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Application of Optimization Theory to the Control of the Optical Fiber Drawing Process

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The optical fiber drawing process is examined and a feedback control loop identified. The incremental dynamic response of each loop component is determined, and the sensitivity of loop response to system parameters is examined. The control loop is optimized, based upon a mean square error criterion with constraints imposed for periodic disturbances. An expression is derived for the effectiveness of the control loop with respect to sources of system disturbance and found to correlate well with experimental results.

I. INTRODUCTION

With the advent of fiber optics technology has come the potential for use of this technology in high quality telecommunications systems. Such systems require sources, detectors, and fibers superior to those used in the present applications. The fibers in high quality systems must have low loss and dispersion and yet be economically produced. One factor influencing transmission loss in the fiber, particularly at splice locations,[1] is the diameter uniformity. Diameter uniformity is directly related to the manufacturing process and is influenced by the environment in which it is drawn[2] as well as the material from which it is drawn. In additon, large variations in diameter occur during the startup operation, resulting in a material loss of up to 10 percent of the potential fiber.

Much of this wastage can be eliminated and a high degree of fiber uniformity maintained by the judicious design and application of a feedback control on the fiber drawing process. Optimization of such a control requires identification of the distributed, nonlinear drawing process and a quantification of the sensitivities of the process to changes in process parameters.

1.1 The fiber drawing process

Optical fiber is formed by locally and symmetrically heating a cylindrical preform, typically 7 to 25 mm in diameter and 30 and 60 cm in length, to a temperature in the neighborhood of 2000°C. As the preform of diameter D_p is fed into the heat zone at a velocity V_p, the fiber is drawn from the molten material at a velocity V_f, as shown in Fig. 1. Due to the temperatures involved and the tolerances required, the fiber cannot be drawn through a die, and consequently the surface of the molten material is a free boundary whose shape is determined by an equilibrium between the velocity shear gradients and the restraining surface tension. The diameter of the fiber, D_f, is determined, then, by the principle of conservation of mass, which may be written as

$$D_f^2 V_f = D_p^2 V_p + \int_{-\infty}^{t} dw, \qquad (1)$$

where w is a random process representing mechanical and thermally-induced disturbances as well as the variations in diameter which occur while the process is establishing its equilibrium condition. The nature of the disturbing influences is illustrated in Fig. 2, where the fiber diameter is plotted for a 500-m length of fiber with constant D_p, V_p, and V_f. An additional source of diameter variation results from changes in preform diameter which are of a slowly varying nature. Once the process "equilibrium" has been established, the noise (diameter vari-

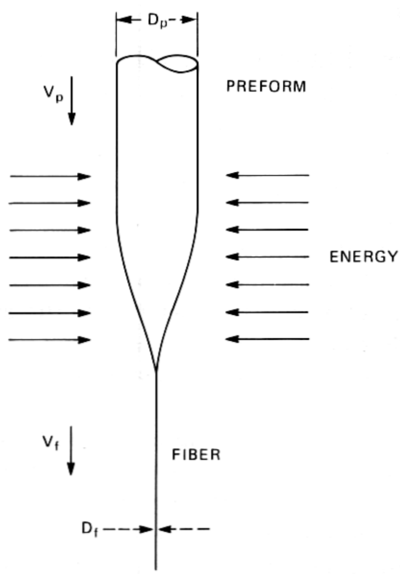

Fig. 1—Geometry of the optical fiber drawing process.

ation) which appears on the fiber can be characterized as band-limited noise riding on a slowly varying bias.

1.2 Identification of process dynamics

Since the fiber diameter is related directly to the two manipulable variables V_f and V_p by the mass conservation principle, and only indirectly to other manipulable variables such as heat source temperature or heat flux, these are the system variables through which a control signal can most effectively be coupled into the process. Experimental results show that the dynamic response of the fiber is two orders of magnitude faster with respect to the drawing velocity than to the feed velocity. Consequently, the control loop shown in Fig. 3 has been determined to be the most effective means of controlling fiber diameter.

