Copyright © 1981 American Telephone and Telegraph Company
THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 60, No. 1, January 1981
Printed in U.S.A.

Absenteeism of Operators: A Statistical Study with Managerial Applications

By Y. VARDI

(Manuscript received July 3, 1980)

The need to assess attendance behavior often arises, at the line-management level, when an employee is considered for a transfer or a promotion. A sound assessment should, of course, take into account the statistical behavior and distributional properties of absenteeism. The first part of this paper is a detailed statistical analysis of attendance records of a sample of 112 telephone operators. We use exploratory and confirmatory statistical techniques to suggest possible theoretical models that can parsimoniously describe the behavior of the variables of interest. Methodological difficulties that often arise in cross-sectional studies and are caused by biased sampling are pointed out and treated. We explore the relation between age and attendance; in particular it is evident that (for this data set) the frequency of "incidental" absences tends to decrease with age, and that the duration of "disability" absences tends to increase with age. In the second part of the paper we suggest an attendance evaluation method based on the statistical analysis of the first part. The method is designed to reflect the current-year attendance as well as a longer-run attendance behavior, interpreted as a personal characteristic, and its properties are demonstrated via examples.

I. INTRODUCTION AND SUMMARY

Management policy regarding absenteeism has two major aspects: a global one spelled out in the various company rules and applied evenly to all employees and a local one, generally less formal, in which line management is concerned about individual's attendance. A question like how many "paid days off" per year an employee should be allowed for unexpected and unavoidable absences is often a subject for union negotiations and is a good example of what we mean by management's global policy. On the other hand, the need to decide whether a given operator has exhibited satisfactory attendance arises when that operator is considered for a transfer or promotion and is a good example of management's local policy. Whether local or global, a sound policy should consider the statistical characteristics and the distributional properties of absenteeism.

Section II gives a detailed statistical analysis of absenteeism (on the basis of a sample of 112 telephone operators). Such an analysis can enhance our understanding of absenteeism, and can be used as a basis for answering questions of the type described above. For example, the distribution of the duration of incidental absences (Section 2.6) and the frequency of incidental absences per year (Fig. 6, or more generally Section 2.6), can be used to answer how many paid days off per year an employee should be allowed. An answer based on such a statistical analysis is more likely to satisfy the true needs of the average employee than any decision which makes no reference to the distributional properties of absenteeism. (Note that the Bell System's allowance for personal time started after our data were taken.)

In Section III we suggest a method for assessing absenteeism, based on our statistical findings of Section II, and discuss its properties. The analysis of Section 2.4 indicates that one year is too short a period to decide whether an operator is intrinsically "good," "bad," etc., regarding attendance. Thus, if management is interested in assessing attendance as a personal characteristic, the follow-up period needs to be longer than one year. The conflict between the viewpoint that past years' attendance should not affect the present evaluation (for any type of performance rating), and the statistical observation that one year is too short a period to assess attendance, are resolved by basing our evaluation method (Section III) on two indices. One index rates the current year attendance, while the other index rates attendance behavior as a personal characteristic, and it depends on the attendance during the three most recent years.

Various aspects of absenteeism have been studied in recent years (particularly in the fields of labor relations, applied and industrial psychology, and management science). The major contributions of our paper to this area of research, and the relation to other studies, as we see them, are summarized below:

(*i*) We suggest an intuitively appealing method for assessing absenteeism, which reflects the current year attendance, as well as attendance behavior, as personal characteristics. With suitable modifications, the method is adaptable to other occupations.

(*ii*) Often in cross-sectional studies a certain sampling bias is introduced because the sampling is done along the time axis. The detailed analysis of Section 2.4 shows how to identify this bias (and in some instances how to estimate the underlying model in the presence of this bias). This technique can be of use to other researchers analyzing cross-sectional data. (A more detailed paper devoted entirely to statistical questions that arise in the analysis of this type of data is forthcoming.)

(*iii*) In the course of our analysis in Section II, we use some graphical techniques that are common tools in exploratory data analysis, but are not yet familiar to most social scientists. These tools are useful in the tedious chore of identifying patterns and models in large data sets, and we hope that exposing them to researchers in the social sciences will help make them popular.

(*iv*) Throughout the paper we distinguish between two types of absences, disability and incidental (definitions in Section II). This classification enables us to shed some light on the relation between absenteeism and age. Several authors have tried to relate absenteeism to age and conflicting findings are often reported. Indeed, in a recent study based on a survey of blue-collar production workers, Nicholson et al. (Ref. 1, pp. 319–320) report on a marked inverse relation (especially for male employees) between absence frequency and age which, as they point out, contrasts the conclusions of Porter and Steers[2] (a review of the literature on the subject of absenteeism and turnover) and Cooper and Payne,[3] that absence frequency increases with age. Our data suggest that for telephone operators (all of whom in our sample are females) the truth lies somewhere in the middle. That is, the frequency of incidental absences is higher for younger operators, while the frequency and duration of disability absences is higher for older operators.

For readers who are interested in aspects of absenteeism that are not directly related to this work (such as economic, psychological, etc.), we include a supplementary reference list (which is by no means complete).

II. DATA ANALYSIS AND STATISTICAL MODELING

2.1 Introduction

We distinguish between two types of absences: incidental absences (IA), which are usually short, more frequent, and (to a certain extent) controllable, and disability absences (DA), which are usually long, less frequent, and uncontrollable. Formally, a DA is any absence that lasts six or more days and is due to an illness (an exception is an on-the-job accident in which case the DA period can be shorter than six days); any other absence is defined as an IA. Periods of attendance at work will be referred to as showing up (SU) periods.

Our data are made up of attendance records of 112 New England Telephone operators, for variable periods $t_1, \cdots, t_{112}$. Out of the 112 records, 6 cover approximately 1 year (between 0.8 and 1.4 years), 63 cover approximately 2 years (between 1.6 and 2.4 years), and 43 cover approximately 3 years (between 2.5 and 3.1 years). Here we take a year to be 240 working days. The attendance records in our sample do not usually start, or end, at a beginning of a DA, IA, or SU period and thus two censored (i.e., incomplete) periods typically exist (these are usually SU periods) for each of the 112 records, one at each end of the record. This situation is demonstrated in Fig. 1, which gives a schematic example of an attendance record in our data. Note that holidays, weekends, vacations, etc., have been deleted from the time axis. The large proportion of censored SU periods, among the total number of SU periods, requires special attention and leads to an interesting analysis.

Frequency of absences, duration of absences, duration of SU periods, relations between absence and age, etc., are all parts of the complete picture of "attendance behavior" of operators. We analyze these variables below. In cases where our analysis suggests possible theoretical models that can adequately describe the behavior of the variables in question, we point out these models.

Our analysis suggests that operators older than 35 are different from operators younger than 35 with regard to certain aspects of absence behavior; for the sake of brevity, we refer to the first group as older operators and to the second group as younger operators.

2.2 Duration of IA's

A histogram of the duration of the 560 observed IA's is given in Fig. 2a. A simple theoretical model that fits these data to a remarkable degree of accuracy is

$$P[\text{duration of } IA = j] \equiv P_j = \begin{cases} p & \text{if } j = 1, \\ (1-p)2^{-j+1}, & \text{if } j = 2, 3, \cdots, \end{cases} \tag{1}$$

with $p = \hat{p} = 346/560 = 0.6179$ (note that if $x_1, \cdots, x_n$ is a random sample with a probability density function (pdf) (1), then the maxi-

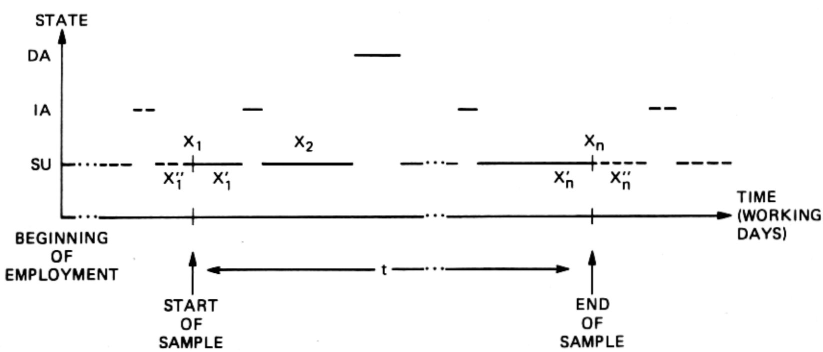
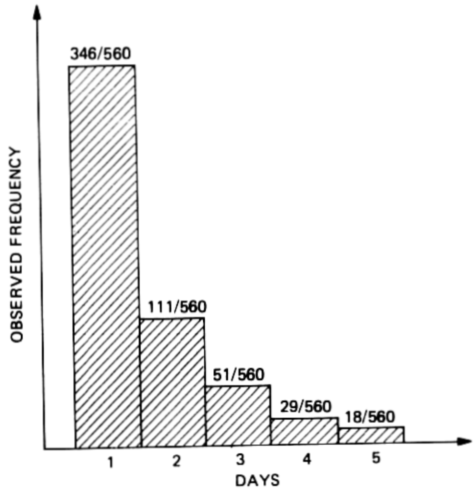


Fig. 1—A schematic description of an attendance record. Note that the first and last SU periods (X_1 and X_n) are censored (only X_1' and X_n' are recorded in the sample).



