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A New Code for Transmission of Ordered Dithered Pictures

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This paper presents a new predictor for coding dithered pictures. A dithered picture, a two-tone picture which gives the illusion of a picture with many shades of grey, is obtained by comparing the grey-level picture with a position-dependent threshold. When the intensity of a picture element (pel) exceeds the threshold, it is classified as white; otherwise it is black. In the new prediction scheme, the color of a pel is predicted from pels having nearly the same threshold level. Therefore, the position of the pels used for prediction varies according to the threshold level. Computer simulations show that prediction errors are reduced by 50 percent for certain classes of originals, and the entropy is reduced by 20 percent compared to the result obtained with a previous predictor. An advantage of this new prediction scheme is that it appears to be less sensitive to picture content.

I. INTRODUCTION

Dithering is an image processing technique that creates a two-level picture that gives the illusion of a multilevel picture by appropriately arranging the spatial density of the two levels (usually black and white) on the picture.[1–5] The dithering technique consists of comparing a multilevel image with a position-dependent threshold and setting pels to white when the input signal exceeds the threshold. Other pels are set to black. The matrix of threshold values (called the dither matrix) is repeated over the entire picture to provide the threshold pattern for the whole image.

The merit of a dither matrix is judged from the quality of its rendition of the original picture. A class of dither matrices of special interest is the ordered dither matrices,[3] which use a simple recursion algorithm to create dither matrices of size $2^n \times 2^n$ to simulate $2^{2n} + 1$ brightness levels.

In the case of the 4 × 4 matrix, the 16 threshold levels are put in the following positions:

$$\begin{vmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{vmatrix}. \tag{1}$$

The ordered dither matrices have the advantage of rendering grey level pictures with as good a subjective quality as with empirical dither matrices.[3] Moreover, we show that ordered dither matrices are especially convenient for the proposed new predictor. Figure 1 shows three dithered pictures, "Karen," "Engineer drawing," and "House," that are used for computer simulations. The originals for these pictures are 10 cm by 10 cm and were scanned to generate an array of 512 by 512 pels. The pels were digitized with a uniform 8-bit PCM code (256 levels). Matrix 1 then becomes

$$\begin{vmatrix} 8 & 136 & 40 & 168 \\ 200 & 72 & 232 & 104 \\ 56 & 184 & 24 & 152 \\ 248 & 120 & 216 & 88 \end{vmatrix}. \tag{2}$$

Note that Figs. 1a, b, and c were made from digitizing the same originals as used in Refs. 4 and 6 but not from the same digitized versions.

Efficiently coding the bits of the dithered image reduces the data rate. If the pels of the dithered picture are sampled in a typical raster scan fashion, the frequent alternations between black and white pels prevent the direct use of run-length coding, which is an efficient redundancy reduction scheme.[4] Others have proposed a modification to the straightforward raster scan sampling or to the direct use of the picture pels. Judice proposed a different sampling order, a bit interleaving code.[4] Netravali et al.[6] devised a predictive coding technique that is an extension of their two-level facsimile coding scheme.[7] In both cases, the binary picture containing the interleaved bits or the prediction errors can be efficiently run-length coded. In Netravali's coding technique, the prediction of the value of a pel ($1 \hat{=}$ white, $0 \hat{=}$ black) is made dependent on the values of four neighbor pels as well as to the threshold level. Figure 2 shows the ordered dither matrix of (2) and the prediction that he used. With a dither matrix of 16 threshold levels, and a prediction from 4 neighboring pels, the prediction table has 256 possible states. This prediction scheme takes advantage of the correlation between adjacent pels, but it can only partially exploit the much stronger correlation that exists between neighboring pels with the same or similar threshold level.

The scheme proposed in this paper predicts a pel according to the

Fig. 1a—Dithered test picture of Karen.

value of nearby pels of the same or similar threshold level. The new predictor gives for two of the three test pictures 50 percent fewer prediction errors compared to the previous one, and 10 percent fewer prediction errors for the third test picture.

II. PREDICTION PRINCIPLES AND ALGORITHMS

2.1 Prediction principle

A pel can best be predicted from the following information:

(*a*) the threshold level of the pel itself,

(*b*) the values of the nearest (already coded) pels with similar threshold levels.

