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An Extension of the CCITT Facsimile Codes for Dithered Pictures

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The International Telegraph and Telephone Consultative Committee (CCITT) has recently recommended two redundancy reduction codes for the digital transmission of two-level facsimile. It is of interest to transmit other types of still pictures using these codes. In this paper we develop a method for bilevel dithered pictures, wherein pictures are preprocessed reversibly to permit efficient coding by the CCITT codes. The preprocessing consists of local rearrangement of picture elements to obtain large contiguous black and white areas. The main advantage of these schemes over the existing ones is that they require the addition of a simple processor to a standardized facsimile coder. One of the schemes has the additional advantage that even in the absence of the postprocessor at the receiver, a distorted but recognizable picture is obtained. A compression ratio of 1.5 to 3.5 is obtained with these methods, which is comparable with other coding schemes.

I. INTRODUCTION

The International Telegraph and Telephone Consultative Committee (CCITT) has recently recommended two redundancy reduction codes for the digital transmission of two-level facsimile over the general switched telephone networks.[1] In the future, most facsimile apparatus using redundancy reduction techniques will use these codes. The first code is a one-dimensional run length code called the modified Huffman code (MHC). The second code is a two-dimensional extension of the modified Huffman code, called the modified READ code (MRC).

Dithering is an image processing technique which creates a two-level picture that gives the illusion of a multilevel picture by appropriately controlling the spatial density of black and white picture elements.[2-6] It is useful in systems that use inherently bilevel displays (e.g., plasma panels). Dithering allows a picture to be transmitted as a two-level picture, thus greatly reducing the number of bits to be transmitted.

The high frequency components of a dithered picture prevent the use of CCITT codes as an efficient redundancy reduction scheme.[5] Judice,[5] Netravali et al.,[7] and Johnsen[8] have proposed several coding techniques for dithered pictures. Judice[5] has proposed a bit interleaving scheme which regroups pels with the same or similar threshold levels, thus allowing the use of run-length coding.

We are interested in coding schemes for dithered pictures that would require only a slight modification to the standardized one-dimensional and two-dimensional CCITT codes. Our aim is to include only a simple preprocessor at the transmitter and a postprocessor at the receiver. The preprocessor would include a dithering processor which transforms the analog signal from the facsimile scanner into the binary data stream corresponding to the dithered image and a precoder which modifies the dithered image in such a way that it can be efficiently coded by the one-dimensional or two-dimensional codes. We only consider reversible transformations. The postprocessor of the receiver must make the inverse transformation. One of the preprocessing schemes that we propose has an interesting property that the ordered picture is visually similar to the original dithered picture. Therefore, a facsimile receiver not containing the postprocessor will be able to reproduce a picture that is in most cases recognizable.

II. ORDERED DITHER

The dithering technique consists of comparing a multilevel image with a position dependent threshold and turning only those picture elements "on" (or "1") where the input signal exceeds the threshold value. The square matrix of threshold values (called the "dither matrix") is periodically repeated over the entire picture to provide the threshold pattern for the whole image.

Several dither matrices can be used. They have been judged on the basis of subjective fidelity of reproduction. A class of dither matrices of special interest are the "ordered dither matrices."[4] A dither matrix of size $n \times n$ simulates $n^2 + 1$ brightness levels where n is a power of 2. Thus, 17 brightness levels can be simulated with a 4×4 ordered dither matrix and 65 brightness levels with a 8×8 matrix. A 4×4 matrix, with 16 threshold levels, is shown in eq. (1),

$$\begin{vmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{vmatrix} \tag{1}$$

Figure 1 shows dithered pictures, "Karen," "Engineering drawing," and "House," used for computer simulations. These pictures are 10 cm

Fig. 1a—Dithered test picture of Karen.

by 10 cm and are scanned to generate an array of 512 by 512. The picture intensity is linearly quantized to 8 bits (256 levels) and then dithered. The following dither matrix, obtained from eq. (1) by expanding the threshold levels to cover the entire span of picture intensity, is used:

$$\begin{vmatrix} 8 & 136 & 40 & 168 \\ 200 & 72 & 232 & 104 \\ 56 & 184 & 24 & 152 \\ 248 & 120 & 216 & 88 \end{vmatrix} \tag{2}$$

We mention that the three pictures of Fig. 1 are made from the same originals as used in Refs. 4 and 6, but they are not the same digitized versions.