(a)

DURATION OF IA (DAYS)	1	2	3	4	5	6 OR MORE
OBSERVED FREQUENCY	0.6179	0.1982	0.0911	0.0518	0.0321	0.0089
P_j ACCORDING TO (1)	0.6179	0.1911	0.0955	0.0478	0.0239	0.0239
DIFFERENCE	0	0.0071	−0.0044	0.0040	0.0082	−0.0150

DURATION OF IA (DAYS)	1	2	3	4	5 OR MORE
OBSERVED COUNT	346	111	51	29	23
EXPECTED COUNT, 560 × P_j	346	107	53	28	27
DIFFERENCE	0	4	−2	1	−4

(b)

Fig. 2—(a) A histogram of the durations of IA's (avg = 1.72, stdv = 1.16). (b) Comparison between the pdf of (1) and the observed durations of IA's.

mum likelihood and the minimum variance unbiased estimator of p is $\hat{p} = \sum_{i=1}^{n} I[X_i = 1]/n$, where $I[\ ]$ denotes the indicator function; in our case this gives $\hat{p} = 346/560$). Figure 2b compares the model of (1) with the observed data, and the adequacy of the model is transparent. Nevertheless, it is interesting to note that a chi-square goodness of fit test with size α does not reject the hypothesis that the durations of IA's have the pdf (1), even when α is as high as 0.90!

A random variable, say X, with the pdf (1) has the following interesting property:

$$P[X = k + j \mid X > k] = 2^{-j}, \qquad j = 1, 2, \cdots, \quad k = 1, 2, \cdots. \tag{2}$$

The interpretation of this property, when X stands for the duration of an IA, is the following: On the second day of an incidental absence, the employee tosses a coin; if the result is heads the employee returns to work on the next day, otherwise he remains absent. The experiment is repeated daily until the first time the result is heads, in which case the employee returns to work on the following day. Should one try to interpret this interesting property, exhibited by the data, in terms of human behavior in regard to short absences?

We remark that the distribution of the duration of IA's for younger operators is approximately the same as for older operators and neither deviate much from (1).

2.3 Duration of DA's

The following are summary statistics for the 78 DA occasions in our data:

lower quartile = 9.0, median = 13.5 upper quartile = 41.0,

mean (with six most extreme observations removed) = 26.1,

standard deviation (with six most extreme observations removed)

= 24.2.

Out of the 78 observations, 18 were incurred by operators younger than 35. Figure 3 compares, by means of box plots,[4] the distributions of the duration of DA's in the three different cases; younger operators (18 DA occasions), older operators (60 DA occasions), and the combined

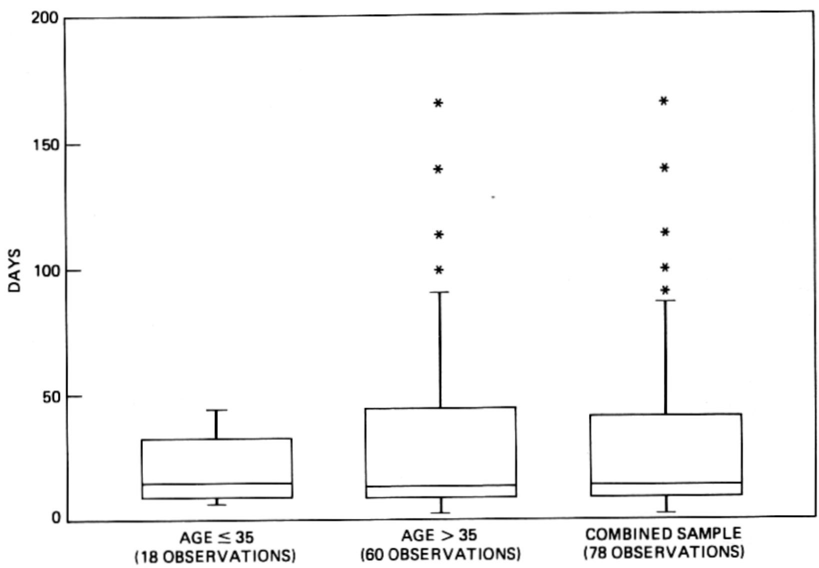


Fig. 3—Box plots for the duration of DA's.

sample (78 DA occasions). The lower and upper sides of each box are the lower and upper quartiles, respectively, and the segment inside the box is the median. If d denotes the distance between the quartiles, then the box whiskers are drawn to the nearest data value with $1.5d$ from the nearest quartile. Points lying outside this range are plotted individually. The figure suggests that long DA's are more frequent among the older operators. For instance, the upper quartile for the older operators is 44.0, with the most extreme observation being 165, while the corresponding figures for the younger operators are 32.0 and 44.0. This difference cannot be accounted for by differences in the total sampling durations, because the distribution of the t_i's is approximately the same for the two age groups.

2.4 Duration of SU periods

We use Fig. 1 as a vehicle to explain some basic concepts regarding the censoring of SU periods. Suppose the length of each individual SU period (X_i in Fig. 1) is distributed according to the cumulative distribution function (cdf) $F(u)$. Then, since the probability of any individual period that covers the point t_0 is directly proportional to its length u, the distribution function of the length of the interval that covers t_0 (X_1, in Fig. 1) is $H(x) = \int_0^x u dF(u)/m$ (m is a normalizing constant that is equal to the mean of F). Given X_1, however, the distribution function of X_1', which is the observable part of X_1, is uniform on the interval $[0, X_1]$ so that (using Bayes' theorem) the unconditional distribution of X_1' is

$$G(y) \equiv P[X_1' \leq y] = m^{-1} \int_0^y [1 - F(u)]\, du. \tag{3}$$

Since (3) is usually derived in the context of renewal processes, in which case an assumption about the independence of different X_i's (SU periods in our application) is built in, it is important to note that this assumption is not used in the derivation of (3) (cf. Ref. 5, p. 66), and therefore it is not assumed in our discussion. In the analysis that follows, however, we assume (unless otherwise stated) that SU periods of different operators have the same cdf F, as long as they are in the same age group.

It is clear that the argument leading to (3) applies also to X_n', so that (3) is the distribution of the censored SU periods. An important property of (3) is

$$G = F \quad \text{if, and only if,} \quad F \text{ is an exponential distribution}$$
$$[\text{i.e.,} \quad F(x) = 1 - e^{-x/\mu}, \quad x > 0, \quad \mu > 0]. \tag{4}$$

Or, in words,

the censored SU's and the completed SU's have the same distribution if, and only if, the distribution of the SU's is exponential. (4′)

2.4.1 Analysis for operators younger than 35

For the younger operators, Fig. 4a compares censored SU's with completed SU's, by means of a Q–Q plot (see Ref. 6, chapter 6). The deviation of the plot from a 45-degree line through the origin is not very large and for practical purposes one can assume that the censored SU's and the completed SU's follow the same distribution. Being more formal, if we test the hypothesis that the two samples have the same distribution, using a Wald–Wolfowitz runs test, we observe 92 runs while the mean and standard deviation under the null hypothesis are 99.0 and 6.0, respectively, so that the hypothesis is not rejected at significance levels of 0.12 or less. Thus, from (4′), we are led to the conclusion that the distribution of the SU's is exponential (or, at least, that this is an adequate description of the data). Figure 4b is a comparison of the combined SU sample (censored and completed) versus quantiles from exponential distribution. The striking closeness to linearity of this plot strongly supports the conclusion that the SU's are exponentially distributed. The estimated mean of the combined sample is 60.2 days, and the standard deviation is 61.2 (which is very close to the mean, as is to be expected from a sample from exponential distribution). In summary, for the purpose of fitting a parsimonious