We use four previous pels for prediction and consider pictures which have been dithered using the 4×4 ordered dither matrix of (2). There are then 16 threshold levels and since each pel used for prediction has only two possible values, 256 states are defined by all the possible

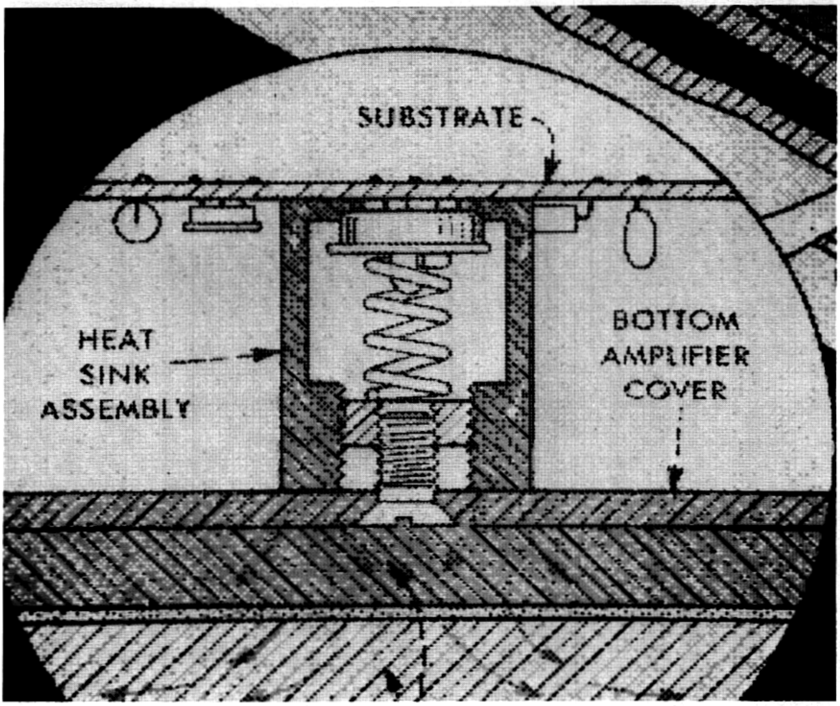

Fig. 1b—Dithered test picture of Engineering drawing.

combinations of the predictor and of the threshold level. The predictor code book contains 256 bits, each indicating whether the pel is predicted to be black or white. The code book is simply obtained by statistical measurement. From these predictions, we generate a new picture where the pels correctly predicted are put to "0" while the pels incorrectly predicted are put to "1." The new picture, called the "error picture," can easily be run-length coded and transmitted efficiently. Knowledge of the predictor code book allows the receiver to reconstruct the original dithered picture. The four pels used for prediction are:

(*a*) the two nearest pels with the same threshold level,
(*b*) the nearest pel with the next higher threshold level,
(*c*) the nearest pel with the next lower threshold level.

For cases (*b*) and (*c*), when two pels satisfy the rule at the same time, the pel with the smallest correlation with the other prediction pels is chosen. Also, for case (*b*), if there are no pels with a higher threshold level, a second pel with the next lower threshold level is used for prediction. A similar situation can appear in case (*c*).

2.2 Prediction algorithm

Let the pel to be predicted, S_{ij}, have position $(i; j)$, where i is the line number, j is the position on the line, and i and j are increasing down and to the right, respectively. The two nearest pels with the same threshold level naturally have coordinates $(i - 4; j)$ and $(i; j - 4)$ since a 4×4 dither matrix is used.

To find the two other pels used for prediction, we must look at the structure of the 4×4 ordered dither matrix. The procedure is described in the appendix. We give here only the result, showing that the coordinates of the four pels used for the prediction are

1. $i; j - 4$,
2. $i - 4; j$,
3. $i - 2; j + 2$,
4. variable.

The position of the prediction pel with variable position is given according to the position of Sij within the dither matrix by