The motor-drawing mechanism is typically a pinch-wheel device in which one wheel is driven by a DC motor. The dynamic response of the mechanism used in this investigation is modeled as

$$\frac{V_f(s)}{u(s)} = \frac{0.31}{0.0045s^2 + 0.030s + 1} \frac{\text{meters/second}}{\text{volt}}, \tag{2}$$

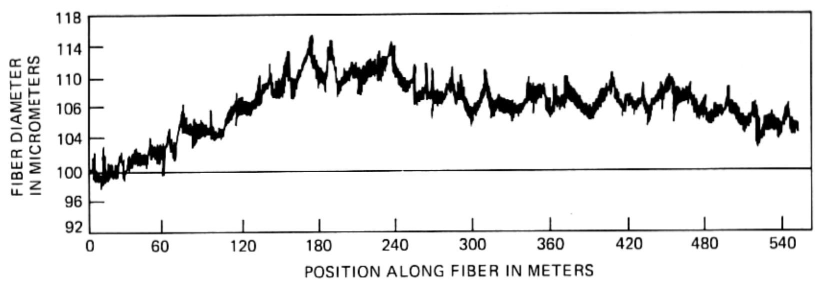

Fig. 2—Diameter profile of uncontrolled fiber with constant feed and drawing speeds.

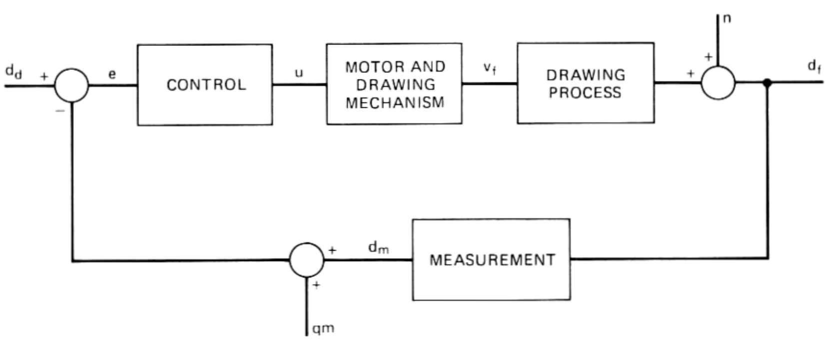

Fig. 3—Block diagram of diameter feedback control loop.

where u is the control signal to the motor. The response of the motor-drawing mechanism is dominated by the inertia of the pinch wheels, while the drawing tension on the fiber has little effect on the response of this component.

The drawing process is a nonlinear, distributed process. Since the diameter is to be regulated about a fixed set point, a linear perturbation model can be used to represent incremental system dynamics. The parameters for this model, as well as the model structure, must be determined experimentally. To obtain response characteristics, the drawing mechanism is excited with a sinusoidal perturbation and the change in draw speed and fiber diameter are measured. Using the technique of Fourier filtering,[3] the relative gain and phase of the drawing mechanism and the process diameter response at each excitation frequency can be determined, and a model can be constructed from the data. It is found that, for nominal velocities up to 1 m/s, the process can be modeled by

$$d_f(s) = \frac{D_f/2V_f}{a_p s^2 + b_p s + 1} v_f(s) + n(s), \tag{3}$$

where d_f and v_f now represent incremental changes in fiber diameter and velocity, D_f and V_f represent the nominal process values, and a_p, b_p are the parameters representing process dynamics. The source of diameter variations, n, is a band-limited, Gaussian process with $E\{n\} = 0$, $E\{n^2\} = \sigma_n^2$. The parameters a_p and b_p are sensitive to certain process parameters and insensitive to others. They have been found to be insensitive to preform diameter, fiber diameter, draw velocity, and the temperature of the heat source over the ranges

$$\begin{aligned} 1950°\text{C} &< T_s < 2150°\text{C} \\ 7\ \text{mm} &< D_p < 19\ \text{mm} \\ 0\ \text{m/s} &< V_f < 1\ \text{m/s} \\ 80\ \mu\text{m} &< D_f < 125\ \mu\text{m}. \end{aligned}$$

The response is generally insensitive to changes in preform feed velocity over the range of speeds commensurate with the above values of D_f, V_f, D_p, but is known to change for draw velocities higher than 1 m/s. The dynamic response parameters are also sensitive to the length of the heat zone as shown in Fig. 4. The long heat zone was obtained in a furnace and the short heat zone with laser heat source.[4]