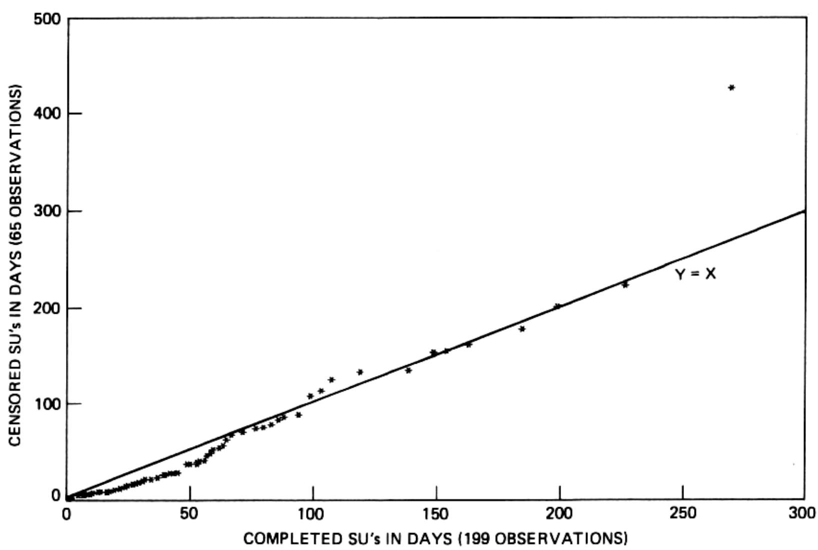


Fig. 4a—Q–Q plot of censored SU's (Y axis, 65 observations) versus completed SU's (X axis, 199 observations), for operators younger than 35.

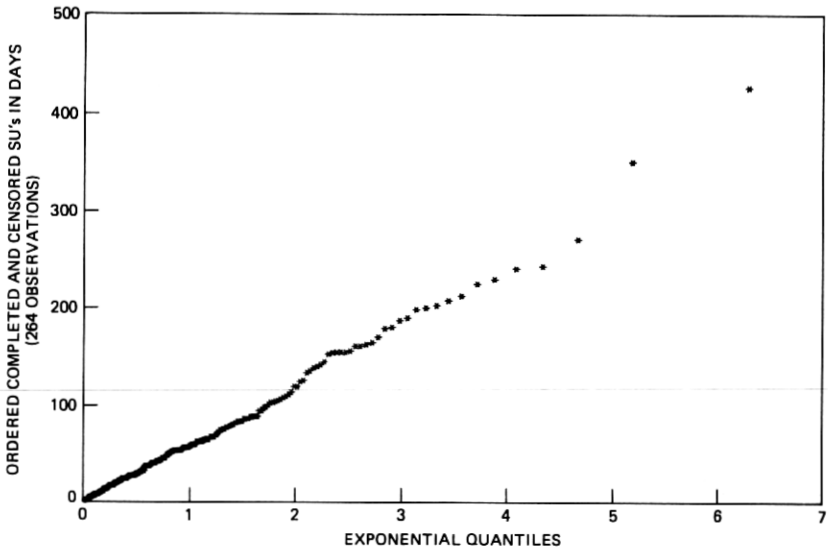


Fig. 4b—Q–Q plot of completed and censored SU periods (Y axis, 264 observations) versus $-\log(1-u)$ (quartiles of standard exponential distribution), for operators younger than 35.

model, one can assume that the periods between consecutive absences, for operators of age 35 or less, follow an exponential distribution with mean = 60 days.

2.4.2 Analysis for operators older than 35

Applying a similar analysis as in the previous case, we point out an interesting "data paradox" exhibited by the two SU samples (censored and completed), and we give possible explanations for this paradoxical behavior of the data.

Figure 5a compares the censored SU's with the completed SU's. The deviation of the Q–Q plot from the 45-degree line through the origin is marked, and it is evident, therefore, that the completed and the censored SU's have different distributions. Specifically, the censored SU's appear to be stochastically bigger than the completed SU's (the Q–Q plot is on or above the 45-degree line through the origin) and for comparison we look also at their summary statistics:

(lower quartile, median, upper quartile, mean, stdv) =

(20.0, 46.0, 88.0, 66.3, 71.2) for the completed SU's, and

(21.0, 47.0, 206.0, 113.3, 130.4) for the censored SU's.

In view of this situation and the assumption that SU's of different operators have the same cdf, (4′) suggests that the distribution of the

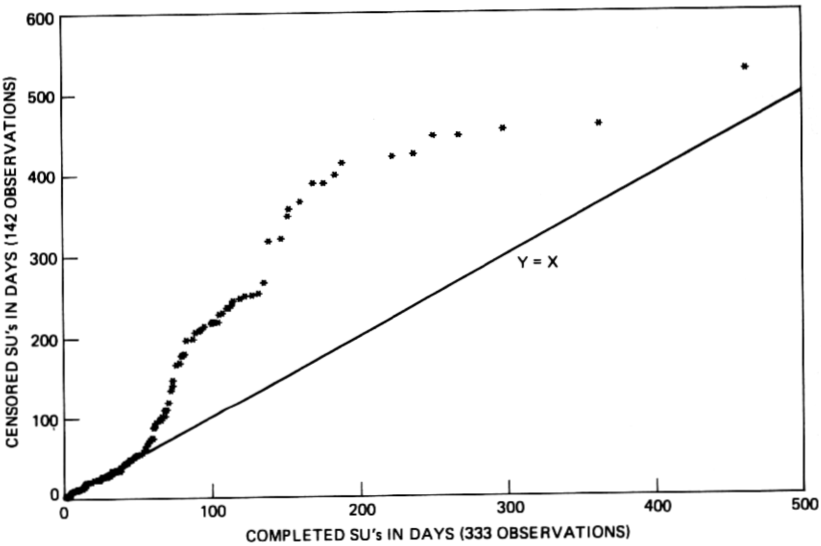


Fig. 5a—Q–Q plot of censored SU's (Y axis, 142 observations) versus completed SU's (X axis, 333 observations), for operators older than 35.

completed SU's cannot be exponential. Figure 5b, however, in which we compare the completed SU's with exponential quantiles, points in the opposite direction. The closeness to linearity of the Q–Q plot (with the exception of the upper 17 points) suggests that the completed SU's do follow an exponential distribution (or perhaps an exponential with a 5-percent contamination).

We give two possible explanations to this data paradox. The first is that intrinsic differences in absence behavior might exist among the 79 operators of age greater than 35, so that any attempt to fit a single cdf to the SU periods of these operators is meaningless [in mathematical language this means that, in eq. (3), different operators are associated with different distribution functions F, while we try to fit a single F], and a more complicated model is needed. One possible model is that operators can be naturally classified into classes according to their attendance behavior (good, bad, etc.). Nevertheless, the sampling periods (t_i's) in our data are not long enough to enable us to decide whether a given operator is intrinsically good, bad, etc., and thus we have not pursued this model.

In the second possible explanation, we show that a certain sampling-bias effect could have been the source of our data paradox. Suppose the cdf of SU periods, which is F of eq. (3), is a mixture of two cdf's, an exponential cdf with mean A, which is small relative to the sampling periods t_i, and a degenerate cdf which assigns a unit mass to a point B

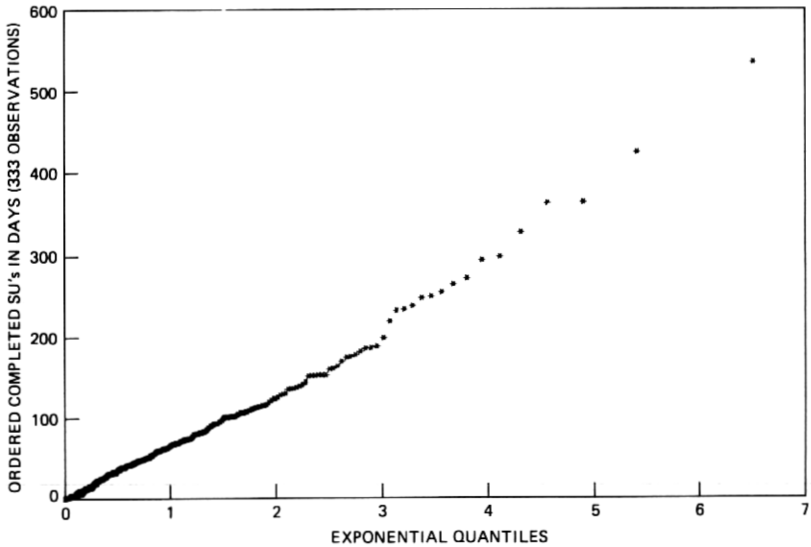


Fig. 5b—Q-Q plot of completed SU's (Y axis, 333 observations) versus $-\log(1-u)$ (quartiles of standard exponential distribution), for operators older than 35.

