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Rain Margin Improvement Using Resource Sharing in 12-GHz Satellite Downlinks

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In this paper we consider the effectiveness of sharing a small pool of reserved time slots of a Time-Division-Multiple-Access (TDMA) frame among a large number of ground stations to overcome rain fading. With this approach, a system dynamically assigns time slots from the reserved pool to ground stations experiencing fade depths above the built-in margin. Powerful error correcting codes can be introduced to occupy the extra time slots, providing 10 *dB or more of extra fade margin. Because a large number of ground stations are competing for the limited reserved pool, blockage can occur if the number of simultaneous fades exceeds the maximum number that can be accommodated. Some factors that influence the effectiveness of resource sharing are the mutual fade statistics at the various sites, the traffic distribution within the network, the number of earth stations, the size of the reserved pool, and the rain outage objective. Since the mutual fade and traffic statistics are unavailable, we develop models that can be used to find a conservative bound on the required size of the reserved pool. The rain model accounts for diurnal, seasonal, and geographical correlation among attenuation events. Results for a maximum resource-sharing gain of* 10 *dB show that reserving six percent of the time slots ensures a realized fade gain in excess of* 9 *dB for a down-link outage objective of* 0.005 *percent if there are more than* 50 *ground stations in the network, each with two percent or less of the traffic.*

I. INTRODUCTION

In an earlier paper,[1] a shared-resource concept was described for increasing the rain fade margin of a digital satellite system by as much as 10 dB above the design fade margin. With this approach, unused time slots of the Time-Division-Multiple-Access (TDMA) frame are made available to ground stations experiencing rain fading above the built-in fade margin, and are relinquished when the fade event has ended. Error-correcting coding is introduced to occupy the extra time slots, thereby reducing the carrier-to-noise ratio (CNR) required to maintain the threshold bit-error-rate (BER). Low rain outage is therefore achieved without radiating excessive down-link power. Not only does this conserve satellite power, but, also, interference into the systems of other users of the geosynchronous orbit is minimized. In such an application, the operating speed of the decoder is much lower than the transponder data rate by virture of the low TDMA duty cycle associated with each ground station.

Because of the infrequency of simultaneous deep fading at multiple sites, a small pool of reserved time slots can often protect a large number of ground stations. The degree of protection so provided is the subject of this current work; we shall evaluate the reduction in rain margin required to achieve a given outage objective when all ground stations in the network are competing for a limited number of shared resources. We restrict our attention to the power-limited down-link since up-link fading can usually be overcome by means of up-link power control.[1] A convolutional code yielding a maximum power saving of 10 dB is assumed throughout. Results are directly applicable to either a wide-area coverage system or a single-scanning beam system,[2] but the modeling and analytical approach can be extended to study TDMA systems that are fixed-beam satellite-switched, multiple scanning beam, or hybrid-fixed scanning beam.[3–6]

Figure 1 shows a typical sequence for interconnecting the various spot-beam footprints. Each interconnection contains one or more time slots during which ground stations within the connected regions communicate on a sequential basis. Although all TDMA time slots can be made available to accommodate normal network traffic demand, the approach taken here specifically reserves a certain number of time slots, shown at the end of the frame, for use exclusively during the rain fade events. By so doing, we have reduced the traffic-handling capability of the system by a small percentage, while guaranteeing that

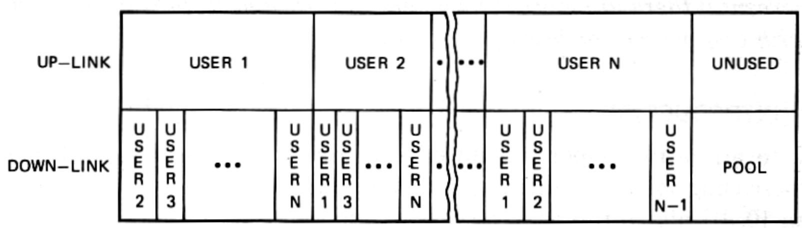

Fig. 1—Typical switching frame showing the interconnections between the system ground stations. An unused pool of reserved time slots is also shown.

some extra time slots will be available as needed to maintain reliability of circuits *already in use* when fade events occur.

The number of reserved slots necessary to provide a specified degree of protection at all stations is dependent upon several factors. Clearly, the utility of resource sharing is dependent upon the joint fade statistics at multiple geographically remote sites; a small number of reserved slots cannot provide protection if the probability of simultaneous excess attenuation at many sites, conditioned upon the occurrence of excess attenuation at any one site, is high. Unfortunately, experimental joint-fade statistics at multiple remote sites are unavailable, and we must resort to modeling to obtain quantitative results.

The utility of resource sharing also depends on the number of ground stations in the network and the traffic distribution among those ground stations. If the number of ground stations is small, then the fraction of TDMA time slots which must be reserved to protect even one site must be large, resulting in an inefficient solution to the rain fade problem. Also, if the traffic distribution is highly nonuniform such that a few ground stations carry a disproportionately large volume of traffic, then again it becomes impractical to reserve enough time slots to protect this small number of large users. In such an event, it might be desirable to protect the large users by some other technique, such as larger antennas or site diversity, and employ resource sharing for the exclusive protection of the much larger number of small users.

Similarly, the effectiveness of resource sharing depends on the geographical distribution of ground stations relative to the profiles of high rain-attenuation regions, and upon the volume of traffic carried by ground stations in high rain-attenuation regions. An additional factor is the relationship between the busy hour, when all time slots not reserved for resource sharing might be expected to be in heavy demand, and the time-of-day occurrence of significant rain-attenuation events.

We shall present both a multiple-site rain-attenuation model and a population-dependent traffic model, upon which is based the subsequent predicted performance of the resource sharing concept. All assumptions implicit in this modeling are addressed in detail in Section II. Section III contains the mathematical analysis of outage based on the rain and traffic models. Section IV contains numerical results of this analysis; the effects of the various factors are presented parametrically. A typical result shows that for a 12/14-GHz network of 100 identical ground stations, a shared resource reservation equal to six percent of the total transponder time slots will provide an outage of ½-hour per year with 9 dB less rain margin than otherwise needed; diurnal, seasonal, and geographical dependencies of joint rain-fade statistics are accounted for in this prediction.

II. JOINT RAIN ATTENUATION AND TRAFFIC MODELS

2.1 Joint rain attenuation model

Figure 2 shows statistics for single-site rain attenuation. It plots the fraction of time that rain fading exceeds the level of the abscissa averaged over one contiguous 12 month interval. This fraction can be interpreted as the probability that any given rain-attenuation level is exceeded. From such single-site curves, we develop a model to be used for predicting joint outages at multiple geographically remote sites.

At first glance, one might assume that if two sites are widely separated, then rain events at the two sites occur independently. However, this cannot be the case because in the 12-GHz satellite band, rain attenuation in excess of 5 dB is typically associated with thunderstorm activity which produces intense rainfall. Periods of thunderstorm activity are typically restricted to a four-month interval lasting from June through September, and to an interval of six hours each day lasting from 1 PM to 7 PM. Thus, if we are told that the rain attenuation at one of the sites is, say, 10 dB, then the probability of simultaneous deep fading at the second site must be higher than its yearly average because, at that moment, we are likely to be in the interval when thunderstorm activity normally occurs. Thus, knowledge

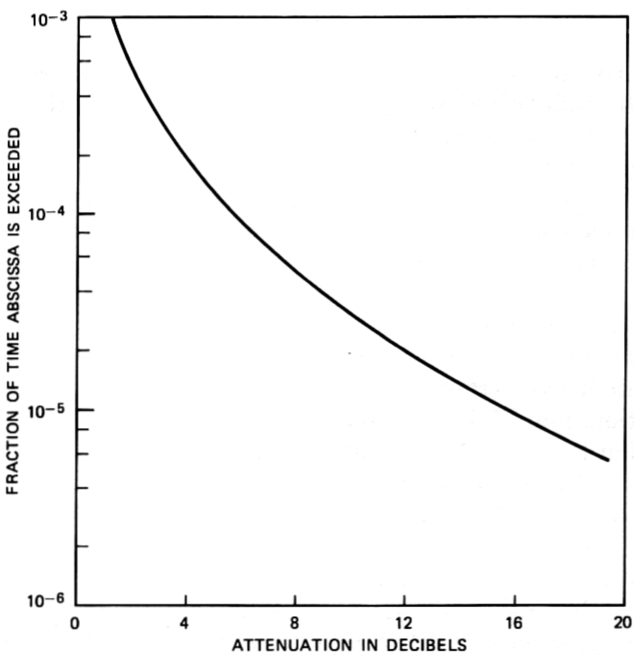

Fig. 2—Typical single-site attenuation curve showing the fraction of time in one year that the attenuation exceeds the abscissa.

of rain attenuation at one site affects the probability of simultaneous attenuation at the second site, and the two events are not independent.

