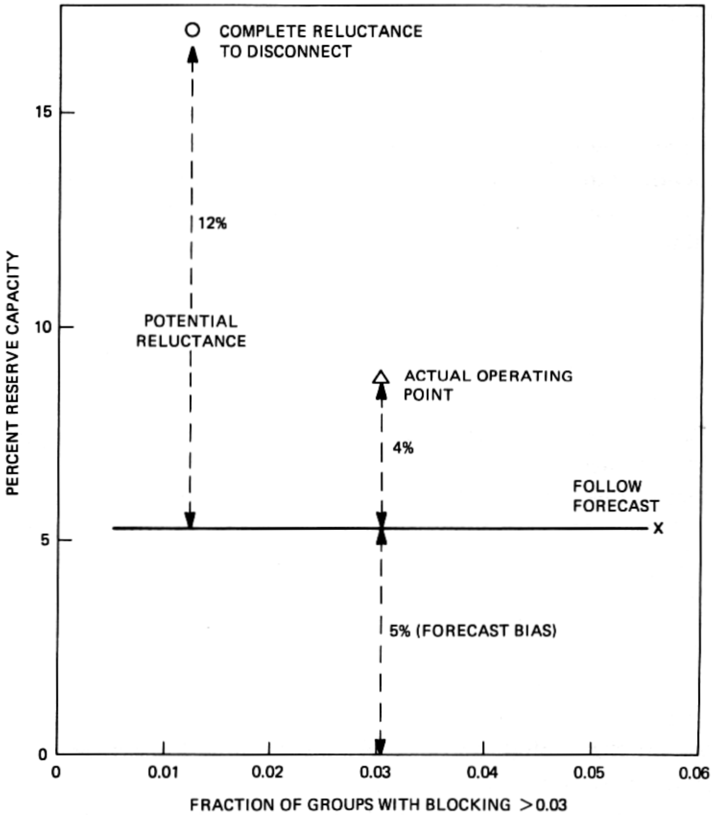


Fig. 11—Performance analysis for a particular trunking entity—with forecast and previous trunks connected information.

In summary, the TPOC model—as discussed for direct final groups—suggests a global or network monitoring technique that allows both comparative evaluations and analysis of key factors. This can be useful, for example, in determining if disconnect policy needs changing. At the same time, those responsible for decisions at the lower echelons can be provided with a set of more detailed displays that highlight the same factors, as well as any individual anomalies in the data that should be investigated. While these displays are not now available, the capability to produce such graphical aids could be an important enhancement to current trunk servicing and forecasting systems.

## V. CONCLUDING REMARKS AND FURTHER WORK

In this paper, we have developed a mathematical model of the trunk provisioning process which allows one to study the relationship of traffic measurement and forecasting errors and disconnect policy to

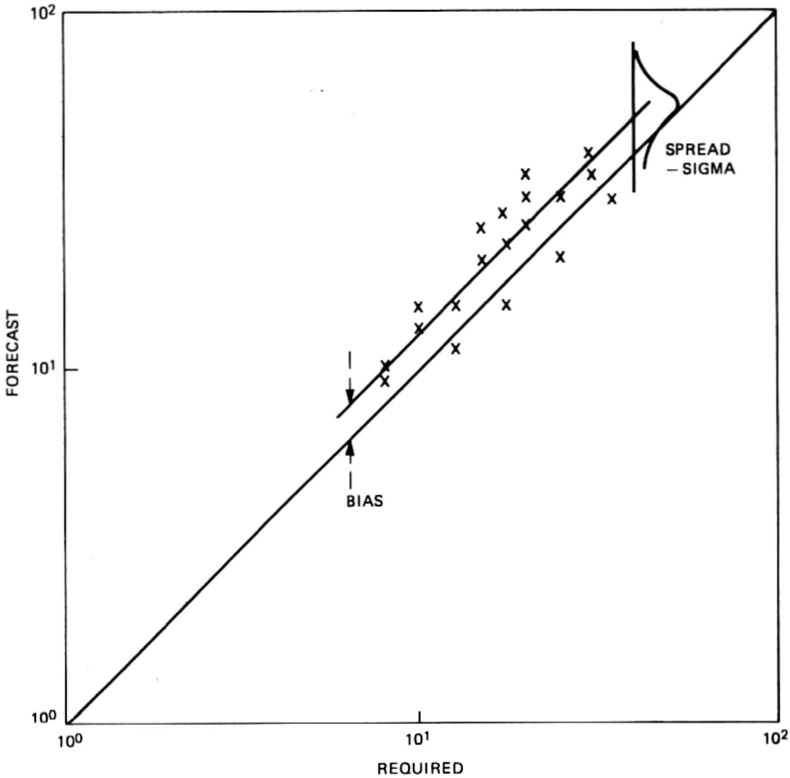


Fig. 12—Trunks forecast vs trunks required.