which is big relative to the t_i's. This means that over a long period of time, a certain proportion, say α, of the SU's have an exponential distribution, while the other SU's last a fixed length B. Now any sampling period of length t satisfying $A \ll t < B$ cannot possibly contain a completed SU period of length B, so that all the completed SU's must be from the exponential population and only censored SU's could possibly be from the B population. In addition to being a model that accommodates the "paradoxical" behavior of our data, this model provides a useful framework for estimation. Under the model's assumptions

$$P[\text{SU} > x] \equiv 1 - F(x) = \alpha e^{-x/A} + (1-\alpha)I[x < B], \qquad (5)$$

where $I[\ ]$ denotes the indicator function, so that, using (5) and (3), the moments of the censored SU's are

$$E[\text{censored SU}]^n = \int_0^\infty X^n \, dG(x)$$

$$= \frac{\alpha(n+1)!A^{n+1} + (1-\alpha)B^{n+1}}{(\alpha A + (1-\alpha)B)(n+1)}, \qquad n = 0, 1, \cdots. \qquad (6)$$

Since our model assumes $A \ll t$ we have, to a good approximation,

$$E[\text{completed SU}]^n \approx \int_0^t x^n A^{-1} e^{-x/A} \, dx \sim n!A^n, \qquad n = 0, 1, \cdots, \qquad (7)$$

and therefore the moments method, applied to (6) and (7), gives

$$
\begin{aligned}
\hat{E}[\text{completed SU}] &= \quad 66.3 \quad = \hat{A}, \\
\hat{E}[\text{censored SU}] &= \quad 113.3 \quad = \frac{2\alpha\hat{A}^2 + (1-\alpha)\hat{B}^2}{2(\alpha\hat{A} + (1-\alpha)\hat{B})}, \\
\hat{E}[\text{censored SU}]^2 &= 29841.05 = \frac{6\alpha\hat{A}^3 + (1-\alpha)\hat{B}^3}{3(\alpha\hat{A} + (1-\alpha)\hat{B})},
\end{aligned}
\tag{8}
$$

which yield the estimates $\hat{A} = 66.3$, $\hat{B} = 500.0$, $\hat{\alpha} = 0.96$.

Though the above model accommodates the type of behavior demonstrated by our data, so do other models based on a contaminated exponential distribution and the question of finding a model that fits our data well has not been answered yet. Toward this end we derived a nonparametric estimate of F, denoted $\hat{F}$, by tailoring the Kaplan–Meier estimator[7] to our application, in which each completed observation has to be counted with multiplicity two. (The exact details of

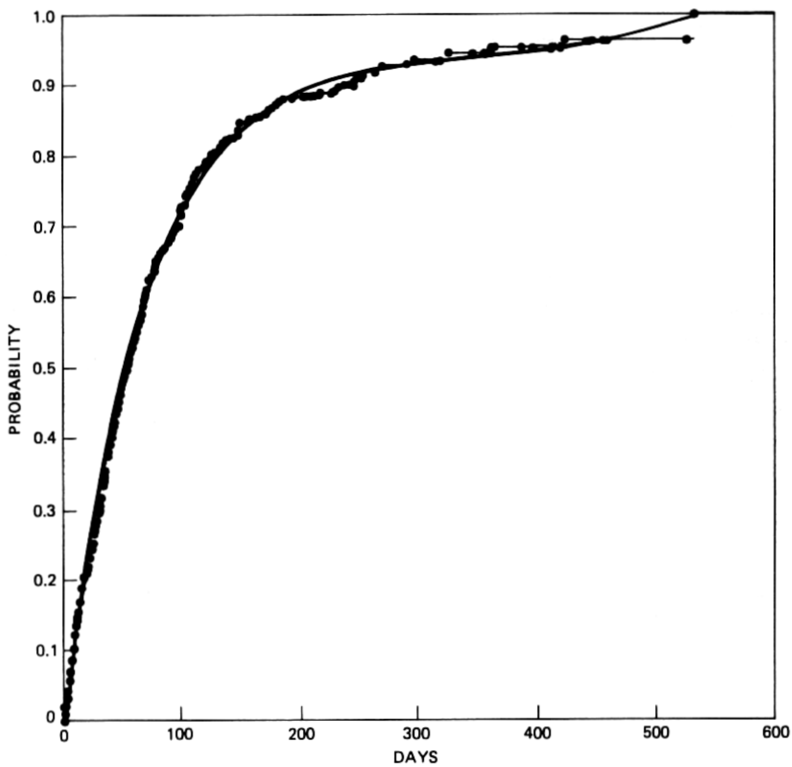


Fig. 5c—The dotted line is $\hat{F}(x) = \hat{P}_{\text{older}}[\text{SU} \le x]$ (using a modification of the Kaplan–Meier estimator, Section 2.4). The solid line is the contaminated exponential distribution of eq. (1).

this estimator will be discussed in a separate paper.) The result is given in Fig. 5c. The contaminated exponential model

$$F(x) = 0.94(1 - e^{-x/70}) + 0.06\left(\frac{x}{540}\right)^6 I[0 \le x \le 540], \qquad x \ge 0, \quad (9)$$

is superimposed on this figure, and it seems to fit the data rather well. Figure 5d compares (9) with $\hat{F}(x)$ by means of a Q–Q plot, and the closeness of the plot to the 45-degree line reassures us about the adequacy of the model.

The behavioral interpretation of (9) is that usually (i.e., 94 percent of the time) the duration of SU periods follows an exponential distribution with mean 70 days, while occasionally (i.e., 6 percent of the time) an SU period can be much longer (perhaps 500 to 540 days). We note that the average SU period, according to (9), is approximately 94 days, which is substantially bigger than the corresponding number for the younger operators (60 days).

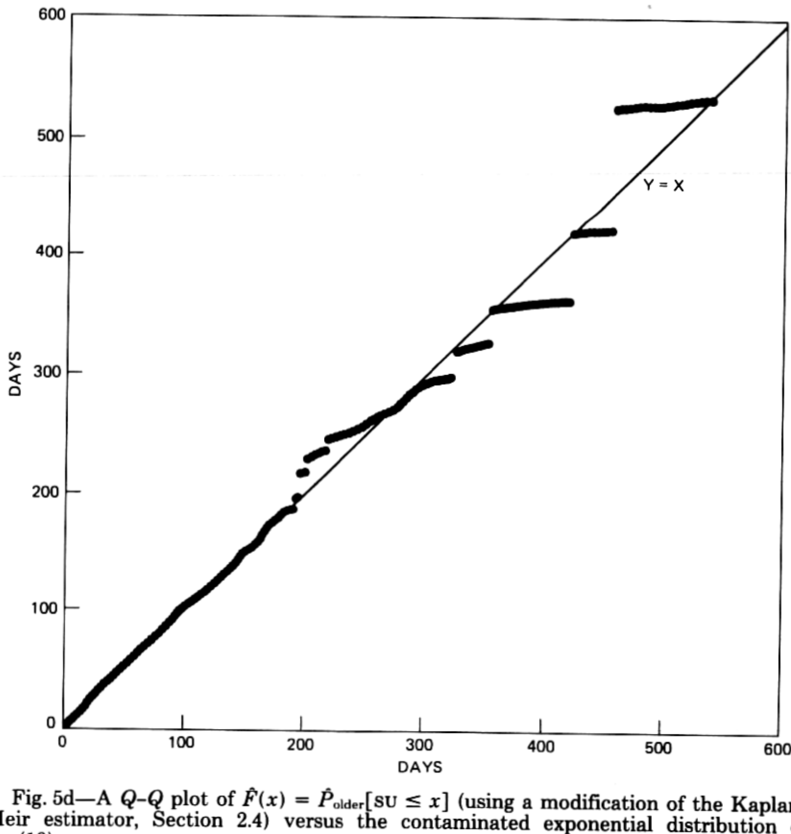


Fig. 5d—A Q–Q plot of $\hat{F}(x) = \hat{P}_{\text{older}}[\text{SU} \le x]$ (using a modification of the Kaplan-Meir estimator, Section 2.4) versus the contaminated exponential distribution of eq. (10).

An important aspect of our data-paradox, and the contaminated exponential model, is that it indicates that a follow-up period of one or two years is not sufficiently long for evaluating the attendance behavior of operators. This observation has implications to our discussion of evaluation procedures.