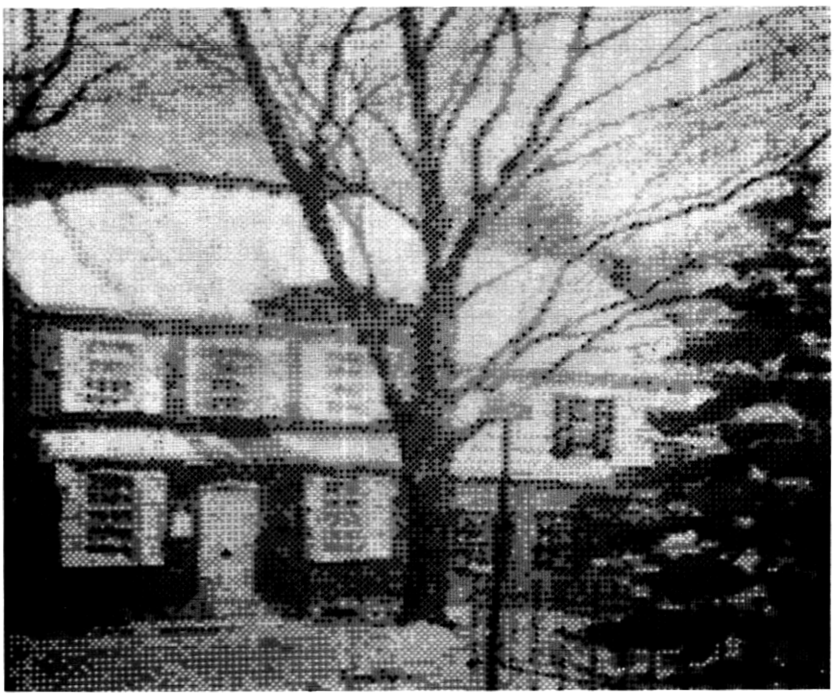

Fig. 1c—Dithered test picture of House.

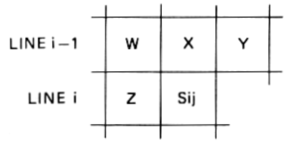

Fig. 2—Picture elements used for prediction, where *Sij* is the picture element to be predicted, and *W*, *X*, *Y*, and *Z* are picture elements used to predict *Sij*.

$$\begin{vmatrix} i-2; j-2 & i-1; j & i-2; j & i-2; j \\ i-3; j+1 & i-3; j-1 & i-2; j & i-2; j \\ i-1; j+1 & i-1; j-1 & i-2; j & i-2; j \\ i-2; j-2 & i-3; j & i-2; j & i-2; j \end{vmatrix} \qquad (3)$$

Figure 3 shows the positions of the four pels used for the prediction.

Note that for the eight pels in the two right rows of the matrix the prediction pel with variable position is the same.

This new predictor can be called a position-dependent predictor since one of the pels used for the prediction is variable. In the case where using a position-dependent predictor is undesirable, the predictor size can be reduced and only three pels can be used, or the fourth pel used in the prediction can be fixed to the most-used position ($i - 2; j$). Both cases lead to a slight increase in the number of prediction errors.

2.3 Extension to other dither matrices

The same prediction principle can be applied to different dither-matrices or different sizes of matrices, but with care. For example, when applying the same rule to an ordered 8×8 dither matrix, three of the four pels of the predictor are fixed and their locations, compared to the pel to be predicted, are in the same direction but twice as far as

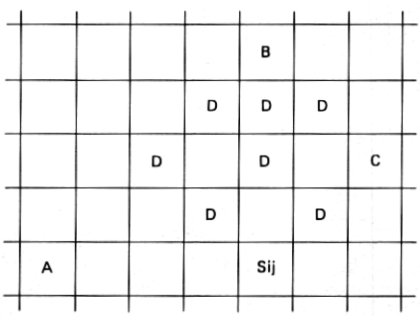

Fig. 3—Position of the pels used for the prediction, where *Sij* is the pel to be predicted, *A*, *B*, and *C* are the three fixed pels used for predicting, and the *D*'s are the different positions occupied by the pel with variable position used for prediction.

for a 4×4 matrix. Their locations are

1. $i; j - 8$,
2. $i - 8; j$,
3. $i - 4; j + 4$.

The prediction pel with variable position again has the same relative position for the right half of the matrix, the coordinates being ($i - 4; j$). For the other half, the position is variable, but often the same within each 2×2 submatrix. The position matrix of the prediction pel with variable position can be constructed by looking at the structure of the 2×2 and 4×4 submatrices.

It is questionable whether such a predictor gives good prediction. The prediction is now made according to the value of the pels twice as far away, as in the case of a 4×4 dither matrix. The error rate is therefore likely to increase. The best solution is to use the same prediction algorithm as for the 4×4 dither matrix, since the 8×8 dither matrix is composed of four 4×4 dither matrices whose threshold levels are only very slightly different. The number of prediction errors would then be only slightly higher than if the picture was dithered with a 4×4 matrix. A slight decrease in prediction errors can be obtained by considering all 64 threshold levels instead of 16 for constructing the prediction table. The prediction table would then contain 1024 states instead of 256.