Due to physical limitations, the fiber diameter is measured at some point below the heat zone. Using a forward scattering interference fringe counting technique,[5] the fiber can be measured with an accuracy of 0.25 μm at a rate of 1000 measurements per second. Due to the high measurement rate, the measurement process has no dynamic response to contribute to the loop dynamics. As a result of the digital fringe

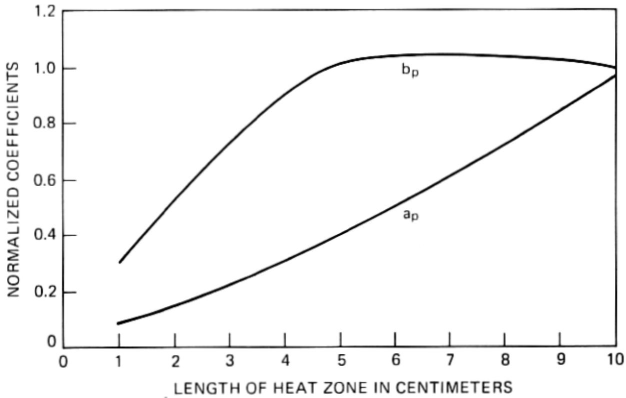

Fig. 4—Variation of process parameters with the length of heat zone.

counting technique, however, a quantization noise, q_m, perturbs the control loop. The effect of q_m can be seen from Fig. 3 to be equivalent to a loop with perfect measurement and a set point which changes $\pm q_m$ at random times. The effect of the noise on the fiber diameter depends upon the rate at which the step changes occur. Experience has shown that the quantization noise becomes significant only when the standard deviation of other system disturbances have been reduced to the level q_m and represents a lower bound of achievable performance for the drawing system.

Since the measurement process must be physically located beneath the heat source, it must also be located some distance away from the point, or region, where the diameter of the molten zone changes in response to variations in the drawing velocity. The distributed nature of the process leads to the expectation that the response to a step change in V_f would result in a distributed change in the boundary of the molten material, as well as a change in fiber diameter. Experience indicates that small perturbations in draw velocity result in diameter variations in the molten zone which are confined to a very small region, near the point where the fiber is formed. Consequently, a "point" at which the molten material changes diameter in response to changes in draw velocity can be defined, and a measurement time delay, T, results which relates the delay distance and nominal drawing velocity.

The resulting model of the measurement process is

$$\frac{d_m(s)}{d_f(s)} = 0.040\ e^{-sT}\ \text{volts}/\mu\text{m}, \tag{4}$$

where the quantization noise has been reassociated with the diameter set point. The delay time is typically 0.04 to 0.10 second for a drawing velocity of 1 m/s.

II. DESIGN OF THE CONTROL

The control problem is to reduce the effects of the noise source, n, upon the fiber diameter d_f. The relationship between the two quantities when the feedback loop of Fig. 3 is used is given by

$$\frac{\delta\, d_f(s)}{n(s)} = \frac{1}{1 + G(s)} = H(s), \tag{5}$$

where $\delta\, d_f(s) = d_f(s) - d_d$, and $G(s)$ is the forward transfer function

$$G(s) = \left.\frac{d_f(s)}{v_f(s)}\right|_{n=0} \times \frac{v_f(s)}{u(s)} \times \frac{u(s)}{d_m(s) - d_d} \times \frac{d_m(s) - d_d}{d_f(s)}. \tag{6}$$

The form of the control circuit is

$$\frac{u(s)}{d_m(s) - d_d} = \frac{K_c}{s}\frac{a_c s + 1}{b_c s + 1}. \tag{7}$$

The integral term is required to remove the slowly varying components of the noise source, and the lead-lag term is used to shape the response curve. Using this control circuit and the component responses (2) to (4), the response curve

$$\left|\frac{\delta d_f(s)}{n(s)}\right|^2_{s=j\omega}$$

as a function of ω is shown in Fig. 5. The response curve shows that low frequency disturbances can be effectively suppressed. There is a loss of control effectiveness as frequency increases to a point where the control loop has no effect upon the disturbances. In the region of the corner frequency ω_c, the noise is amplified. Figures 5 and 6 illustrate the role of the control circuit parameters upon the response curve. The suppression of low frequency variations is affected only by the loop gain. The degree of response peaking around the corner frequency is affected by the gain as well as the lead-lag network parameters. Some peaking resulting from high loop gain must be allowed, since it can only partially be compensated for by adjustment of a_c and b_c. It has been found that +2 dB is an acceptable peak gain for the response curve, from the standpoint of the stochastic disturbances.