reserve capacity and service. We reemphasize that this is a first-cut model based on many simplifying assumptions.

Direct final groups were considered since they represented the simplest case.* The results were displayed as trade-off curves of percent reserve capacity vs percent of groups with blocking >3 percent. The curves were parameterized by $\sigma_x$, the normalized error in forecast load.

These TPOC curves were found to be relatively insensitive to reluctance policy, probability distribution of errors, and growth.† This robustness of the curves is useful not only from the point of view of convenience of presentation but also because of our uncertainty concerning these quantities. The basic TPOC concepts were validated using field data collected on approximately 250 direct final trunk groups. In

* It might be noted that they also represent a large fraction of the total Bell System trunks and groups.

† Where one operates on the curve depends on the reluctance policy and growth.

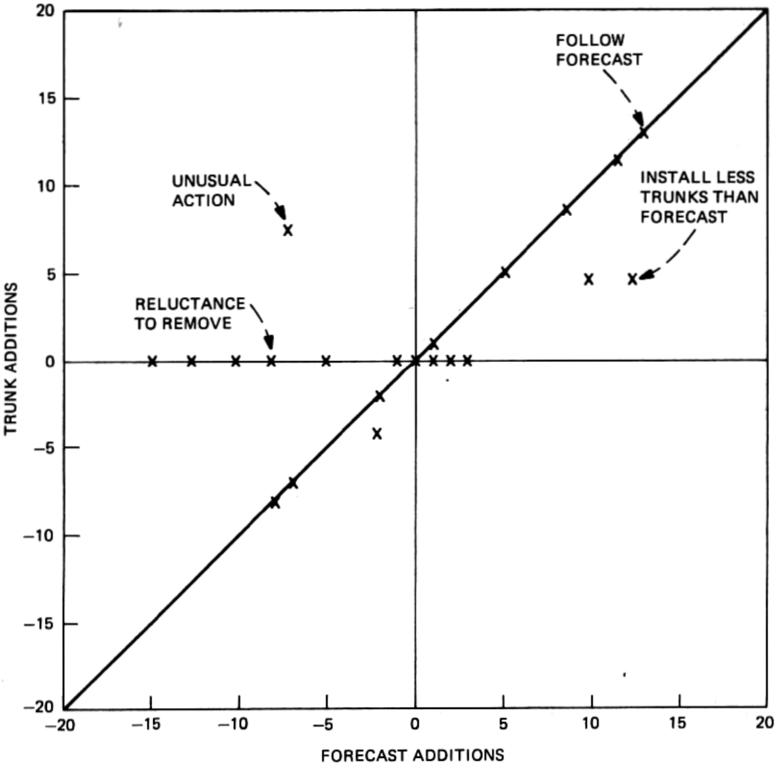


Fig. 13—Trunk additions vs forecast additions.

the course of this effort, the TPOC model was extended to include nonzero mean forecast error, and the effect of day-to-day variations and to reflect the fact that estimates of reserve capacity and service were themselves corrupted by various errors. It may be noted the nonzero mean forecast error enters the TPOC model as a persistent error for a single trunk group over a sequence of years. While the data were obtained for a set of trunk groups over one year, the model still yielded good accuracy.

Much remains to be done to further develop and extend the concepts we have described. We indicate here only a few of the many areas worthy of further study. The results we have described assume an equilibrium condition corresponding to reluctance and true growth, which are constant from year to year. These assumptions could be relaxed and transient solutions developed in more detail than we have done. Extension of the modeling approach to other probability-engineered groups would seem like an obvious next step, while the appropriate framework for viewing network clusters involving high-usage groups would require more work. In addition, further work needs to be done to characterize the dynamics of growth factor errors which can result in biases in a particular year.