Suppose, however, that we assume that all of the probability of Fig. 1 is attributed to the period of thunderstorm activity. Then, if we restrict our attention to this period only (it is only during this period when resource sharing is needed to combat fading), the probability that attenuation exceeds a given level is about 12 times the yearly averaged value of Fig. 2 (4 out of 12 months per year, 6 out of 24 hours per day). Let $p_1(A)$ be the yearly averaged probability that attenuation exceeds level A at site No. 1. Thus, at a given instant of time t,

$$p_1(A \mid t \text{ within thunderstorm period}) = \alpha p_1(A), \tag{1}$$

$$p_1(A \mid t \text{ outside thunderstorm period}) = 0. \tag{2}$$

In (1), for the illustrative example given above, $\alpha = 12$. Choosing a larger value of α enhances the probability of multiple simultaneous fades since the yearly-averaged probability is then attributed to a narrower interval. In what follows, we conservatively assume the thunderstorm periods to be 3 out of 12 months and 4 out of 24 hours, yielding $\alpha = 24$. The factor α applies at all sites in the satellite network.

Now, given that t is within the thunderstorm-activity period, the events of attenuation exceeding level A at site 1 and level B at site 2 may be assumed to be independent if the two sites are widely separated. For example, knowledge that attenuation exceeds level A in New York during the thunderstorm-activity period provides no information concerning the event that attenuation exceeds level B in Denver, since different independent storms are involved. Thus, for t within the period of thunderstorm activity,

$$p_{1,2}(A, B \mid t) = p_1(A \mid t) p_2(B \mid t) \tag{3}$$

$$= \alpha^2 p_1(A) p_2(B), \tag{4}$$

where $p_{1,2}(A, B \mid t)$ is the probability that attenuation at site 1 exceeds level A and attenuation at site 2 exceeds B simultaneously at time t. Equation (4) is readily generalized for the case of an arbitrary number of widely separated sites.

For two closely spaced sites, another degree of attenuation event correlation is assumed beyond the seasonal and diurnal correlations just addressed. Here, attenuation at the two sites may be produced by the same storm. We assume that, for closely spaced sites, all fades in excess of some thunderstorm characteristic level A_0 always occur within an H-hour interval of each other, where H is much larger than the typical several minute duration of deep fades. Suppose that a fade of level $A > A_0$ occurs at some site. Then, at a neighboring site, the probability that a fade of level $B > A_0$ is simultaneously occurring is given by $C(B)$, a fade level dependent constant over the H-hour window. At this second site, the yearly averaged probability of a fade of level $B > A_0$ is attributed exclusively to the H-hour intervals surrounding the events $A > A_0$ at the original site. Then,

$$p_2(B) = \kappa \frac{H}{T} C(B), \tag{5}$$

where T is the number of hours in a year and κ is the average number of events per year that the attenuation level exceeds A_0. Now,

$$p_{1,2}(A, B \mid t) = p_2(B \mid A, t) p_1(A \mid t). \tag{6}$$

For $A, B > A_0$,

$$p_2(B \mid A, t) = C(B) = \frac{T}{\kappa H} p_2(B). \tag{7}$$

Thus, defining $\beta = T/\kappa H \alpha$,

$$p_{1,2}(A, B \mid t) = \beta \alpha^2 p_1(A) p_2(B). \tag{8a}$$

For L closely spaced sites, (8) generalizes to

$$p_{1,2,\cdots,L}(A_1, A_2, \cdots, A_L) = \beta^{L-1} \alpha^L p_1(A_1) \cdots p_L(A_L). \tag{8b}$$

In the following, we will allow the "geographical correlation factor" β to vary between 1 and 6. The extreme value $\beta = 1$ implies that $\kappa H \alpha = T$, and that, within the thunderstorm period, fades occur independently. Recognizing that α is equal to the number of hours in a year divided by the annual number of hours in the thunderstorm-activity period, we see that for $\beta = 1$, the average number of fades per year multiplied by the uncertainty window H equals the number of hours in the thunderstorm-activity period. Hence, knowledge that a fade of level $A > A_0$ is occurring at some site does not restrict the interval within the thunderstorm-activity period when a fade of level $B > A_0$ may occur at a neighboring site. Similarly, the extreme value $\beta = 6$ implies, for example, that knowledge of deep fading at a given site pinpoints the two out of four thunderstorm hours per day and the average of one out of three days during the thunderstorm period when intense rainfall might occur in the region surrounding that site. Conditioned upon an attenuation event at some site, the probability of an attenuation event at a second site within the surrounding region is then six times higher than would be expected from diurnal and seasonal-correlation considerations alone.

The conservatism of the values $\alpha = 24$ and $\beta = 6$ can be demonstrated by applying eq. (8a) to experimental attenuation data obtained with site diversity such as appears in Ref. 7. Averaging (8a) over one full year, we obtain

$$p_{1,2}(A, B) = \frac{1}{\alpha} p_{1,2}(A, B/t) = \beta \alpha p_1(A) p_2(B). \tag{9}$$

For attenuation greater than 5 dB, the site diversity measurements are somewhat more optimistic then predicted by eq. (9). This observation, coupled with ground station separation much wider than used for the site diversity measurements and the physical interpretations given to α and β above, confirms the conservatism of the approach.

To apply the above rain-attenuation event model, we need to know the yearly-averaged attenuation statistics at the location of each site in the network. The following simplification is invoked: We divide the continental United States into three regions such that the yearly averaged attenuation statistics that apply at a representative site in one region are typical for all sites in that region (see Fig. 3). Los Angeles is chosen as representative of the western region (Region 1), where rain attenuation occurs infrequently. New York is chosen as representative of the northeast central region (Region 2), throughout which rain attenuation is moderate. Finally, Atlanta is chosen as representative of the southeast central region (Region 3), where rain attenuation occurs frequently. When applying the model, we make the pessimistic assumption that the "geographical correlation factor," β, applies at all sites throughout an entire region; for sites located in different regions this factor is neglected. Thus, for this model, "closeness" of two sites is defined by whether both sites are within the same region. The errors incurred because of sites located near each other but on opposite sides of a regional boundary are more than offset by the large "correlation distances" assumed.

2.2 *The traffic model*

The traffic model used in this analysis is based on a rank ordering of the 100 most populous continental United States cities, as shown in

Fig. 3—Three regional maps of the United States. Regional boundaries are selected such that an attenuation curve for a representative ground station in each region applies throughout that region.

Table I.[4] We assume that the traffic between any two of these cities is inversely proportional to the product of their indices. Traffic between two cities closer than 500 miles is excluded from the network. Under these assumptions, Region 1 offers 24 percent of the total satellite traffic, while Regions 2 and 3 offer 63 percent and 13 percent, respectively.[4]

The number of ground stations serving a given region is assumed to be proportional to the traffic offered by that region. For example, if the network contains 100 ground stations, then 24 are located in Region 1, 63 are in Region 2, and 13 are in Region 3. All ground stations are assumed to carry identical traffic cross sections. Thus, for the purposes of analysis, the traffic differences among regions or among cities within each region are accounted for by assigning more or fewer identical capacity ground stations, as the case may be, to accommodate the traffic. The modeling and analysis can be extended to networks containing a mixture of large and small traffic ground stations if the large users are protected by site diversity and if resource sharing is applied to protect only the larger number of small users. This extension has not been carried out, however, and the numerical results to be presented are valid only for the former case.

Two additional assumptions are also made. First, all ground stations in a given region are assumed to be identical, i.e., have the same built-in rain fade margin; the built-in margins of ground stations in different regions need not be the same, however. Second, we conservatively assume that the traffic-busy period, when all time slots not reserved for resource sharing are in full-time use, coincides with the thunderstorm-activity period. Thus, the results are intentionally made to be pessimistic since it is precisely when resource sharing is needed that extra time slots not contained in the reserve pool are unavailable; during traffic off-peak hours, when additional slots are available, resource sharing is not needed because rain fading does not occur.