The performance of the control loop can be optimized by choice of the parameters K_c, a_c, b_c to minimize the performance index

$$J = \int_0^\infty |H(j\omega)|^2\, d\omega, \tag{8}$$

subject to the constraint

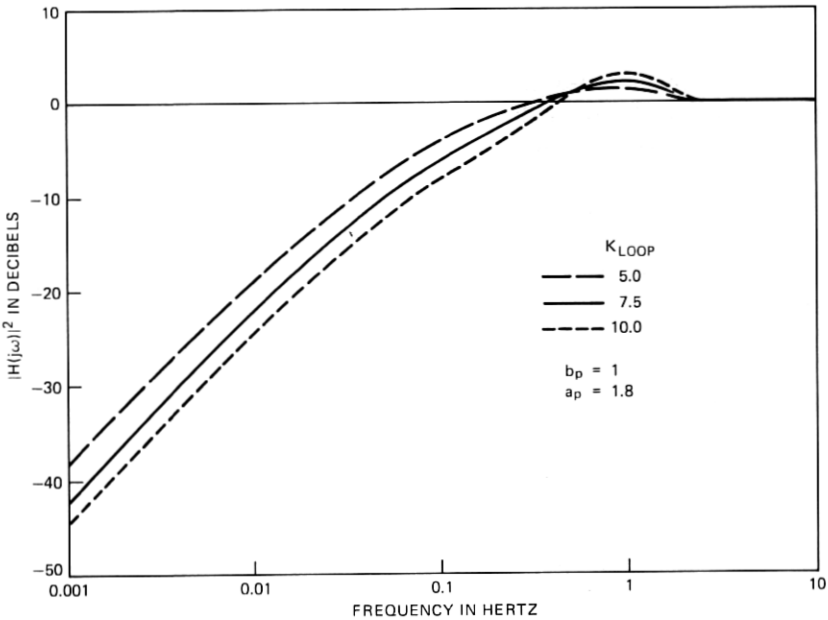

Fig. 5—Sensitivity of control loop performance to loop gain.

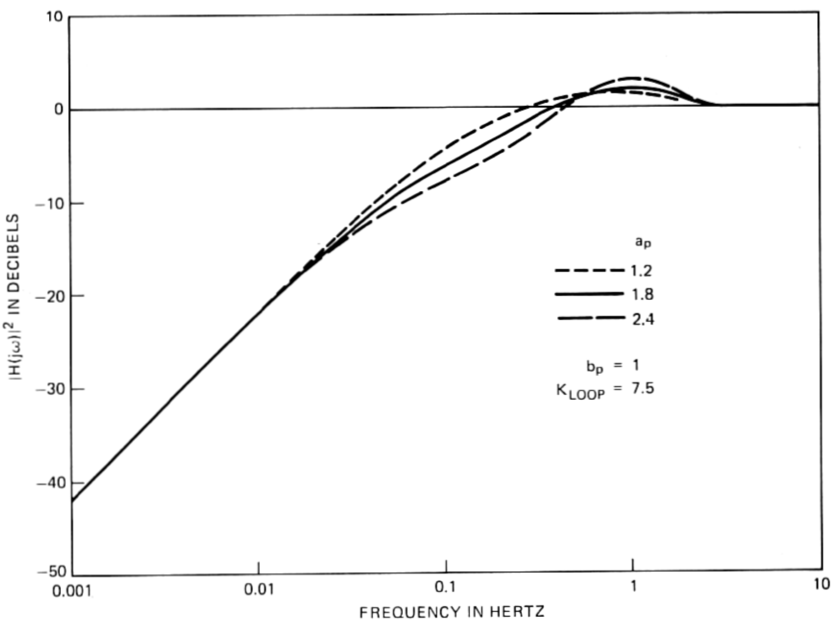

Fig. 6—Sensitivity of control loop performance to lead-lag compensation.