2.5 *Frequency of absences and total time lost (TTL) due to absences*

Figure 6 gives box plots of the frequency of IA's (occasions per year) for the two age groups and for the combined sample. (The nonoverlapping of the notches in the first and second boxes indicates a difference at the rough 5-percent significance level between the two medians.[4]) One can immediately see that younger operators tend to have substantially more IA's. Note again that this difference cannot be accounted for by differences in the total sampling durations, because the distribution of the t_i's is approximately the same for the two age groups.

The situation regarding DA's is somewhat reversed, as one can see from Tables I and II. For example, while the proportion of the younger operators in the sample is 29 percent, the proportion of the DA occasions incurred by them is only 23 percent. We also see in Table II that the ratio "TTL due to DA" to "TTL due to IA" is 0.022/0.021 for younger operators, while it is 0.045/0.014 for older operators. This, plus the fact that the probability distribution of the DA duration for older operators has a substantially longer tail than the corresponding quantity for younger operators (Fig. 3), explains the fact that the TTL

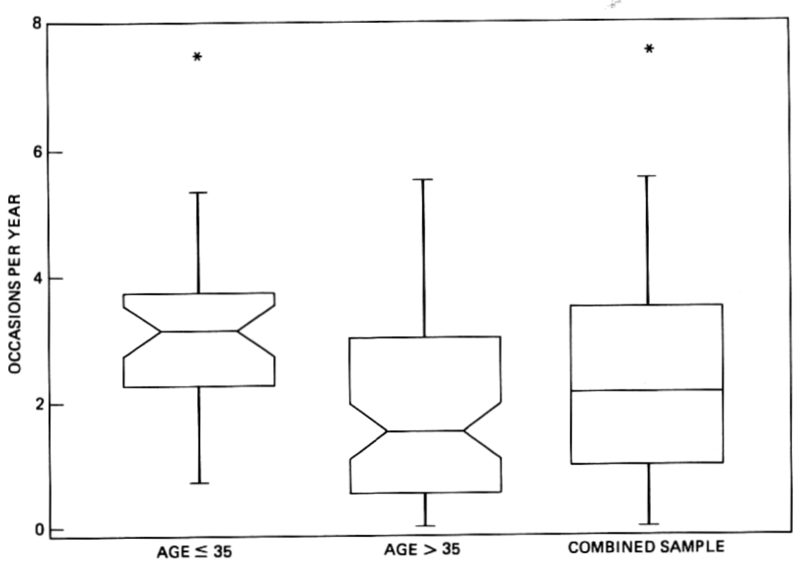


Fig. 6—Box plots for the frequency of IA (occasions per year).

Table I—DA occasions and age

	No. of Individuals with Occasions of DA	No. of Occasions of DA	No. of Operators in the Entire Sample
Age ≤ 35	13 (25%)	18 (23%)	33 (29%)
Age > 35	40 (75%)	60 (77%)	79 (71%)
	53 (100%)	78 (100%)	112 (100%)

caused by absences is somewhat higher for older operators (5.9 percent) than for younger operators (4.2 percent).

The first line of Table II shows that a period that starts at the end of the IA and ends at the end of the following IA (including the possible DA's) lasts, on the average, 72 days for younger operators, and 107 days for older operators.

2.6 The number of IA occasions (IAO) over a fixed period of time

For later applications we want to derive an estimate for the probability distribution of the number of IAO over a fixed period of time, for an arbitrary operator in our sample. To keep the analysis and the presentation simple we ignore, for the time being, the differences between younger and older operators. Later we will comment on the corresponding analysis when the difference in attendance between the two age groups is taken into account. It is well known (e.g., Ref. 5, p. 104) that under fairly weak assumptions about the statistical behavior of the periods between consecutive IA's, the quantity $\sqrt{t}[N(t)b/t]$ has a limiting distribution as $t \to \infty$. Here $N(t)$ denotes the number of IAO over a time period of length t and b is the average number of IAO per unit time. We take this theoretical model as a framework for

Table II—Age comparison of certain absence characteristics (DA = disability absence, IA = incidental absence, TTL = total time lost)

	Age ≤ 35	Age > 35	Combined Sample
$\frac{\text{total sampling periods*}}{\text{total no. of absence occasions}}$	$\frac{16601}{232} = 71.56$	$\frac{43439}{406} = 106.99$	$\frac{60040}{638} = 94.11$
$\frac{\text{no. of DA occasions}}{\text{no. of IA + DA occasions}}$	$\frac{18}{232} = 0.078$	$\frac{60}{406} = 0.148$	$\frac{78}{638} = 0.122$
$\frac{\text{TTL due to IA}}{\text{total sampling periods}}$	$\frac{342}{16601} = 0.021$	$\frac{620}{43439} = 0.014$	$\frac{962}{60040} = 0.016$
$\frac{\text{TTL due to DA}}{\text{total sampling periods}}$	$\frac{363}{16601} = 0.022$	$\frac{1941}{43439} = 0.045$	$\frac{2304}{60040} = 0.038$
$\frac{\text{TTL due to absences}}{\text{total sampling periods}}$	$\frac{705}{16601} = 0.042$	$\frac{2561}{43439} = 0.059$	$\frac{3266}{60040} = 0.054$

* Total sampling periods = the sum of all the observation periods across operators (i.e., sum of the t's of Fig. 1).

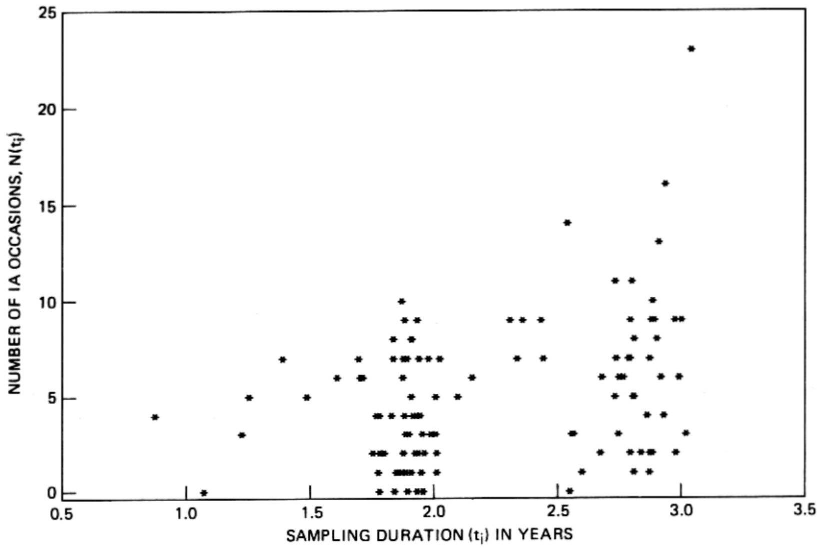


Fig. 7—A scatter plot of the number of IA occasions, $N(t_i)$ (Y axis) versus the sampling duration, t_i years (X axis), for the 112 operators.

producing estimates of the distribution of $N(t)$ for a given t. A scatter plot of $(t_i,\ N(t_i))$, $i = 1, \cdots, 112$, is given in Fig. 7, and one can see immediately that $VAR(N(t))$ increases with t (this is usually referred to as heteroscedasticity), as to be expected from our model. Note that

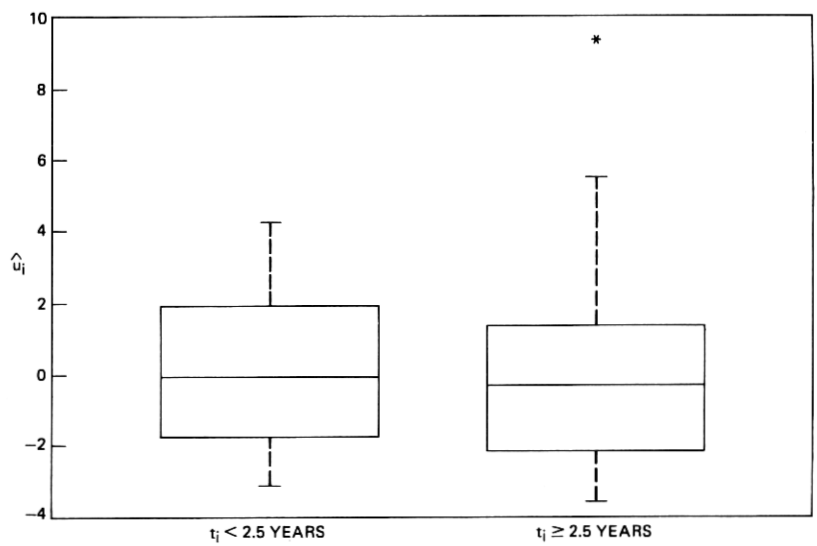


Fig. 8—Comparison of $\hat{u}_i = \sqrt{t_i}[N(t_i)/t_i - \hat{b}]$ for small and large values of t_i.