To capitalize on the ideas we have discussed, some of the obvious problem areas which present themselves involve the development of improved forecasting capabilities, disconnect policies (i.e., "reluctance"), and data screening techniques. As mentioned at the end of Section 4.3, these ideas have already been used to study disconnect policies.

Finally, we note that, to draw conclusions about the value of specific proposed improvements (e.g., an improved measurement system), the overall forecast error must be decomposed into its component parts and each of these components must be quantified for each alternative under consideration. In addition, it should be kept in mind that casual attempts to plot reported trunking data on existing TPOC curves can be misleading because of factors that might be influencing the data but left out of the curves (trunks installed as a result of new machine cutovers, etc.).

Additional work (e.g., further modeling and statistical analysis of usage errors) is needed to more accurately quantify the effect of improvements in measurements (or forecasting) when one is dealing with a nonuniform* collection of trunk groups.

Even with the additional work that is warranted, it is seen from Section IV that the TPOC model is a powerful aid to identify issues and to suggest improvements and ways of evaluating them.

## VI. ACKNOWLEDGMENTS

We thank Anne Carlson, Joanne Darcy, Nancy Kalb, Dave Jagerman, and Sharon Miller for their valuable contributions. We also received much valuable information from a number of other people at Bell Labs and AT&T and Pacific Telephone (from whom we obtained data for validation).

## APPENDIX

### *Model Development*

### *A.1 Model and definitions*

Our model for the provisioning of direct final groups with no day-to-day load variation assumes that peg count, overflow, and usage are

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* Nonuniformity can result from dealing with groups with different measurement devices (for example, ESS and TURS). Furthermore, even with the same measurement devices (e.g., all TUR measurements), one can expect differences in accuracies (such as well-maintained TUR vs poorly maintained TUR).

measured during a base period, such as a busy season. The offered load is estimated according to eq. (3), and the next period's (e.g., next busy season's) offered load is found by multiplying by a projection ratio. The estimated number of trunks required for the next busy season is such that the projected offered load would cause a blocking probability of 0.01. Once the number of trunks required for next year, $\hat{N}_{i+1}$, has been estimated, the number actually provided, $N_{i+1}$, is chosen using eq. (1), or eq. (8), depending on the provisioning policy modeling. This model assumes trunks are provided only once each period with measurements from the last period.

In order to write the model equations we need some definitions.

$u_i$ = Mean of the true traffic usage during study period $i$.
$\hat{u}_i$ = Measured traffic usage during study period $i$.
$m_i$ = Actual maintenance usage during study period $i$.
$\hat{m}_i$ = Measured maintenance usage during study period $i$.
$U_i$ = Mean of the true total (maintenance plus traffic) usage during study period $i$.
$\hat{U}_i$ = Measured total (maintenance plus traffic) usage during study period $i$.
$eu_i = \hat{u}_i - u_i$, usage error due to TUR wiring, etc. with separate traffic usage measurement.
$eU_i = \hat{U}_i - u_i - m_i$, usage error due to TUR wiring, etc. with joint usage measurement.
$a_i$ = Mean of the true offered load during study period $i$.
$\hat{a}_i$ = Estimate of $a_i$ based on measurements during study period $i$.
$\tilde{a}_{i+1}$ = Estimate of $a_{i+1}$ based on measurements during study period $i$.
$M_i$ = Number of traffic trunks required in study period $i$. (e.g. $B(M_i, a_i) = 0.01$).
$\hat{M}_{i+1}$ = Estimate of $M_{i+1}$ based on measurements during study period $i$.
$N_i$ = Number of trunks in place in study period $i$ (includes maintenance).
$\hat{N}_{i+1}$ = Number of trunks estimated as required for period $i + 1$.
$g_i = a_{i+1}/a_i$, the traffic growth.
$\hat{g}_i$ = The estimate of $g_i$.
$\hat{B}_i$ = Measured average fraction overflowing during study period $i$.
$B_i$ = Mean of the fraction overflowing during study period $i$.
$eB_i = \hat{B}_i - B_i$.
$eg_i = \hat{g}_i - g_i$.