III. RAIN OUTAGE ANALYSIS

3.1 General approach

Consider the three-regional map shown in Fig. 3. Let there be N_1 ground stations or sites located in Region 1, each of which has a built-in fade margin of A_1 dB. Similarly, let there be N_2 and N_3 sites located in Regions 2 and 3, respectively, with fade margins of A_2 and A_3 dB. The extra fade margin provided by resource sharing if extra time slots are available is M dB, that is, M dB is the maximum extra margin provided by the coding approach employed. All sites carry the same volume of traffic, and enough time slots are reserved to accommodate K simultaneous fades. We wish to find the yearly-averaged probability that a given site is operational when all sites in the network are competing, as needed, for the reserved resource-sharing slots. With no

Table I—Rank-ordering of the 100 largest U.S. cities by population

Rank	City	Region	Relative Traffic	Rank	City	Region	Relative Traffic	Rank	City	Region	Relative Traffic	Rank	City	Region	Relative Traffic
1	New York	1	60	26	San Bernadino	3	4	51	Ft. Lauderdale	2	2	76	Canton	1	1
2	Los Angeles	3	43	27	Indianapolis	1	4	52	Greensboro	2	2	77	Davenport	1	1
3	Chicago	1	25	28	San Jose	3	3	53	Salt Lake City	3	2	78	El Paso	3	1
4	Philadelphia	1	17	29	New Orleans	2	4	54	Allentown	1	1	79	New Haven	1	1
5	Detroit	1	18	30	Tampa-St. Pete	2	3	55	Nashville	1	2	80	Tucson	3	1
6	San Francisco	3	15	31	Portland	3	3	56	Omaha	1	2	81	W. Palm Beach	2	1
7	Washington	1	10	32	Phoenix	3	3	57	Grand Rapids	1	2	82	Worcester	1	1
8	Boston	1	10	33	Columbus	1	3	58	Youngstown	1	2	83	Wilkes-Barre	1	1
9	Pittsburgh	1	10	34	Providence	1	2	59	Springfield	1	1	84	Peoria	1	1
10	St. Louis	1	10	35	Rochester	1	2	60	Jacksonville	2	2	85	Utica	1	1
11	Baltimore	1	8	36	San Antonio	2	3	61	Richmond	1	2	86	York	1	1
12	Cleveland	1	8	37	Dayton	1	3	62	Wilmington	1	1	87	Bakersfield	3	1
13	Houston	2	8	38	Louisville	1	3	63	Flint	1	2	88	Little Rock	1	1
14	Minneapolis	1	7	39	Sacramento	3	2	64	Tulsa	1	2	89	Columbia	2	1
15	Dallas	2	7	40	Memphis	1	3	65	Orlandoa	2	2	90	Lancaster	1	1
16	Seattle	3	6	41	Fort Worth	2	3	66	Fresno	3	1	91	Beaumont	2	1
17	Anaheim	3	5	42	Birmingham	2	3	67	Tacoma	3	2	92	Albuquerque	3	1
18	Milwaukee	1	5	43	Albany	1	2	68	Harrisburg	1	1	93	Chattanooga	1	1
19	Atlanta	2	5	44	Toledo	1	2	69	Charlotte	2	1	94	Trenton	1	1
20	Cincinnati	1	5	45	Norfolk	1	2	70	Knoxville	1	1	95	Charleston	2	1
21	San Diego	3	4	46	Akron	1	2	71	Wichita	1	1	96	Binghamton	1	1
22	Buffalo	1	4	47	Hartford	1	2	72	Bridgeport	1	1	97	Greensville	2	1
23	Miami	2	5	48	Oklahoma City	1	2	73	Lansing	1	1	98	Reading	1	1
24	Kansas City	1	4	49	Syracuse	1	2	74	Mobile	2	1	99	Austin	2	1
25	Denver	3	4	50	Gary	1	2	75	Ventura	3	1	100	Shreveport	2	1

loss in generality, we find this probability for a site located in Region 1; a simple permutation of indices enables this result to be applied in either of the two remaining zones.

At any site within Region 1, three disjoint events may occur at any point in time: (1) the fade depth F may be less than A_1, (2) F may be between A_1 and $A_1 + M$, and (3) F may exceed $A_1 + M$. Call these events E_1, E_2, and E_3. Then

$$P(\text{operational} \mid t) = \sum_{l=1}^{3} P(\text{operational} \mid E_l, t) P(E_l \mid t). \tag{10}$$

Clearly,

$$P(\text{operational} \mid E_1, t) = 1, \tag{11}$$

$$P(\text{operational} \mid E_3, t) = 0, \tag{12}$$

$$P(E_1 \mid t) = \begin{cases} 1 - \alpha p(A_1), & t \text{ within thunderstorm period,} \\ 1, & \text{otherwise,} \end{cases} \tag{13}$$

$$P(E_2 \mid t) = \begin{cases} \alpha p(A_1) - \alpha p(A_1 + M), & t \text{ within thunderstorm period,} \\ 0, & \text{otherwise.} \end{cases} \tag{14}$$

Thus, for t within the thunderstorm period,

$$P(\text{operational} \mid t) = 1 - \alpha p(A_1) + \alpha P(\text{operational} \mid E_2, t)[p(A_1) - p(A_1 + M)], \tag{15}$$

and for t outside the thunderstorm period,

$$P(\text{operational} \mid t) = 1. \tag{16}$$

From (15) and (16), we obtain the result that, averaged over an entire year,

$$P(\text{not operational}) = p(A_1) - P(\text{operational} \mid E_2, \hat{t})[p(A_1) - p(A_1 + M)], \tag{17}$$

where $\hat{t} = \{t \text{ within thunderstorm-activity period}\}$.

3.2 Derivation of $P(\text{operational} \mid E_2, t)$

Now, for t within the thunderstorm period, and conditioned on the event E_2, a particular site will be operational if the number of other sites which need to use the reserve pool is less than $K - 1$. Also, if the number of other sites which need to use the pool is equal to $j \geq K$, then the probability that a particular site is one of the K sites permitted to use the pool is equal to $K/(j + 1)$.

Conditioned on the event E_2 at a particular site, the probability that i_1 additional sites in Region 1 need to use the reserved pool, where $0 \le i_1 \le N_1 - 1$, is given by

$$P(i_1 \text{ in Region } 1 \mid E_2) = {}_{N_1-1}C_{i_1}[(\alpha\beta\rho_1)^{i_1}(1 - \alpha\beta\rho_1)^{N_1-1-i_1}], \quad (18)$$

where

$$\rho_1 = P(A_1) - P(A_1 + M) \quad (19)$$

and

$$_nC_m = \frac{n!}{m!(n-m)!}. \quad (20)$$

Conditioned on the event E_2 at a particular site, the probability that i_2 sites in Region 2 need to use the reserved pool, where $0 \le i_2 \le N_2$, is given by the unconditional probability during the thunderstorm-activity period since the events are assumed to be independent. For $i_2 \ge 1$, we find this probability by first defining the three events:

L_1: {a particular set of $i_2 - 1$ sites in Region 2 need to use the reserved pool},

L_2: {an additional site in Region 2, not included in the particular set of L_1, needs to use the reserved pool},

L_3: {none of the remaining $N_2 - i_2$ sites of Region 2 need to use the reserved pool}.

Then,

$$P(L_1 \cap L_2 \cap L_3) = P(L_1 \cap L_3 \mid L_2)p(L_2) \quad (21)$$

$$= P(L_1 \mid L_2)P(L_3 \mid L_2)P(L_2) \quad (22)$$

$$= (\alpha\beta\rho_2)^{i_2-1}(1 - \alpha\beta\rho_2)^{N_2-i_2}(\alpha\rho_2), \quad (23)$$

where

$$\rho_2 = P(A_2) - P(A_2 + M). \quad (24)$$

Thus, for $i_2 \ge 1$,

$$P(i_2 \text{ in Region } 2 \mid E_2) = {}_{N_2}C_{i_2}(\alpha\rho_1)(\alpha\beta\rho_2)^{i_2-1}(1 - \alpha\beta\rho_2)^{N_2-i_2}$$

$$= \frac{1}{\beta}\left[{}_{N_2}C_{i_2}(\alpha\beta\rho_2)^{i_2}(1 - \alpha\beta\rho_2)^{N_2-i_2} \right]. \quad (25)$$

The probability that none of the sites in Region 2 need to use the reserved pool is given by

$$P(0 \text{ in Region } 2 \mid E_2) = 1 - \sum_{i_2=1}^{N_2} P(i_2 \text{ in Region } 2 \mid E_2). \quad (26)$$

Substituting (25), and invoking the binomial sum formula,

$$P(0 \text{ in Region } 2 \mid E_2) = 1 - \left(\frac{1 - (1 - \alpha\beta\rho_2)^{N_2}}{\beta}\right). \tag{27}$$

Similarly, for Region 3 and for $i_3 \geq 1$,

$$P(i_3 \text{ in Region } 3 \mid E_2) = {}_{N_3}C_{i_3}(\alpha\rho_3)(\alpha\beta\rho_3)^{i_3-1}(1 - \alpha\beta\rho_3)^{N_3-i_3} \tag{28}$$

and

$$P(0 \text{ in Region } 3 \mid E_3) = 1 - \left(\frac{1 - (1 - \alpha\beta\rho_3)^{N_3}}{\beta}\right), \tag{29}$$

where

$$\rho_3 = P(A_3) - P(A_3 + M). \tag{30}$$

Clearly, conditioned on the thunderstorm period, the event {i_2 sites in Region 2 need to use the reserved pool} and the event {i_3 sites in Region 3 need to use the reserved pool} are independent, $0 \leq i_2 \leq N_2$, $0 \leq i_3 \leq N_3$.