$$\left\{\omega \overset{\max}{=} \omega_0 : 0 < \omega < \infty\right\} \quad |H(j\omega)|^2 \le 2 \text{ dB}. \tag{9}$$

The performance index (8) is a measure of the relative noise power transmitted to the fiber diameter through the control loop. Minimization of (8) is equivalent to minimizing

$$J_1 = E\left\{\delta d_f^2(t)\right\}, \quad |n(\cdot)| = 1, \tag{10}$$

for $n(\cdot)$ modeled by white noise. This relationship is established by the relationship between the power spectral densities

$$\Gamma_{d_f} = |H(j\omega)|^2 \Gamma_n, \tag{11}$$

resulting in

$$J_1 = \int_{-\infty}^{\infty} \Gamma_{d_f} d\omega = 2\sigma_n^2 J. \tag{12}$$

Due to the constraint (9), which seeks to reduce the effect of periodic disturbances upon the loop design, optimization is most effectively performed in the frequency domain.

The optimum performance for a typical set of drawing system parameters is shown in Fig. 7, for the case of two measurement delay times. Referring to Figs. 5 to 7, of all the system parameters, the effectiveness of the control loop is most sensitive to loop gain and the measurement delay time. Examination of the phase characteristics of the system components reveals that over the range of controllable disturbances the measurement delay contributes the largest phase lag to the loop dynamics. Consequently, it is imperative in drawing system design to minimize the measurement delay. The loop response is sensitive to loop gain because the destablizing tendencies of the measurement delay pull the root loci toward the $j\omega$ axis.[6]

To determine the effectiveness of the control, account must be taken of both the process-related noise, n, and the measurement quantization noise q. The fiber diameter variation is related to these two quantities in the frequency domain by the expression

$$\delta d_f(s) = H(s)n(s) + G(s)H(s)q(s). \tag{13}$$

The power spectral densities are related by

$$\Gamma_{d_f}(\omega) = |H(j\omega)|^2 \Gamma_n(\omega) + |F(j\omega)|^2 \Gamma_q(\omega), \tag{14}$$

where $F(s) = G(s)H(s)$. The noise generating process associated with the fiber drawing process has been experimentally determined and can be modeled as

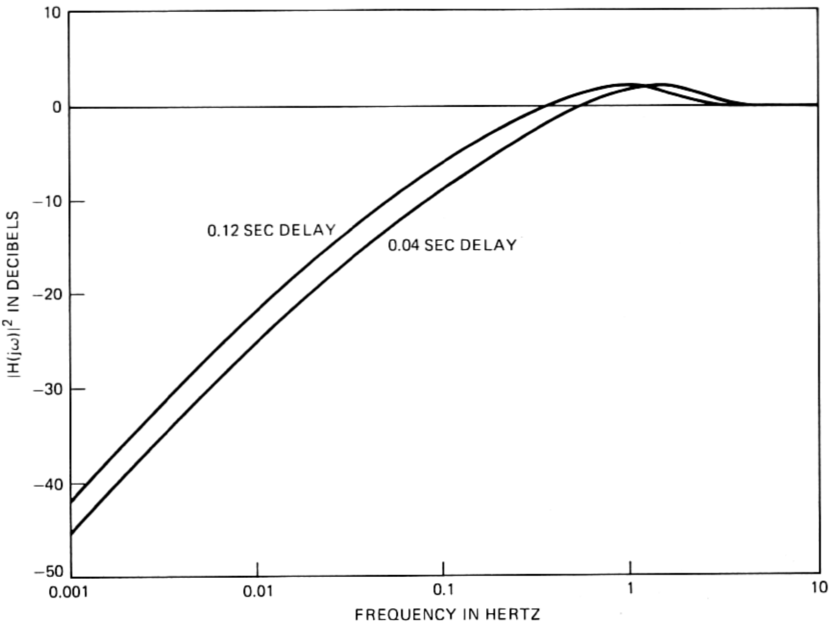

Fig. 7—Sensitivity of control loop performance to measurement time delay. System parameter values are $a_p = 0.0032$, $b_p = 0.080$.

$$\Gamma_n(\omega) = \frac{\frac{\omega_n}{\pi}\sigma_n^2}{\omega^2 + \omega_n^2}, \tag{15}$$

where ω_n is the corner frequency of the noise spectrum.