our model implies that the mean and the variance of $N(t)$ are approximately linear in t, for large values of t. The regression estimate of b in the model

$$N(t_i)/\sqrt{t_i} = b\sqrt{t_i} + U_i \qquad (10)$$

is $\hat{b} = 2.24$ (with a t value of 15.36). Figure 8 compares (by means of box plots) the residuals, $\hat{U}_i$'s, of the regression (10) for periods of length $t_i < 2.5$ years with the $\hat{U}_i$'s for periods of length $t_i \geq 2.5$ years. The choice of $t = 2.5$ as a cutoff point seems natural from the distribution of t's (see second paragraph of Section 2.1); other t's in the neighborhood of 2.5 gives similar results. The comparison shows that the heteroscedasticity of the data (Fig. 7) is eliminated and the $\hat{U}_i$'s can be considered as random variables satisfying $VAR\ \hat{U}_i = \sigma_u^2 > 0$, independent of t, so that the empirical distribution of the $\hat{U}_i$'s can be used to estimate the distribution of $N(t)$.

Theoretically, if (i) our assumptions (e.g., all operators behave according to the same probability law) were completely realistic and (ii) t were very large, then the distribution of U would be close to a normal distribution and we could use this fact to estimate the distribution of $N(t)$. Since, however, neither (i) or (ii) is entirely correct, we do not rely on the asymptotic normality of U. Instead we use the empirical distribution of the $\hat{U}_i$'s as an estimate of the distribution of U, and hence obtain an estimate for the distribution of $bt + \sqrt{t}U$. In practice, however, since $N(t)$ is restricted to the nonnegative integers,

Table III—$\hat{P}[N(t) = j]$, estimated probability that the number of IA occasions, over a period of length t, equals j

j \ t	1 year	2 years	3 years
0	0.29	0.09	0.01
1	0.13	0.16	0.08
2	0.14	0.11	0.08
3	0.12	0.07	0.09
4	0.17	0.11	0.11
5	0.08	0.07	0.07
6	0.03	0.10	0.07
7	0.02	0.12	0.10
8	0.02	0.07	0.05
9	0.0	0.04	0.06
10	0.0	0.02	0.13
11	0.0	0.02	0.05
12	0.0	0.02	0.04
13	0.0	0.0	0.02
14	0.0	0.0	0.02
15	0.0	0.0	0.02
Total	1.00	1.00	1.00

we look at

$$m_i(t) = \max\{0, [\hat{b}t + \sqrt{t}\,\hat{U}_i + \tfrac{1}{2}]\}, \tag{11}$$

where $[x]$ denotes the integer part of x, and we estimate the distribution of $N(t)$ by

$$\hat{P}_t(j) \equiv \hat{P}[N(t) = j] = (\text{number of } m_i(t) = j)/112. \tag{12}$$

Table III gives $\hat{P}_t(j)$ for $t = 1, 2, 3$, years.

Comment: In view of the difference between younger and older operators, it would have been more appropriate to estimate $P[N(t) = j]$ separately for younger and for older operators, and then to use their relative weights in the entire sample to obtain a final estimate. That is,

$$\hat{P}[N(t) = j] = \frac{33}{122}\hat{P}_{\text{younger}}[N(t) = j] + \frac{79}{112}\hat{P}_{\text{older}}[N(t) = j].$$

The actual values of the estimates using this method are close to the values of the estimates we obtained (Table III) without partitioning the sample, and therefore we do not give the details of this calculation. Note that the estimates of (12) are motivated by a model that imposes

Table IV—$\hat{P}[L(t) \leq j]$, estimated probability that the TTL from IA's, over a period of length t, is at most j days

t	1 year	2 years	3 years
0	0.29	0.09	0.01
1	0.37	0.19	0.06
2	0.45	0.26	0.11
3	0.52	0.32	0.16
4	0.60	0.38	0.21
5	0.68	0.43	0.26
6	0.75	0.48	0.31
7	0.81	0.53	0.36
8	0.86	0.58	0.41
9	0.90	0.63	0.46
10	0.93	0.68	0.50
11	0.95	0.73	0.54
12	0.97	0.77	0.58
13	0.98	0.81	0.62
14	0.99	0.84	0.66
15	1.00	0.87	0.70
16		0.89	0.74
17		0.91	0.78
18		0.93	0.81
19		0.95	0.84
20		0.96	0.87
21		0.97	0.90
11		0.98	0.92
12		0.99	0.94
24		1.00	0.96

very few assumptions on the data. Other possible frameworks for estimation, which impose more conditions on the data (e.g., Poisson arrivals of the IA's) result in estimates for which we feel that the assumptions, rather than the data, determine the actual values of the estimates. However, with more absenteeism data (in particular, longer t_i's) it is possible to identify a useful parametric model for estimating $P[N(t) = j]$.

Let $L(t)$ denote the TTL from the $N(t)$ occasions of IA. Clearly

$$L(t) = X_1 + X_2 + \cdots + X_{N(t)}, \tag{13}$$

where X_i denotes the duration of the ith IA. Assuming that the X_i's are independent of $N(t)$, we have

$$P[N(t) = n, L(t) = l] = P\left[\sum_{1}^{n} X_i = l\right] P[N(t) = n]. \tag{14}$$

Combining the estimates (12) and (14) with (15), we obtain the joint probabilities of $N(t)$ and $L(t)$ for t = 1, 2, 3 years. The marginal distributions of $N(t)$ and $L(t)$ (Tables III and IV, respectively) are then used to construct Tables Va, b, and c, as described below.

2.7 Constructing Tables Va, b, and c

Tables Va, b, and c are the building blocks of our proposed evaluation procedure (Section III) and understanding their construction enables the user to interpret the ratings R_a, R_b, and R_c which make up the attendance evaluation scheme.

To each possible value of $N(1)$, the number of IA's in a single year, and to each possible value of $L(1)$, the total number of days lost in these IA's, we attach a grade and a score. Values of $N(1)$ which lie in the lower 5 percent of the distribution of $N(1)$, which is given in Table

Table Va—Scoring table on the basis of one-year attendance

		Number of Occasions								
	t \ n	0	1	2	3	4	5	6		
Number of days	0	100							E	100
	1		74						G	74
	2		68	55					G	62
	3		60	49	43				F	49
	4		56	46	40	32			F	43
	5		52	43	37	30	23		F	37
	6		48	39	34	27	21	0	F	31
	7		42	34	30	24	18	0	P	24
	8		38	31	27	22	17	0	P	20
	9		34	28	24	20	15	0	P	16
	10		30	24	21	17	13	0	P	12
	11		24	20	17	14	11	0	P	8
	12		0	0	0	0	0	0	U	0
		E	G	F	F	P	P	U		
		100	74	49	37	24	14	0		

Table Vb—Scoring table on the basis of two-years attendance

Number of days t \ Number of Occasions n	0	1	2	3	4	5	6	7	8	9	10		
0	100											E	100
1		94										V	94
2		83	74									G	74
3		81	71	65								G	69
4		78	69	63	56							G	64
5		74	66	60	54	49						G	59
6		71	63	58	51	47	42					G	54
7		68	60	55	49	45	40	34				F	49
8		64	57	52	46	42	38	32	28			F	44
9		61	54	49	44	40	36	31	26	22		F	39
10		57	50	46	41	37	33	29	25	20	0	F	34
11		52	46	42	38	34	31	26	23	19	0	F	29
12		47	42	39	34	31	28	24	21	17	0	P	24
13		45	40	37	33	30	27	23	20	16	0	P	22
14		43	38	35	31	29	26	22	19	15	0	P	20
15		41	36	33	30	27	24	21	18	15	0	P	18
16		39	34	31	28	26	23	20	17	14	0	P	16
17		36	32	29	26	24	21	18	16	13	0	P	14
18		34	30	27	24	22	20	17	15	12	0	P	12
19		31	27	25	22	20	18	15	13	11	0	P	10
20		0	0	0	0	0	0	0	0	0	0	U	0
	E	V	G	G	F	F	F	P	P	P	U		
	100	94	74	62	49	41	33	24	18	12	0		

III, are given the grade Excellent and their scores vary between 100 and 95; values of $N(1)$ which lie between the 6th and the 25th percentile of the distribution of $N(1)$ are given the grade Very Good and their scores vary between 94 and 75, etc. [The particular score depends on how many values of $N(1)$ fall in this range. For example, if only one value of $N(1)$ lies between the 6th and 25th percentile, its score is 94 (e.g., the rightmost column of Table Vb); if there are two values, their scores are 94 and $84 \doteq 94 - (1/2)(94 - 75)$ (e.g., the lower-most row of Table Vc); if there are three values, they get the scores 94, $88 \doteq 94 - (1/3)(94 - 75)$ and $81 \doteq 94 - (2/3)(94 - 75)$, and so on.] We treat $L(1)$ similarly, using the estimated distribution in Table VI. Table VI gives the details of the grading and scoring method, and it is used for $N(t)$ and $L(t)$, $t = 1, 2, 3$.