Returning to (15), for t within the thunderstorm period, the probability that a particular site is operational, given attenuation between A_1 and $A_1 + M$ dB, can be expressed as the union of the events $\{V_{i_1,i_2,i_3,s}\}$:

$$\{V_{i_1,i_2,i_3,s}\} = \{\mathscr{I}_{i_1} \cap \mathscr{I}_{i_2} \cap \mathscr{I}_{i_3} \cap S\}, \qquad i_1 = 0, \cdots, N_1 - 1, \quad i_2 = 0, \cdots, N_2, \quad i_3 = 0, \cdots, N_3, \tag{31}$$

where

$$\{\mathscr{I}_{i_1}\} = \{i_1 \text{ additional sites in Region 1 need to use the reserved pool}\}, \tag{32}$$

$$\{\mathscr{I}_{i_2}\} = \{i_2 \text{ sites in Region 2 need to use the reserved pool}\}, \tag{33}$$

$$\{\mathscr{I}_{i_3}\} = \{i_3 \text{ sites in Region 3 need to use the reserved pool}\}, \tag{34}$$

$$\{S\} = \{\text{the particular site has been assigned time slots from the reserved pool}\}. \tag{35}$$

Clearly, for $i_1' \neq i_1''$ or $i_2' \neq i_2''$ or $i_3' \neq i_3''$, the events $\{V_{i_1',i_2',i_3',s}\}$ and $\{V_{i_1'',i_2'',i_3'',s}\}$ are disjoint, and the probability of the union of events (31) is equal to the sum of the probabilities of all events in that union. Let us define the function

$$f(m) = \begin{cases} 1, & m \leq K - 1, \\ \dfrac{K}{m}, & m \geq K. \end{cases} \tag{36}$$

Then, we conclude that for $t = \hat{t}$,

$$
\begin{aligned}
P(\text{operational} \mid E_2, \hat{t})
&= \sum_{i_1=0}^{N_1-1} \sum_{i_2=1}^{N_2} \sum_{i_3=1}^{N_3} \{ f(i_1 + i_2 + i_3) P(i_1 \text{ in Region } 1 \mid E_2) \\
&\quad \cdot P(i_2 \text{ in Region } 2 \mid E_2) P(i_3 \text{ in Region } 3 \mid E_2) \} \\
&\quad + \left[\frac{(1 - \alpha\beta\rho_2)^{N_2} + \beta - 1}{\beta} \right] \sum_{i_1=0}^{N_1-1} \sum_{i_3=1}^{N_3} \{ f(i_1 + i_3) \\
&\quad \cdot P(i_1 \text{ in Region } 1 \mid E_2) P(i_3 \text{ in Region } 3 \mid E_2) \} \\
&\quad + \left[\frac{(1 - \alpha\beta\rho_3)^{N_3} + \beta - 1}{\beta} \right] \sum_{i_1=0}^{N_1-1} \sum_{i_2=1}^{N_2} \{ f(i_1 + i_2) \\
&\quad \cdot P(i_1 \text{ in Region } 1 \mid E_2) P(i_2 \text{ in Region } 2 \mid E_2) \} \\
&\quad + \left[\frac{(1 - \alpha\beta\rho_2)^{N_2} + \beta - 1}{\beta} \right] \left[\frac{(1 - \alpha\beta\rho_3)^{N_3} + \beta - 1}{\beta} \right] \\
&\quad \cdot \sum_{i_1=0}^{N_1-1} \{ f(i_1) P(i_1 \text{ in Region } 1 \mid E_2) \},
\end{aligned} \tag{37}
$$

where $P(i_1 \text{ in Region } 1 \mid E_2)$, $P(i_2 \text{ in Region } 2 \mid E_2)$, and $P(i_3 \text{ in Region } 3 \mid E_3)$ are given, respectively, by eqs. (18), (25), and (28).

Substituting (37) into (17) yields the desired result for the yearly-averaged probability that a particular site is unavailable versus the built-in fade margins A_1, A_2, and A_3.

IV. RESULTS

We now apply the model of Section II and the analysis of Section III to investigate the utility of resource sharing for a 12-GHz satellite. Initially, we shall assume a satellite location of 100 degrees West longitude. A total of 100 ground stations is assumed, and the thunderstorm-activity factor $\alpha = 24$. Results are obtained for various values of K and β. The yearly-averaged attenuation data used for Regions 1, 2, and 3 are based upon S. Lin's attenuation model[8, 9] for converting long-term rain-rate data, obtained from the United States Weather Bureau, into attenuation predictions. Figure 4 shows derived plots for Los Angeles (Region 1), New York (Region 2), and Atlanta (Region 3). Sensitivity of the results to regionally dependent built-in fade margins, number of ground stations, thunderstorm-activity factor, and satellite orbital position are investigated later on. A coding gain of 10 dB is assumed.

4.1 Baseline results

Figure 5 shows predicted values of yearly fractional outage with resource sharing for a site in Region 2 versus the built-in rain margin,

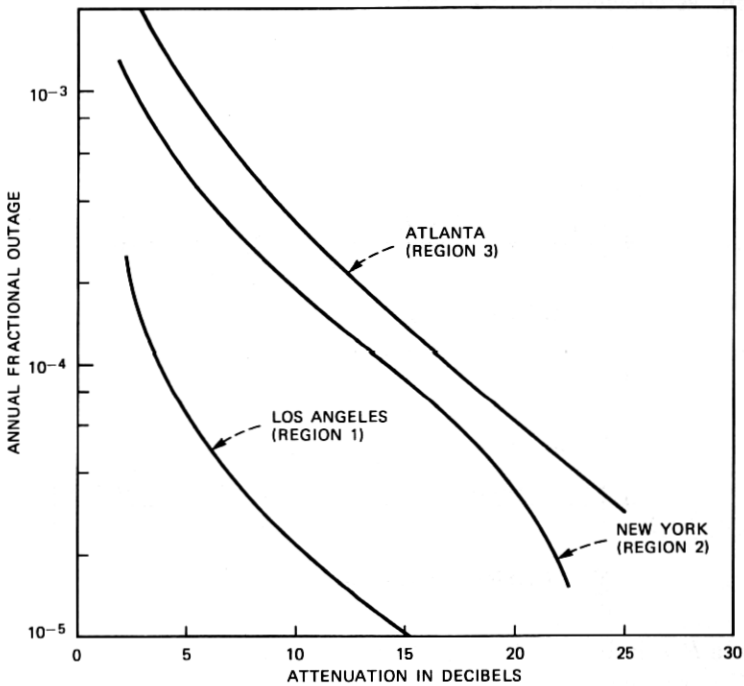

Fig. 4—Yearly average 12-GHz attenuation curves for Los Angeles (Region 1), New York (Region 2), and Atlanta (Region 3). The satellite is at 100°W longitude.

assumed to be common at all ground stations (i.e., all ground stations have the same size antenna). The network contains 100 ground stations, and the "geographical correlation factor," β, is assumed equal to unity, that is, within a given region, the probability of attenuation during the thunderstorm period conditioned upon an attenuation event at any site is equal to the unconditional probability. The number of simultaneous fades which can be accommodated, K, varies between 1 and 3. Similar curves for $\beta = 2$, 4, and 6 appear in Figs. 6, 7, and 8, respectively.

Figures 5 through 8 show that resource sharing is of increasing utility as the outage objective becomes more stringent. For example, for $K = 2$, $\beta = 2$, the fade margin gain is 8.8 dB for a down-link outage of 0.01 percent, and increases to 9.8 dB for an outage of 0.005 percent. We see also that the ability to accommodate only a single fade ($K = 1$) restricts the fade margin resource-sharing gain at a down-link outage objective of 0.005 percent to 9.6 dB under the best of conditions ($\beta = 1$). For less favorable conditions ($\beta = 6$), the gain shrinks to 7.6 dB. Thus, the ability to accommodate only a single fade may severely influence the utility of resource sharing. However, if two simultaneous fades can be accommodated, then even for the unfavorable condition

$\beta = 4$, the fade margin gain is 9.4 dB; whereas for $\beta = 1$, the gain becomes 10 dB. If three simultaneous fades can be accommodated, then, even for the extremely unfavorable case $\beta = 6$, a fade margin gain of at least 9.5 dB can be achieved. For this last case, the outage objective of 0.005 percent can be achieved with a built-in margin of 8.8 dB, to compare against 18.4 dB required in the absence of resource sharing.