The quantization noise can be modeled as a random telegraph signal[7] for which the power spectral density function is

$$\Gamma_q(\omega) = \frac{\lambda q_m^2}{\lambda^2 + \pi^2\omega^2}, \tag{16}$$

where λ is the mean number of switchings per unit time. The control function can be approximated as

$$|H(j\omega)|^2 = \frac{\omega^2}{\omega^2 + \omega_c^2}$$

$$|F(j\omega)|^2 = \frac{\omega_c^2}{\omega^2 + \omega_c^2}, \tag{17}$$

where ω_c is the corner frequency of the closed loop transfer function. The variance of the fiber diameter is then determined by computing the auto-covariance function[8] with zero lag:

$$\sigma_d^2 = \frac{1}{1 + \frac{\omega_c}{\omega_n}} \sigma_n^2 + \frac{1}{1 + \frac{\lambda}{\pi \omega_c}} q_m^2 . \quad (18)$$

Equation (18) illustrates an interesting tradeoff. Whereas it is desirable to reduce the process-induced fiber diameter variation by increasing ω_c insofar as possible, it is done at the expense of allowing additional quantization noise to affect the fiber diameter. The latter source may only be reduced by increasing the resolution of the measurement system.

Experimentally, the effectiveness of the control loop is illustrated in Fig. 8 by the distribution of mean and standard deviations measured on 500-m lengths of controlled and uncontrolled fibers, after the process has reached equilibrium. The mean diameter of the uncontrolled fibers varies over a 4-μm range due to variations in preform diameter, nominal drawing velocities, etc. Using the feedback control, the mean diameter is held to within 0.1 μm of the set point. This deviation from set point is due, in part, to the 0.25 μm measurement resolution. The standard deviations of uncontrolled fibers are distributed over a wide range with the majority of the samples having a

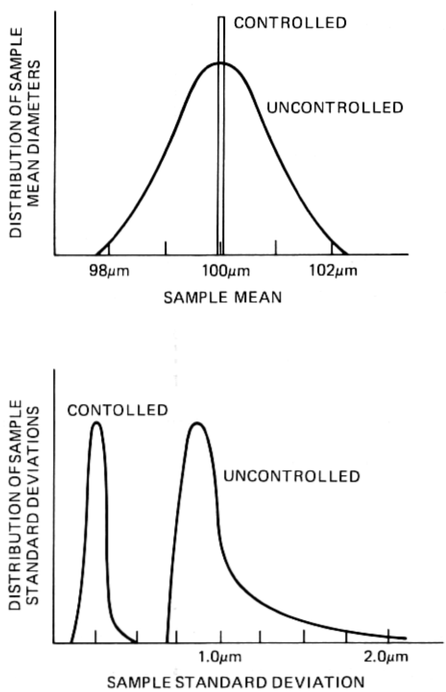

Fig. 8—Measured statistical characteristics of controlled and uncontrolled fibers.

standard deviation of 0.8 μm. Use of the feedback control improves fiber quality with the result that standard deviations below 0.30 μm can be repeatably attained.

Using experimentally determined system parameters, the controlled variance of the fiber diameter given in eq. (18) can be compared to experimental values. If $\omega_n = 0.02$, $\sigma_n = 0.8$ μm, $q_n = 0.25$ μm, $\lambda = 5$, and from Fig. 7 $\omega_c = 0.2$, eq. (18) yields a theoretical standard deviation of 0.27 μm. This value is very close to the median of the experimentally measured sample standard deviations.

III. CONCLUSION

Optimization theory has been applied to the optical fiber drawing process, resulting in a diameter feedback control loop which effectively reduces fiber diameter variations. In addition to examining the sensitivities of control loop performance to system component parameters, the sensitivities with respect to the noise sources were also examined. The expression derived to describe system performance with respect to process-induced diameter variation and measurement quantization noise showed good agreement with experimental results.

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