### *A.2 Estimating $\hat{N}$ from lumped traffic and maintenance usage*

Estimating $\hat{N}$, in terms of its mean and variance, requires estimating $a_i$. The estimate of $a_i$ depends on whether or not maintenance usage is measured separately.* If it is not, then $\hat{U}_i$ is measured and we estimate $a_i$ by

$$\hat{a}_i = \frac{\hat{U}_i}{1 - \hat{B}_i} = \frac{m_i + u_i + eU_i}{1 - [B(N_i - m_i, a_i) + eB_i]} \tag{9}$$

$$\approx a_i \left[ 1 + \frac{m_i}{u_i} + \frac{eU_i}{u_i} + \frac{eB_i}{1 - B_i} \right]. \tag{10}$$

The estimate of the next period's offered load is

$$\tilde{a}_{i+1} = \hat{g}_i \hat{a}_i \approx a_{i+1}(1 + x_i), \tag{11}$$

where

$$x_i = \frac{m_i}{u_i} + \frac{eU_i}{u_i} + \frac{eB_i}{1 - B_i} + \frac{eg_i}{g_i}. \tag{12}$$

Since $B(\hat{N}_{i+1}, \tilde{a}_{i+1}) = 0.01$ is the design criterion, expanding and regrouping terms yields

$$\frac{\hat{N}_{i+1}}{M_{i+1}} = 1 + \left( -\frac{B_2}{B_1} \frac{a_{i+1}}{M_{i+1}} \right) x_i = 1 + cx_i, \tag{13}$$

where

$$B_1 = \frac{\partial B}{\partial N} \bigg|_{M_{i+1}, a_{i+1}}, \quad B_2 = \frac{\partial B}{\partial a} \bigg|_{M_{i+1}, a_{i+1}}, \qquad c = -\frac{B_2}{B_1} \frac{a_{i+1}}{M_{i+1}}. \tag{14}$$

By (14), $c$ depends only on $M_{i+1}$. It is 0.647 for $M_{i+1} = 10$ and increases toward 1 as $M_{i+1}$ increases.

To find the mean of $\hat{N}/M$, we assume that all measurements and estimates are unbiased. That, together with (12), (13), and the definitions, gives

$$E\left( \frac{\hat{N}_{i+1}}{M_{i+1}} \right) = 1 + c \frac{Em_i}{u_i} = 1 + d \frac{Em_i}{M_i}, \tag{15}$$

where

$$d \triangleq c \frac{M_i}{u_i} = -\frac{B_2}{B_1} \frac{a_{i+1}}{u_i} \frac{M_i}{M_{i+1}}. \tag{16}$$

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* Frequently, maintenance usage is not measured separately.

By their definitions, $u_i$ depends only on $M_i$ so $d$, which is approximately 1, depends only on $M_i$ and $M_{i+1}$.

To find the variance of $\hat{N}/M$ we assume that the terms in (12) are statistically independent. That gives

$$\mathrm{Var}\left(\frac{\hat{N}_{i+1}}{M_{i+1}}\right) = c^2\sigma_x^2, \tag{17}$$

where

$$\begin{aligned}
\sigma_x^2 &= \sigma_{U_i}^2 + \sigma_{B_i}^2 + \sigma_{g_i}^2 \\
\sigma_{U_i}^2 &= \sigma_{m_i}^2 + \sigma_{u_i}^2 \\
\sigma_{m_i}^2 &= \mathrm{Var}\left(\frac{m_i}{u_i}\right), \qquad \sigma_{u_i}^2 = \mathrm{Var}\left(\frac{eU_i}{u_i}\right), \qquad \sigma_{B_i}^2 = \mathrm{Var}\left(\frac{eB_i}{1-B_i}\right), \\
\sigma_{g_i}^2 &= \mathrm{Var}\left(\frac{eg_i}{g_i}\right).
\end{aligned} \tag{18}$$

In the rest of the paper we assume that

$$\mathrm{Var}\,\frac{eU_i}{u_i} = \mathrm{Var}\,\frac{eu_i}{u_i},$$

since neither contains variation due to maintenance usage, and all the other errors have the same sources.

A similar analysis is possible for the case when the traffic usage is measured separately from the maintenance usage.

### A.3. Solution of model equations

The solution of eq. (2) for the distribution of $N_i/M_i$ has been obtained in several ways. Analytic approximations and characterizations of properties of the solution have been developed by D. L. Jagerman, while the actual TPOC curves presented in this paper were obtained by employing simulation techniques to solve (2).

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