Figure 9 shows results obtained in Region 3. Again, a 12-GHz satellite located at 100 degrees West longitude and 100 identical ground stations, all with a common fade margin, are assumed. Curves in this illustration apply for $\beta = 1$, 2, 4, and 6 and for $K = 2$, 3, and 4, and show that at an outage objective of 0.005 percent, the fade margin gain is 10 dB for all cases considered. This outage objective can be achieved with a built-in margin of 11.3 dB, to be compared against 21.3 dB required in the absence of resource sharing.

We note that this last result applies only when a built-in fade margin

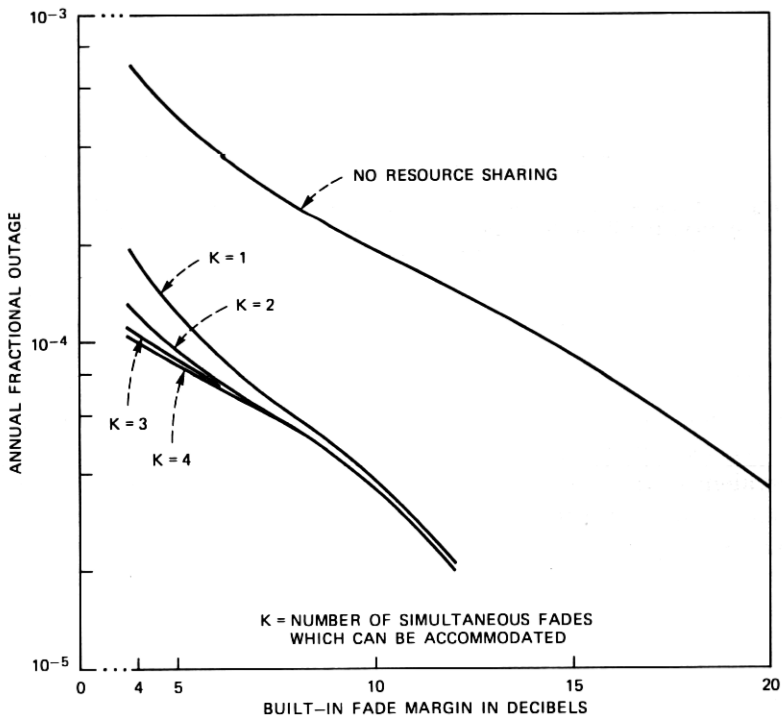

Fig. 5—Rain outage experienced with resource sharing at a site in Region 2 (northeast) versus the built-in fade margin. A maximum possible power saving of 10 dB is assumed. Time-of-day and seasonal thunderstorm activity factor $\alpha = 24$. The geographical factor $\beta = 1$. (Attenuation events at different sites within the thunderstorm activity period are independent.) The satellite is at 100°W longitude, and the network contains 100 identical ground stations.

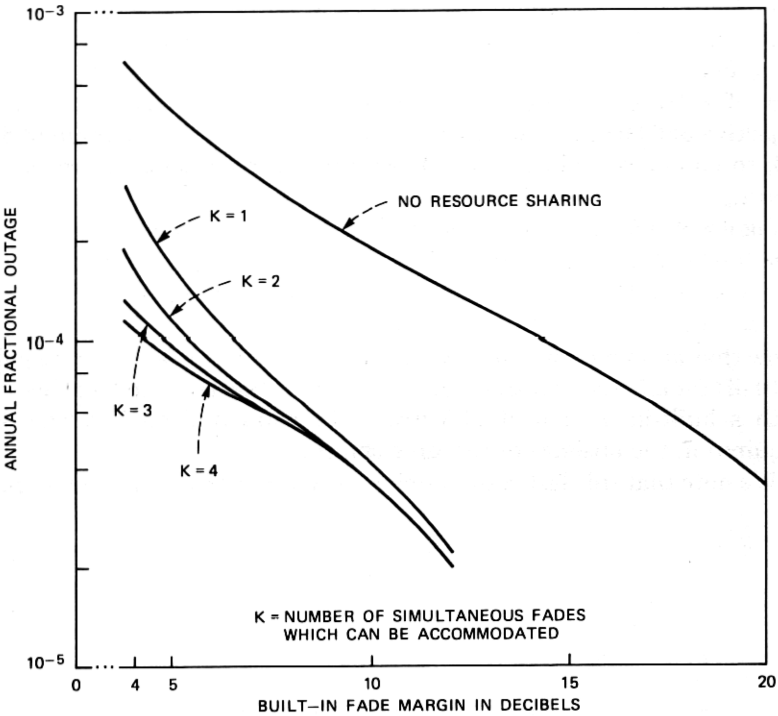

Fig. 6—Same as Fig. 5, except the geographical factor $\beta = 2$. (Given an attenuation event at one site within the thunderstorm activity period, it is twice as likely that a simultaneous attenuation event occurs at any other site within the same region.)

of 11.3 dB is provided at all sites in Regions 1, 2, and 3. From Figs. 5 to 8, we see that in Region 2, a built-in fade margin of 11.3 dB provides an outage lower than the required 0.005 percent. (No results are given for Region 1 because it was found that the objective of 0.005 percent was always readily achieved by virtue of the low level of attenuation prevalent in that region). Thus, we are motivated to consider a case wherein sites with different size antennas are deployed in different regions. The goal here is to optimize the system (i.e., provide the smallest antenna possible in each region) such that a down-link outage objective of precisely 0.005 percent is achieved everywhere.

4.2 Results for nonuniform rain margins

We consider a system wherein the antenna gain at each site in Region 3 exceeds the antenna gain for Region 2 by 2 dB, and the antenna gain at each site in Region 1 is less than the antenna gain in Region 2 by 2 dB. The results are shown in Fig. 10. Again, a 12-GHz satellite at 100 degrees West longitude and a network of 100 ground stations are assumed. Outage is plotted as a function of the built-in margin in Region 2. Parameters are $K = 2$ or 3, $\beta = 1$ or 4. We see that if $K \geq 3$, the outage objective of 0.005 percent can be achieved in Region 3 if the Region 2 built-in margin is 9.3 dB, even for $\beta = 4$. At this level, the outage actually achieved in Region 2 is 0.0044 percent for $\beta = 4$, $K = 3$; an outage of 0.005 percent could have been achieved with a built-in margin of 8.8 dB (within 0.05 dB of that needed to achieve 0.005 percent in Region 3). Thus, this system is close to optimum, except that again, the outage achieved in Region 1 is far lower than the objective. This indicates that the antennas used in Region 1 are still far too large if outage is the only consideration.

For the case of a 500-MHz satellite transponder radiating a power level of 30 watts and a satellite aperture of 15 feet diameter, it has been estimated that a 5 meter diameter earth-station antenna would be needed in Region 2 to provide a rain margin of 16 dB.[2] Extrapolating to the parameters of Fig. 10 by holding the satellite power, bandwidth and aperture fixed, we find that an antenna diameter of 2.5 meters would be needed in Region 2 to provide a built-in margin of 9.3 dB; the Region 1 antenna diameter assumed in Fig. 10 is 2 dB smaller or 2 meters. Thus, it may be impractical to deploy a smaller antenna in

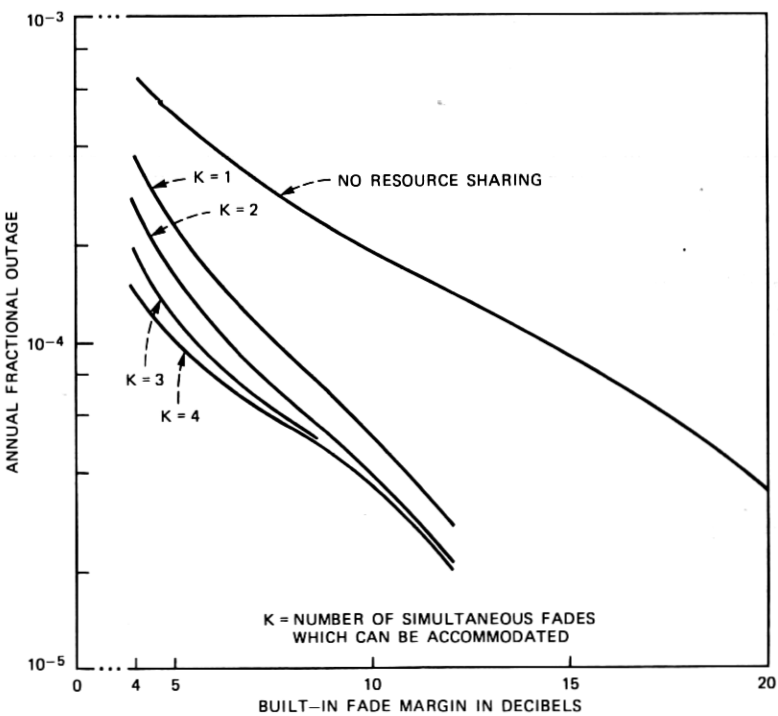

Fig. 7—Same as Fig. 5, except the geographical factor $\beta = 4$.