In the case of other dither matrices (nonordered) the prediction principle is the same but the algorithm is different, since the position of the nearest pels with similar threshold level changes. In most cases, two of the four pels used for prediction would have a variable position, thus showing the advantage of using the ordered dither matrices with this prediction scheme.

This prediction scheme can be extended by adding more pels to the predictor; for example, using pels closer to the pel to be predicted, thereby decreasing the number of prediction errors along a sharp edge.

III. SIMULATION RESULTS

Computer simulations are performed using the three dithered pictures of Fig. 1. The results are compared to those obtained using the technique proposed by Netravali et al.[6] Two measures of performances are made: counting the number of prediction errors and measuring the entropy of the run-length statistics of the picture containing the prediction errors. The entropy is converted into bits per pel. The run-length entropy measured is the classical run-length entropy, given for example in Ref. 6.

Table I gives the number of prediction errors with the new predictor. For comparison, the number of prediction errors with the technique in Ref. 6 is also given. Compared with the predictor of Ref. 6, the number of prediction errors is reduced by 53 percent for "Karen" and "House" and 10 percent for "Engineering drawing."

Table II gives the entropy comparisons. Four entropies are given. The first is the entropy of the run-length statistics of the prediction error when using separate prediction tables for each picture. The second is the same entropy in the case where a single prediction table is used for all three pictures (the prediction table is optimized to the sum of the three pictures).

The third entropy is the entropy of the run-length statistics of the prediction errors when the prediction errors are ordered according to the probability of error[6] (good-bad ordering). In the good-bad ordering the prediction errors of the pels of a line are filled in the right side of a line if their error probabilities are high, while they are put on the left side when their error probabilities are low.[6] The goodness threshold used is 0.05.

The fourth entropy is the same as the latter but with a single prediction table for all three pictures.

The entropy measurements show the great improvements obtained by this new prediction technique compared to the results from Ref. 6. For "Karen" and "House" the reduction in the run-length entropy is 25 to 30 percent, but it is limited to 3 percent for "Engineering drawing." When a single prediction table is used, the reduction in entropy is 28 percent for "Karen" and "House" but it reaches 10 percent for "Engineering drawing." In the case of good-bad ordering, the decrease in entropy with the new prediction technique compared to Ref. 6 is 17 to 24 percent for "Karen" and "House" while an increase of 2 percent appears for "Engineering drawing." When a single prediction table is used for all pictures, the decrease is 24 percent and 17 percent for "Karen" and "House" and 3 percent for "Engineering drawing."

The gain obtained with the new predictor for "Karen" and "House" is very different for "Engineering drawing." It can be explained by the picture characteristics. The "Karen" and "House" pictures are really half-tone pictures with mostly gradual changes in brightness while "Engineering drawing" is a graphical picture containing mostly steep

Table I—Comparison of the number of prediction errors

Pictures	Prediction Errors with New Predictor	Prediction Error with Predictor of Netravali
Karen	12,613	26,557
Engineering drawing	25,742	28,526
House	12,106	26,156

Table II—Comparison of entropies (in bits/pel)

	Karen	Engineering Drawing	House
I. New Predictor			
Run-length coding with separate prediction table for each picture	0.221	0.396	0.214
Run-length coding with a single prediction table	0.228	0.403	0.216
Run-length coding with good-bad ordering with separate prediction table for each picture	0.196	0.376	0.187
Run-length coding with good-bad ordering and a single prediction table	0.202	0.380	0.191
II. Predictor of Netravali			
Run-length coding with separate prediction table for each picture	0.317	0.409	0.286
Run-length coding with a single prediction table	0.317	0.446	0.303
Run-length coding with good-bad ordering and separate prediction table for each picture	0.259	0.367	0.225
Run-length coding with good-bad ordering and a single prediction table	0.278	0.393	0.253

changes in grey level. These steep changes are well predicted with Netravali's predictor and therefore practically no improvements are obtained with the new predictor.

An advantage of this new technique is that the predictor is quite independent of the characteristics of individual pictures. The entropy with the predictor optimized for the sum of the three pictures is less than 2 percent higher than with individual predictors for each picture. With Netravali's predictor the entropy is sometimes 10 percent higher when the predictor is optimized for the sum of the three pictures.

The advantage of using a good-bad ordering is smaller with the new predictor, the decrease in entropy being about 10 percent. We emphasize however, that the good-bad ordering leads to a nearly monotonically decreasing run-length distribution and therefore the runs are easy to code, while without ordering, the run-length distribution is very irregular. Straightforward run-length codes can be devised which code the pictures obtained after prediction and reordering with an efficiency of about 90 percent.