An exception to Table VI is made when $N(t) = 0$ [and hence $L(t) = 0$], in which case the grade is Excellent and the score is 100 regardless of whether 0 is in the lower 5 percent of the distribution of $N(t)$ [note that $\hat{P}(N(1) = 0) = 0.29$ and $\hat{P}(N(2) = 0.09)$; see Table III]. The scores (and grades) associated with each value of $N(1)$ and $L(1)$ are written on the margins of Table Va and each entry in the body of the table is the geometric mean of the marginal scores; for example, $R_a(N(1) = 3, L(1) = 4) = \sqrt{37 \times 43} \doteq 40$. The reason for choosing the geometric mean to combine the marginal scores is to achieve the desirable shape of the equicontours of the resulting table. More specifically, we observe the following attractive properties: (*i*) The ratings decrease along the west-east and north-south directions. (*ii*) Each entry in the table is slightly greater than (or equal to) its north-east neighboring entry. (This implies that a reduction in the number of occasions of IA's is desirable even at the expense of a slight increase in the TTL.) (*iii*) An operator is rated Unsatisfactory whenever at least one margin is rated as such.

Tables Vb and Vc are constructed similarly with the obvious substitutions of ($N(2)$, $L(2)$) and ($N(3)$, $L(3)$) for ($N(1)$, $L(1)$).

III. EVALUATING ATTENDANCE AT WORK

3.1 Introduction

The attendance behavior of an operator is one of the most important components in the operator's overall performance, so it is evaluated regularly. In particular, it is weighed very carefully when the operator is considered for a transfer or promotion. So far, however, attendance has been assessed in local terms (compared to other operators in the office) and naturally this is done in a subjective and informal way. Though the informality is an advantage both for management and employees, this is not so for the subjectivity of the evaluation. A

Table Vc—Scoring table on the basis of three-years attendance

Number of days t \ Number of Occasions n	0	1	2	3	4	5	6	7	8	9	10	11	12	13		
0	100														E	100
1		94													V	94
2		91	86												V	89
3		89	84	79											V	84
4		86	81	76	72										V	79
5		83	79	74	70	66									G	74
6		81	77	72	68	64	60								G	70
7		79	74	70	66	62	57	53							G	66
8		76	72	68	64	60	55	52	48						G	62
9		74	70	66	62	58	53	50	46	42					G	58
10		71	67	63	60	56	51	48	45	41	36				G	54
11		68	64	60	57	53	49	46	43	39	34	30			F	49
12		65	61	58	54	51	47	44	41	37	33	28	23		F	45
13		62	59	55	52	49	45	42	39	36	31	27	22	0	F	41
14		59	56	52	49	46	43	40	37	34	30	26	21	0	F	37
15		56	53	49	47	44	40	38	35	32	28	24	20	0	F	33
16		52	49	46	44	41	38	35	33	30	26	23	19	0	F	29
17		47	45	42	40	37	34	32	30	27	24	21	17	0	P	24
18		44	42	39	37	32	35	30	28	26	22	19	16	0	P	21
19		41	39	36	34	32	30	28	26	24	21	18	15	0	P	18
20		38	35	33	31	29	27	25	24	22	19	16	13	0	P	15
21		34	32	30	28	26	24	23	21	19	17	15	12	0	P	12
22		29	27	26	24	23	21	20	18	17	15	13	10	0	P	9
23		24	22	21	20	19	17	16	15	14	12	10	8	0	P	6
24		0	0	0	0	0	0	0	0	0	0	0	0	0	U	0
	E	V	V	G	G	G	F	F	F	F	P	P	P	U		
	100	94	84	74	66	58	49	43	37	31	24	18	12	0		

Table VI—Grades and scores for $N(t)$ and $L(t)$

Percentile range	Grade	Score Range
0–5	Excellent	100–95
6–25	Very-good	94–75
26–50	Good	74–50
51–75	Fair	49–25
76–95	Poor	24–5
96–100	Unsatisfactory	0

scheme that allows an objective and consistent evaluation of attendance would be of potential use to line management.

In Section II we studied in detail the statistical aspects of absenteeism. Our analysis, in particular Section 2.4, suggested that if one is interested in attendance behavior as a personal characteristic, then one year is too short for evaluating it. The far past, on the other hand, bears little relevance to recent attendance behavior and thus should not be included in the attendance evaluation. In this section we suggest an evaluation method based on the present and near past (three most recent years) that reflects the current year attendance as well as attendance behavior in a more general sense. We recall, however, from the analysis of Section II that disability absences (DA's) are intrinsically different from incidental absences (IA's). The high variation in the distribution of the duration of DA's and their low frequency of occurrences make it hard to give meaningful statistical guidelines as to what can be considered good, bad, etc., behavior regarding DA's. Furthermore, management can do practically nothing to control DA's. We therefore base our attendance evaluation on IA's only.

In a sensitive issue such as absenteeism from work, the numerical values of the attendance rating do not always tell the whole story. Any method for evaluation might occasionally misjudge good employees, if it is used in a formal and rigid manner. Thus, the best way to avoid these effects is to use it as an informal tool. One has to keep in mind that for every absence there is a reason, and these reasons are not reflected in the formal attendance ratings.

3.2 The evaluation procedure

The proposed scheme is best explained with an example. Consider an operator who started to work on January 1970 and whose IA occurrences and total time lost (TTL) are given in Table VII.

Table VII—Record of IA's

Year	1970	1971	1972	1973	1974	1975	1976	1977	1978
No. of IA occasions	1	4	2	0	0	4	2	0	1
TTL due to IA's	1	6	3	0	0	5	4	0	1

Table VIII—Example of the evaluation scheme (Lines 4, 5, 6 are read off Tables Va, b, c)

	1970	1971	1972	1973	1974	1975	1976	1977	1978
1. $(N(1), L(1))$	(1, 1)	(4, 6)	(2, 3)	(0, 0)	(0, 0)	(4, 5)	(2, 4)	(0, 0)	(1, 1)
2. $(N(2), L(2))$	–	(5, 7)	(6, 9)	(2, 3)	(0, 0)	(4, 5)	(6, 9)	(2, 4)	(1, 1)
3. $(N(3), L(3))$	–	–	(7, 10)	(6, 9)	(2, 3)	(4, 5)	(6, 9)	(6, 9)	(3, 5)
4. R_a	74	27	49	100	100	30	46	100	74
5. R_b	–	45	36	71	100	54	36	69	94
6. R_c	–	–	48	53	84	70	53	53	74
7. R	74	36	44	75	95	51	51	74	81

To evaluate the operator's attendance we propose the following simple procedure:

(*i*) Determine the first three lines of Table VIII as follows: For each year use Table VII to calculate $N(t)$ and $L(t)$ $(t = 1, 2, 3)$, the number of IA occasions, and the TTL due to IA's during the t most recent years, respectively.

(*ii*) For each year, read the ratings associated with $(N(1), L(1))$, $(N(2), L(2))$, and $(N(3), L(3))$ from Tables Va, b, and c. These ratings are written in lines 4, 5, and 6 of Table VIII, respectively, and their interpretation [in terms of percentiles of the marginal distributions of $N(t)$ and $L(t)$, $t = 1, 2, 3$] is described in Section 2.7.