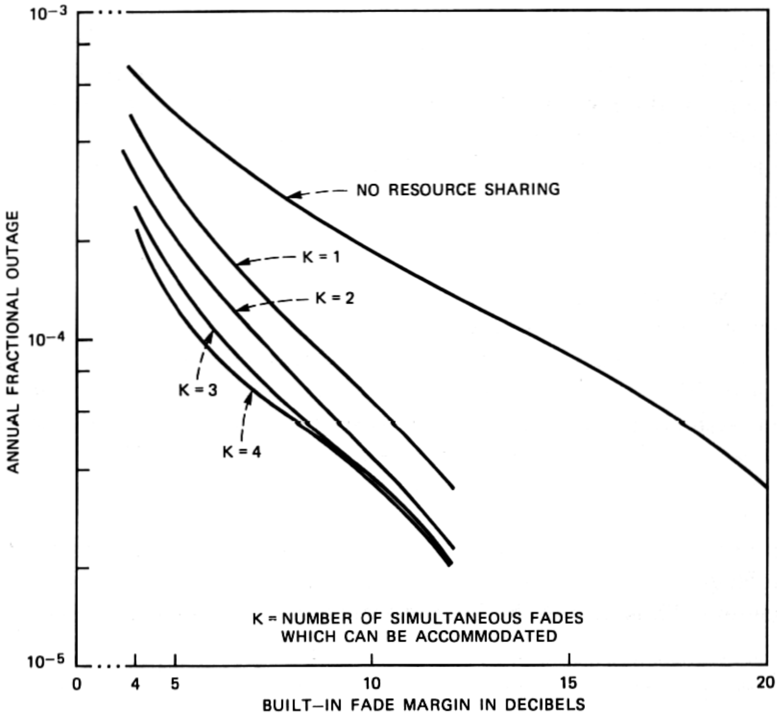

Fig. 8—Same as Fig. 5, except the geographical factor $\beta = 6$.

Region 1 without seriously interfering with other satellites in the geosynchronous orbit. Rather, it might be more advantageous to maintain a larger-than-needed antenna in Region 1 to enable a reduction in the required uplink transmitter power. In summary, the illustration portrayed in Fig. 10 is nearly optimum in a practical sense, and the outage objective of 0.005 percent is achieved everywhere by providing antennas which yield a built-in margin of 7.3 dB in Region 1, 9.3 dB in Region 2, and 11.3 dB in Region 3. The capability to accommodate $K = 3$ simultaneous fades is required for a network with 100 ground stations.

4.3 Sensitivity to number of ground stations

In Fig. 11, we consider a network containing 200 ground stations and plot the achievable outage in Region 2 versus the built-in margin, assumed to be the same at all 200 ground stations. Results for geographical factors β of 1, 2, and 4 and $K = 2$ to 5 appear. Because the number of ground stations has increased by a factor of two above previous cases considered, the number of simultaneous fades which must be accommodated is generally higher to achieve the same resource-sharing advantage. However, this factor is generally smaller

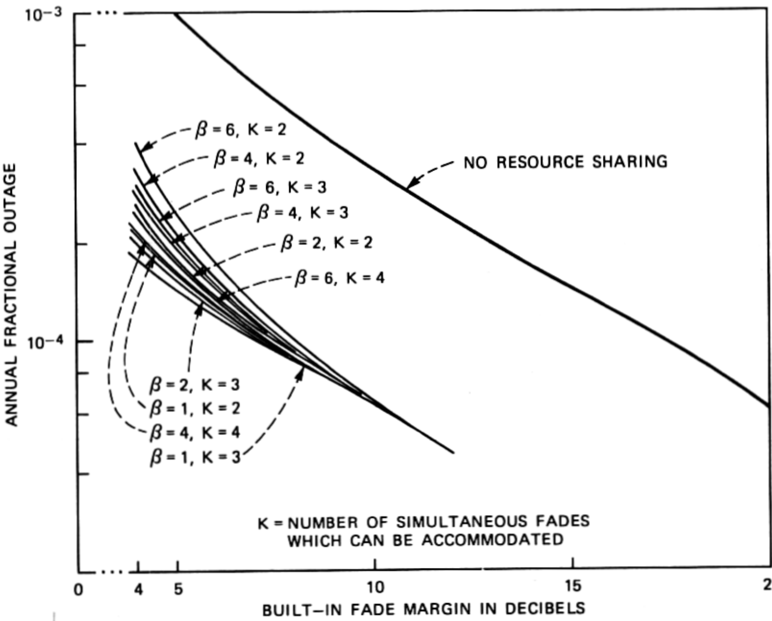

Fig. 9—Rain outage experienced with resource sharing at a site in Region 3 (southeast) versus built-in fade margin. The geographical factor $\beta = 1$, 2, 4, and 6. Other conditions are the same as for Fig. 5.

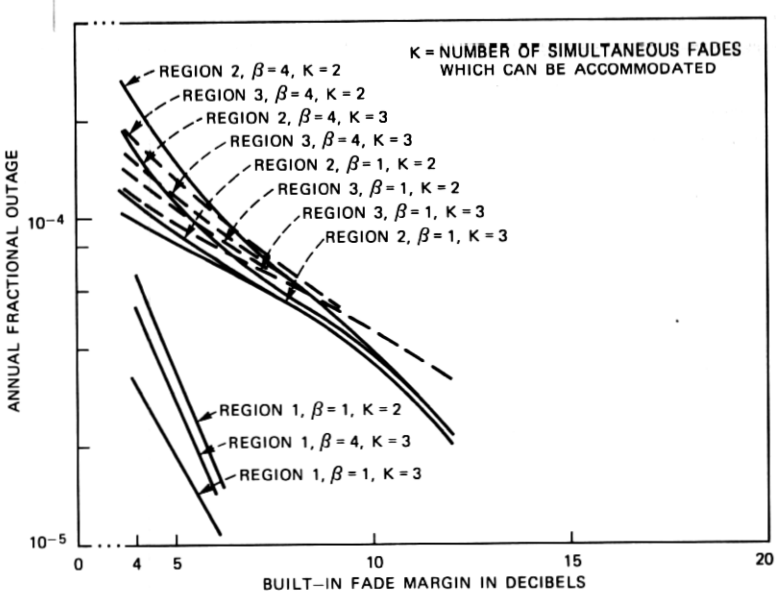

Fig. 10—Rain outage experienced with resource sharing for sites in Regions 1 (west), 2 (northeast), and 3 (southeast) versus built-in fade margin at sites in Region 2. Sites in Region 1 have 2 dB less built-in fade margin than sites in Region 2. Sites in Region 3 have 2 dB more fade margin than sites in Region 2. The geographical factor $\beta = 1$ and 4. Other conditions are the same as for Fig. 5.

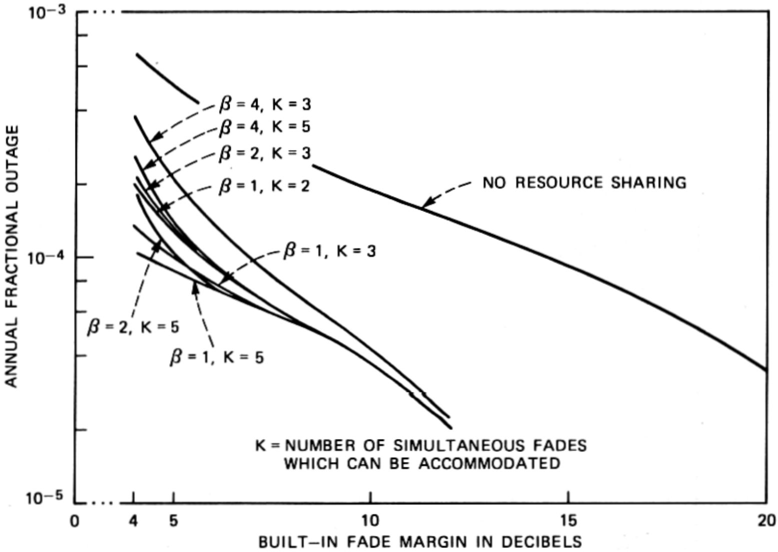

Fig. 11—Rain outage experienced with resource sharing at a site in Region 2 (northeast) versus built-in fade margin. The network contains 200 identical ground stations, and $\beta = 1$, 2, and 4. Other conditions are the same as for Fig. 5.

than two, implying that the resource-sharing overhead, expressed as the required number of reserved slots divided by the total number of accesses or interconnections, decreases as the number of users is increased.

Similarly, as shown in Fig. 12, the number of simultaneous fades which must be accommodated to achieve a given level of resource-sharing advantage decreases for a network containing 50, rather than 100, ground stations. Again, results for Region 2 are shown, and a 12-GHz satellite located at 100 degrees West longitude along with a common fade margin at all sites are assumed. The resource-sharing overhead is generally higher for the 50 station network than for the 100 ground station network.