IV. SUMMARY AND CONCLUSION

We have described a new predictive coding scheme for dithered pictures where the pels used for prediction have variable positions. The prediction is made according to the value of nearby pels that have nearly the same threshold level and according to the value of the threshold of the pel to be coded. We obtain a decrease of 20 to 30 percent in the run-length entropy compared to an earlier prediction scheme in the case of dithered pictures of natural scenes. The bit rate is reduced to 0.18 to 0.23 bits/pel. We obtain a slightly higher entropy when we fix the position of the pels used for prediction. In the case of graphical pictures, the entropy is about the same as before. The new predictor has the advantage of being nearly independent of the characteristics of individual pictures. In the case of a coding system, the run-length distributions must be used to propose a code whose performances should be near those given by the entropy measures. This prediction scheme was conceived for images dithered with a 4 × 4 ordered dithered matrix, but can be easily applied to other dither matrices. We envision extensions that would improve the performance in regions that have sharp changes in brightness.

APPENDIX

Determination of the Four Neighbor Pels used for Prediction

The two nearest pels with the same threshold level naturally have coordinates $(i - 4; j)$ and $(i; j - 4)$ since a 4 × 4 dither matrix is used.

To find the two other pels used for prediction, we examine the structure of the 4 × 4 ordered dither matrix. Let us number the four 2 × 2 matrices contained in the 4 × 4 dither matrix as 0, 1, 2, and 3 according to the increase in their average threshold value. Figure 4 shows their relative positions. We must also remember that the 4 × 4 matrix is surrounded by identical matrices whose pels can also be used as prediction pels.

For a pel in submatrix 1, the neighboring pel with the next-higher threshold level will be in the same relative position in submatrix 2 with coordinates $(i - 2; j)$. The neighbor with the next-lower threshold level will be in the same relative position in submatrix 0. That neighbor can be either in submatrix 0 of the same 4 × 4 matrix or in submatrix 0 of the matrix adjacent to the right. Both cases lead to the same distance from the neighbor to the pel to be predicted, but the latter case leads to the smallest correlation with the other prediction pels and is therefore chosen. That prediction pel has position $(i - 2; j + 2)$.

Similarly, if the pel to be predicted is in submatrix 2, the two pels

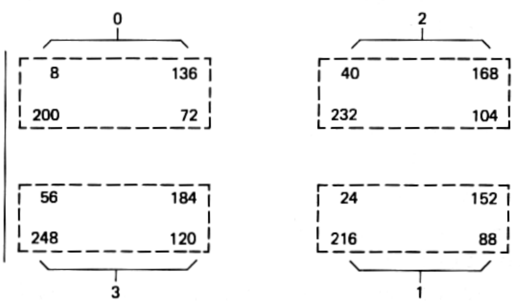

Fig. 4—4 × 4 dither matrix with the numbering of its 4 submatrices 0, 1, 2, and 3.

with the next threshold level chosen for the prediction have coordinates $(i-2; j)$ and $(i-2, j+2)$.

For a pel in submatrix 3, the pel with the next-lower threshold level will be in the same relative position in submatrix 2. As earlier, there are two possibilities, but correlation comparisons leads us to choose the pel with coordinate $(i-2; j+2)$. The neighbor with the next-higher threshold level will be in submatrix 0, but in each case in a different relative position, as can be verified from Fig. 4. For the pel with threshold level 248, pels with a higher threshold level do not exist. Therefore, the two nearest pels with the next lower-threshold level are chosen as prediction pels.

Similarly, if the pel to be predicted is in submatrix 0, the pel with the next-higher threshold level chosen for prediction has coordinates $(i-2, j+2)$, while the pel with the next-lower threshold level has in each case a different relative position. In this case, when the pel with threshold level 8 is predicted, the two nearest pels with the next-higher threshold level are chosen as prediction pels.

In summary, the positions of the pels used in the prediction are:

1. $i; j-4$,
2. $i-4; j$,
3. $i-2; j+2$,
4. variables.

The position of the prediction pel with variable position is given according to the position of *Sij* within the dither matrix by

$$\begin{vmatrix} i-2; j-2 & i-1; j & i-2; j & i-2; j \\ i-3; j+1 & i-3; j-1 & i-2; j & i-2; j \\ i-1; j+1 & i-1; j-1 & i-2; j & i-2; j \\ i-2; j-2 & i-3; j & i-2; j & i-2\ j \end{vmatrix}$$

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