III. REORDERED PEL CODING

Since the codes to be used are already known, the goal of the ordering is to transform the picture to minimize the coding length. In the one-dimensional case, the runs must be as long as possible. In the

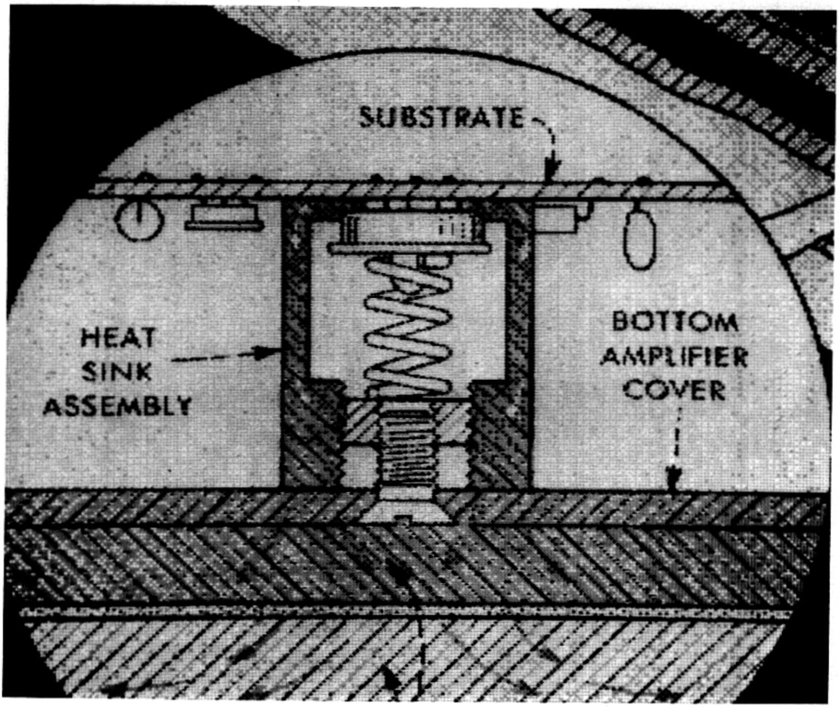

Fig. 1b—Dithered test picture of Engineering drawing.

two-dimensional case, the vertical correlation between runs must also be maximized.

The pel reordering is related to the bit interleaving method proposed by Judice.[5] The main difference is that now the reordering of the pels is done locally. The ordering is made within a few 4 × 4 blocks at a time, typically 1 to 4 blocks. The difference can be seen from the relation between neighboring pels of the ordered pictures. In Judice's schemes, neighboring pels are pels with similar threshold levels of adjacent blocks, while in the local ordering scheme, neighboring pels are pels of nearly the same threshold level of the same or neighboring blocks. Therefore, bit interleaving exploits better the correlation among pels with similar threshold levels, while pel ordering exploits better the correlation among the neighboring pels.

We propose two closely related kinds of pel reordering. One of them is optimized for run-length coding, while the other is optimized for the two-dimensional codes. They have both been conceived for 4 × 4 ordered dither matrices, but in the case of larger dither matrices, simple modifications can be made.