Figure 13 is a composite plot for Region 2 showing the resource-sharing fade margin gain at an outage of 0.005 percent versus the required overhead for $\beta = 1$, 2, and 4 and for a total number of 50, 100, and 200 ground stations. The four-for-one time slot expansion of Ref. 1, which allows use of a rate ⅓ convolutional code to provide a coding gain of 10 dB along with extended synchronization preamble, is assumed. Letting the number of ground stations be represented by N, the overhead η, expressed as a percentage, is given by

$$\eta = \frac{3K}{N + 3K} \times 100. \tag{38}$$

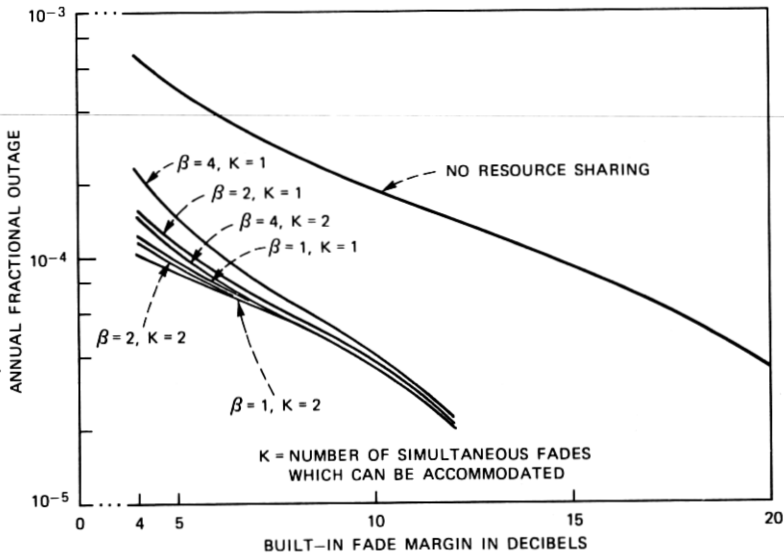

Fig. 12—Same as Fig. 11, except that the network contains 50 identical ground stations.

Again, a 12-GHz satellite located at 100 degrees West longitude is assumed. A similar composite plot can be derived for Region 3. We see that for $\beta \leq 4$, an overhead of 4.3 percent will ensure a fade margin gain greater than 9 dB if the network contains 200 ground stations. An

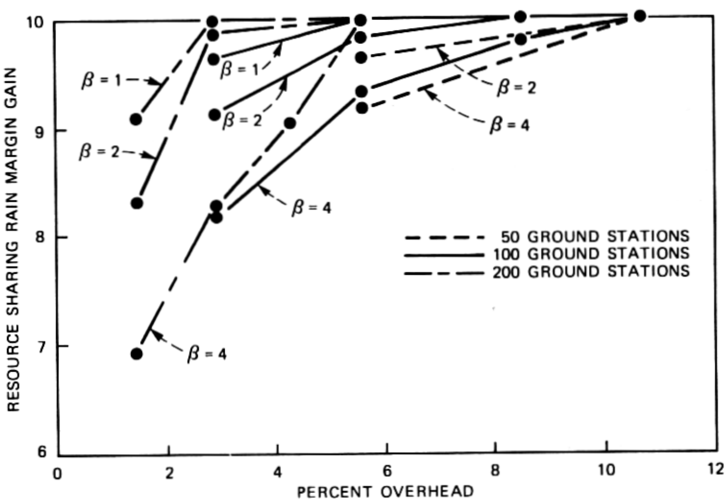

Fig. 13—Composite plot for a site in Region 2 (northeast) showing the resource-sharing fade margin gain at an outage of 0.005 percent versus the TDMA overhead for networks of 50, 100, and 200 ground stations. A maximum possible power saving of 10 dB is assumed. Time-of-day and seasonal thunderstorm activity factor $\alpha = 24$ and the geographical factor $\beta = 1$, 2, and 4. The satellite is at 100°W longitude.

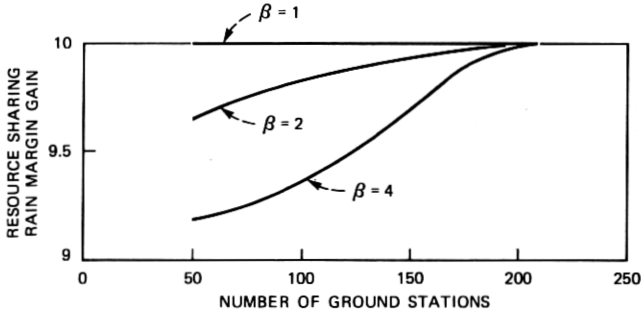

Fig. 14—Composite plot for a site in Region 2 (northeast) showing the resource sharing fade margin gain at an outage of 0.005 percent and a TDMA overhead of 5.66 percent versus the number of ground stations in the network. A maximum possible power saving of 10 dB is assumed. Time-of-day and seasonal thunderstorm activity factor $\alpha = 24$ and the geographical factor $\beta = 1$, 2, and 4.

overhead of 5.66 percent is needed to achieve similar results for a network of 100 ground stations. Of course, the required overhead in all cases is reduced if smaller values of β apply.

Figure 14 plots a family of curves for a site in Region 2, showing the resource-sharing fade margin gain, for an outage of 0.005 percent, as a function of the number of ground stations in the network. The overhead is kept constant at 5.66 percent ($K = 1$ for 50 ground stations, $K = 2$ for 100 ground stations, and $K = 4$ for 200 ground stations), and the family parameter is β. Again, we see that resource sharing becomes more effective as the number of ground stations in the network increases.

4.4 Sensitivity to α

In Fig. 15, we investigate the effect of the thunderstorm-activity period factor, α, on the utility of resource sharing. For these curves, we assume that $\alpha = 12$, rather than the value of 24 used for all previous results, implying that the period of heavy attenuation is concentrated over an interval twice as large in time. Thus, at any site, the probability that attenuation exceeds some value A, conditioned on the thunderstorm period, is half its former value. Again, a 12-GHz satellite located at 100 degrees West longitude is assumed, and results apply at a ground station in Region 2. A total of 100 ground stations is assumed, and all ground stations have a common built-in fade margin. We see that, at an outage of 0.005 percent, the ability to accommodate two simultaneous fades provides a fade margin advantage in excess of 9.8 dB for β as high as four. Thus, under the less pessimistic and more realistic assumption that $\alpha = 12$ (4 out of 12 months/year, 6 out of 24 hours/day) rather than 24, we see that the resource-sharing advantage is very close to the maximum possible for an overhead of about six percent. We note, however, that the gain over $\alpha = 24$ is only 0.4 dB (see Fig. 7).

4.5 Sensitivity to satellite longitude

Finally, in Fig. 16, we plot the yearly average attenuation statistics (based upon Lin's model) for sites in Los Angeles (Region 1), New York (Region 2), and Atlanta (Region 3) for a 12-GHz satellite located at 130 degrees West longitude rather than 100 degrees West as assumed before. Because of the greater slant range to the satellite, the attenuations experienced at sites in Regions 2 and 3 are greater; again, Region 1 experiences very little attenuation. We use these statistics to derive the curves of Fig. 17 which show the fractional outage versus the built-in margin for a site in Region 2 and for various values of K and β. A total of 100 ground stations of common fade margin is assumed, and $\alpha = 24$. Figure 17 shows that for $\beta \leq 4$, capability for accommodating only one fade is sufficient to provide 10 dB of extra protection via resource sharing at an outage objective of 0.005 percent. Because the yearly averaged attenuation statistics are nearly the same in Regions 2 and 3 for this satellite longitude, it is safe to assume that Fig. 17 is reasonably representative of a site in Region 3 as well. Also, because of this similarity between Regions 2 and 3, it appears that resource sharing is more effective for a satellite location at 130 degrees

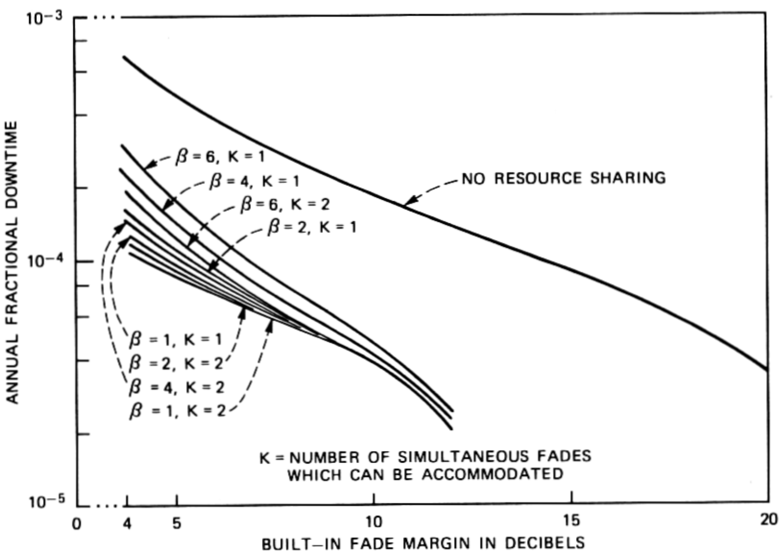

Fig. 15—Rain outage experienced with resource sharing at a site in Region 2 (northeast) versus the built-in fade margin. Time-of-day and seasonal thunderstorm activity factor $\alpha = 12$, and the geographical factor $\beta = 1, 2, 4$, and 6. Other conditions are the same as for Fig. 5.