(*iii*) Determine line 7 of Table VIII, the attendance index of the jth year, according to the following formula:

$$R_j = \begin{Bmatrix} \text{avg}(R_{a,j}, R_{b,j}, R_{c,j}), & \text{if} \quad R_{a,j-1} \geq R_{a,j} \\ \max\{R_{j-1}, \text{avg}(R_{a,j}, R_{b,j}, R_{c,j})\}, & \text{if} \quad R_{a,j-1} < R_{a,j} \end{Bmatrix}.$$

For example, in calculating R_{1975} we first compare $R_{a,1975}$ with $R_{a,1974}$. Since $R_{a,1974} = 100 \geq 30 = R_{a,1975}$, we take $R_{1975} = \text{avg}(R_{a,1975}, R_{b,1975}, R_{c,1975}) = (30 + 54 + 70)/3 = 51$. On the other hand, in calculating R_{1976}, comparing $R_{a,1975}$ with $R_{a,1976}$ shows that $R_{a,1975} = 30 < 46 = R_{a,1976}$, so that $R_{1976} = \max\{51, (46 + 36 + 53)/3\} = \max\{51, 45\} = 51$.

(*iv*) The formal evaluation consists of two indices, R_a (line 4) which is the current year rating and R (line 7) which can be considered as an index for attendance behavior (here we view attendance behavior as a personal characteristic of the operator), or in short attendance index.

3.3 Properties of the proposed procedure

(*i*) While the current year rating, R_a, reflects the attendance in the most recent year, the attendance index, R, takes the near past into account, enabling the operator to build up credit. For instance, while 1975 itself was a Fair year ($R_a = 30$), in the example of Table VIII the attendance index, R, for 1975 was Good ($R = 51$). This is due to the perfect attendance during the previous two years. And indeed, if in 1975 this operator was considered for promotion, then the score 51 is a better indicator of her attendance behavior (considered as a personal characteristic) than her current year rating of 30. Similarly, the effect of bad attendance cannot be entirely erased in a single year of perfect attendance, as can easily be seen in the years 1972 and 1973.

By the nature of its definition, R is much smoother than R_a and is a better indicator of attendance. To reemphasize this point, consider an operator whose attendance record fluctuates from $(N(1) = 0, L(1) = 0)$ to $(N(1) = 6, L(1) = 12)$ to $(N(1) = 0, L(1) = 0), \cdots$, etc. The current-year rating then fluctuates from $R_a = 100$ (Excellent) to $R_a = 0$ (Unsatisfactory) while the attendance index fluctuates from $R = 58$ (Good) to $R = 9$ (Poor), which seems more appropriate overall.

(*ii*) If $R_{a,j-1} < R_{a,j}$, then $R_{j-1} \le R_j$, or, in other words, if the current year rating has improved, then the attendance index will not decrease. This follows from the definition of R and is done to avoid negative reinforcement. The situation is exemplified in moving from 1975 to 1976 in Table VIII. There we have $R_{a,1976} = 46 > 30 = R_{a,1975}$ (an improvement in the current-year rating), so we take $R_{1976} = 51 = R_{1975}$ despite the fact that $\text{avg}(R_{a,1976}, R_{b,1976}, R_{c,1976}) = 45 < 51$.

Since the procedure allows the operator to build up credit, the reverse situation does not hold and one can have $R_{a,j-1} > R_{a,j}$ (deterioration in the current-year rating) with $R_{j-1} < R_j$ (improvement in the attendance index). This is exemplified in the ratings of 1977 and 1978 in Table VIII. And indeed, even though the attendance in 1978 was worse than the attendance in 1977, the period 1976–1978 as a whole reflects better attendance than the period 1975–1977.

IV. REMARKS

(*i*) Though the technical details (such as the length of the periods to be used for rating, and the specific values in Tables Va, b, and c) are tuned to telephone operators (more specifically to our sample), the method itself can be adapted to other occupations. In occupations with substantially higher absence rate, such as auto workers [see, for example, the data collected from 60 blue-collar employees of an automobile-parts foundry, reported in Morgan and Herman (Ref. 8, pp. 739)], periods of 1, 2, and 3 years are too far in the past to affect the current attendance index and should be replaced with shorter periods (e.g., 6, 12, and 18 months).

(*ii*) As pointed out by a Bell Laboratories referee, the choice of the scoring bands in Table VI is somewhat arbitrary, and these bands differ from the HOLU (high, objective, low, unsatisfactory) bands that were recommended by the AT&T Measurements Task Force. Since, however, our main contribution here is the general approach for evaluating attendance (i.e., weighing the recent past in the attendance index) rather than the particular details, we prefer to leave the exposition as is.

(*iii*) In view of the difference between younger and older operators in regard to IA's, note that the proposed scheme is tuned to a population of approximately 30-percent younger operators and 70-percent older operators (as in our sample). This proportion emphasizes the better behavior of the older operators without setting unattainable standards for the younger operators.

V. ACKNOWLEDGMENT

This study was initiated through the encouragement of J. Suzansky of AT&T, who supplied unpublished analyses of previous data which suggested that operator absence behavior might be susceptible to statistical modeling. I am grateful to AT&T for interesting me in the problem and for their interest in the study. Paul Tukey helped me with the computer, particularly in transcribing the raw data and creating the data base. His help is greatly appreciated. Thanks are also due to Yoav Vardi of Cleveland State University, for bringing some of the references to my attention.

REFERENCES

1. N. Nicholson, C. A. Brown, and J. K. Chadwick-Jones, "Absence from Work and Personal Characteristics," J. Appl. Psychol., *62,* No. 3 (1977), pp. 319–27.
2. L. M. Porter and R. M. Steers, "Organizational, Work, and Personal Factors in Employee Turnover and Absenteeism," Psychol. Bull., *80* (1973), pp. 151–176.
3. R. Cooper and R. L. Payne, "Age and Absence: A Longitudinal Study in Three Firms," Occup. Psychol., *39* (1961), pp. 31–35.
4. R. McGill, J. W. Tukey, and W. A. Larsen, "Variations of Box Plots," The American Statistician, *32,* No. 1 (1978), pp. 12–16.
5. D. R. Cox, *Renewal Theory*, London: Methuen and Co. Ltd., 1962.
6. R. Gnanadesikan, *Methods For Statistical Data Analysis of Multivariate Observations*, New York: Wiley, 1977.
7. E. L. Kaplan and P. Meier, "Nonparametric Estimation from Incomplete Observations," J. Am. Stat. Assoc., *53* (1958), pp. 457–481.
8. L. G. Morgan and J. B. Herman, "Perceived Consequences of Absenteeism," J. Appl. Psychol., *61,* No. 6 (1976), pp. 738–742.

SUPPLEMENTARY REFERENCES

R. W. Beaty and J. R. Beaty, "Longitudinal Study of Absenteeism of Hard Core Unemployed," Psychol. Rep., *36,* No. 2 (1975), pp. 395–406.

H. Behrend and S. Pocock, "Absence and the Individual: A Six-Year Study in One Organization," Int. Labor Rev., *114,* No. 3 (1976), pp. 311–327.

F. Dansereau, Jr., J. A. Alutto, and S. Markham, "An Initial Investigation into the Suitability of Absenteeism Rates as Measures of Performance," Acad. Manage. Proc. (1977), pp. 230–234.

J. N. Hedges, "Absence From Work—Measuring the Hours Lost" (a special labor force report), Mon. Labor Rev. (October 1977), pp. 16–23.

T. A. Jeswald, "The Cost of Absenteeism and Turnover in a Large Organization," in W. C. Hamner and F. L. Schmidt, eds., *Contemporary Problems of Personnel*, Chicago: St. Claire Press, 1974.

G. Johns, "Attitudinal and Nonattitudinal Predictors of Two Forms of Absence from Work," Acad. Manage. Proc. (1978), pp. 69–73.

G. Latham and E. Pursell, "Measuring Absenteeism from the Opposite Side of the Coin," J. Appl. Psychol., *60,* No. 3 (1975), pp. 369–371.

M. G. Miner, "Job Absence and Turnover: A New Source of Data," Mon. Labor Rev. (October 1977), pp. 24–31.

J. Newman, "Predicting Absenteeism and Turnover. A Field Comparison of Fishbein's Model and Traditional Job Attitude Measures," J. Appl. Psychol., *59,* No. 5 (1974), pp. 610–615.

N. Nicholson, "Management Sanctions and Absence Control," Hum. Relat., *29,* No. 2 (1976), pp. 131–151.

E. Pedalino and V. Gamboa, "Behavioral Modification and Absenteeism: Intervention in One Industrial Setting," J. Appl. Psychol. *59,* No. 6 (1974), pp. 694–498.

A. Stecker, "Two Factor Behavior Theory of Industrial Absenteeism," Diss. Abstr. Int. *34* (3-A) (Sept. 1973), p. 951.