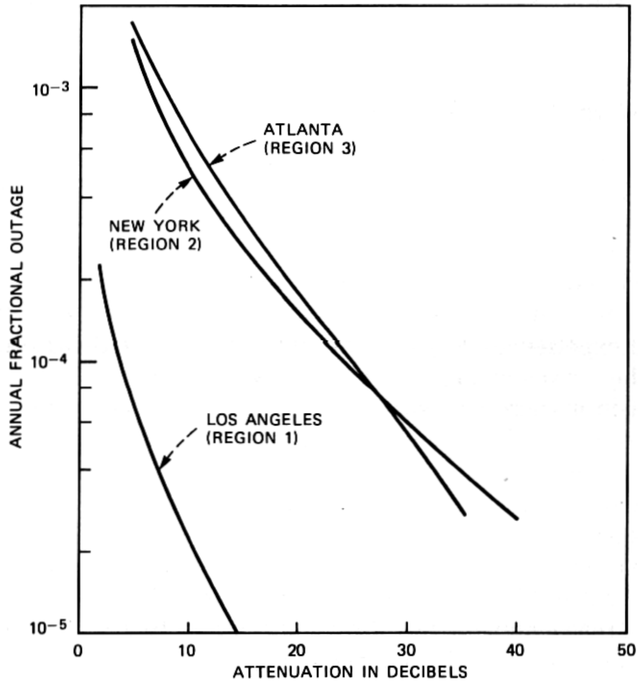

Fig. 16—Yearly average 12-GHz attenuation curves for Los Angeles (Region 1), New York (Region 2), and Atlanta (Region 3). The satellite is at 130°W longitude.

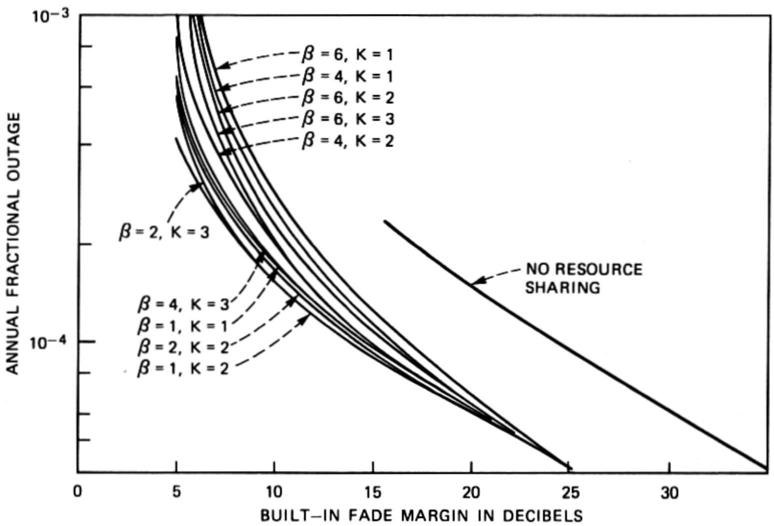

Fig. 17—Rain outage experienced with resource sharing at a site in Region 2 (northeast) versus the build-in fade margin. The satellite is at 130°W longitude, and the time-of-day and seasonal thunderstorm activity factor $\alpha = 24$. Other conditions are the same as for Fig. 5.

West, compared against 100 degrees West, although the required built-in margin is much greater. This is because, when we position the satellite at 100 degrees West, the higher attenuation experienced in Region 3 relative to Region 2 presents more competition for the reserved time slots, which is disadvantageous for a site in Region 2.

V. SUMMARY AND CONCLUSION

In this paper, we have attempted to evaluate the effects of the size of the reserved pool, the number of ground stations, correlation among attenuation events, rain outage objective, and satellite position on the utility of resource sharing to provide 10 dB of extra down-link margin against rain fading. A single wideband satellite transponder such as might be associated with a scanning spot-beam system was assumed, but the joint rain attenuation and traffic modeling and the analytical approach can be extended to study multiple spot beam frequency reuse concepts. In general, it was found that for the cases examined, a pooled resource overhead equal to six percent of the available TDMA time slots is adequate to ensure a fade margin gain in excess of 9 dB for an outage objective of 0.005 percent if there are more than 50 ground stations in the network. The difference between the actual gain realized and the maximum gain of 10 dB possible with the coding approach employed arises from the effects of many ground stations in competition for a small number of shared resources.

The traffic model employed for this study assumes that all ground stations carry similar traffic cross sections. Although the analytical approach can be modified to reflect the effects of users with different traffic cross sections, such an approach might become numerically unwieldy if the number of user classes assumed becomes too large. However, a simple overbound on the required overhead can readily be obtained if for a given number of users, we apply the results obtained by assuming a smaller number of users. For example, suppose the network contains 200 ground stations, and the amount of traffic carried by the ground stations are within a factor of four of each other. Then, the required overhead calculated for a network of 50 ground stations is an overbound of that needed for the 200 ground station network.

The predicted results depend, of course, on the rain model assumed; modeling is necessary because statistics of multiple fade events for a large number of geographically dispersed sites are unavailable. Fortunately, in a TDMA system designed on the resource-sharing concept, it is a simple matter to alter the size of the reserved pool in response to real operating experience.

Three final observations will be offered. First, it appears from these results that resource sharing is most effective in the southeastern portion of the United States because the traffic demand from that region is smaller than that presented by other regions. This is indeed fortunate because it is precisely in that region, where attenuation is high, that resource sharing can offer the most saving. Second, if the satellite receives a western orbital-slot assignment, resource-sharing again assumes an important role because of the large rain margins (and, therefore, the large satellite radiated power or large ground stations) required to provide a suitably low rain outage. Finally, we note that resource sharing can provide high-system reliability (low-rain outage) while maintaining a sufficiently low-satellite effective radiated power to satisfy interference constraints imposed by the presence of other users of the geosynchronous orbit. It is perhaps in this last regard that resource sharing will prove most valuable, by enabling coexistence of spot-beam systems with existing wide-area coverage systems.

REFERENCES

1. A. S. Acampora, "A Shared Resource TDMA Approach to Increase the Rain Margin of 12/14 GHz-Satellite Systems," B.S.T.J., *58,* No. 9 (November 1979), pp. 2097–111.
2. D. O. Reudink and Y. S. Yeh, "A Scanning Spot Beam Satellite System," B.S.T.J., *56,* No. 8 (October 1977), pp. 1549–60.
3. A. S. Acampora and B. R. Davis, "Efficient Utilization of Satellite Transponders via Time-Division Multibeam Scanning," B.S.T.J., *57,* No. 8 (October 1978), pp. 2901–14.
4. A. S. Acampora, C. Dragone, and D. O. Reudink, "A Satellite System with Limited-Scan Spot Beams," IEEE Trans. Commun., *Com-27,* No. 10 (October 1979), pp. 1406–15.
5. D. O. Reudink and Y. S. Yeh, "The Organization and Synchronization of a Switched Spot-Beam System," Fourth Int. Conf. Digital Satellite Commun., Montreal, October 1978.
6. D. O. Reudink, A. S. Acampora, and Y. S. Yeh, "Methods for Achieving High Capacity Universal Service Satellites," Nat. Telecom. Conf., Birmingham, December 1978.
7. S. H. Lin, H. J. Bergman, and M. V. Pursley, "Rain Attenuation on Earth-Satellite Paths—Summary of 10-Year Experiments and Studies," B.S.T.J., *59,* No. 2 (February 1980), pp. 183–228.
8. S. H. Lin, "Nationwide Long-Term Rain-Rate Statistics and Empirical Calculation of 11-GHz Microwave Attenuation," B.S.T.J., *56,* No. 9 (November 1977), pp. 1581–1604.
9. S. H. Lin, "Empirical Rain Attenuation Model for Earth-Satellite Paths," IEEE Trans. Comm., *COM-27,* No. 5 (May 1979).