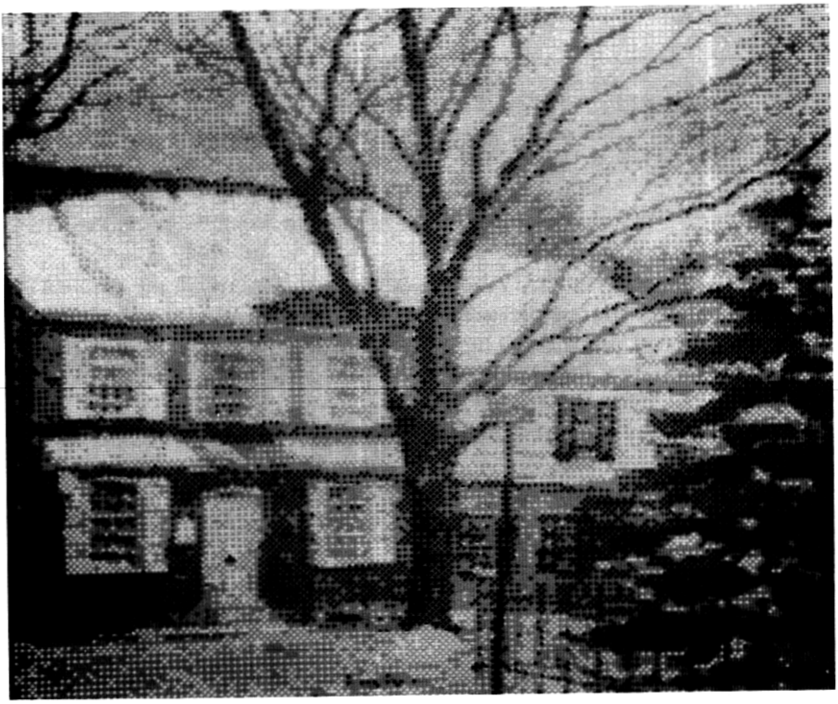

Fig. 1c—Dithered test picture of House.

3.1 Pel reordering for run-length coding

Let us consider the 4×4 dither matrix of eq. (1). To have as few runs as possible, the pels with nearly the same threshold levels should be contiguous on the same line. One way to reduce the number of runs, is to reorder the dithered pels for two consecutive blocks slightly differently, for example,

$$\begin{array}{cccc|cccc} \multicolumn{4}{c|}{\text{Block } A} & \multicolumn{4}{c}{\text{Block } B} \\ 0 & 1 & 2 & 3 & 3 & 2 & 1 & 0 \\ 4 & 5 & 6 & 7 & 7 & 6 & 5 & 4 \\ 8 & 9 & 10 & 11 & 11 & 10 & 9 & 8 \\ 12 & 13 & 14 & 15 & 15 & 14 & 13 & 12 \end{array} \tag{3}$$

Another possibility is to reorder a 4×4 block into a 2×8 block. Two blocks must therefore be reordered at the same time. For reasons of efficiency, four blocks, $\frac{A \mid C}{B \mid D}$, are reordered as shown in Table I.

Figures 2 and 3 show the ordered picture of Karen with 4×4 and 2×8 reordering. It can be seen that these reordered pictures can be

Table I—Reordering of four blocks $\begin{smallmatrix} A & C \\ B & D \end{smallmatrix}$

A		B		D		C	
0	1	1	0	0	1	1	0
2	3	3	2	2	3	3	2
4	5	5	4	4	5	5	4
6	7	7	6	6	7	7	6
8	9	9	8	8	9	9	8
10	11	11	10	10	11	11	10
12	13	13	12	12	13	13	12
14	15	15	14	14	15	15	14

run-length coded efficiently because there are many long runs. Also note that the reordered pictures are visually degraded versions of the original dithered pictures. The 4 × 4 reordering gives smaller degradation than the 2 × 8 ordering.

3.2 Pel reordering for two-dimensional coding

Pel reordering for two-dimensional coding should increase the vertical correlation between runs, i.e., the reordered picture should have mostly vertical black and white strips. This is accomplished within each block by moving the pels most likely to be white to the left side and the pels most likely to be black to the right side and reversing this

Fig. 2—Karen after 4 × 4 reordering for run-length coding.

Fig. 3—Karen after 2 × 8 reordering for run-length coding.

process for the next block. The reordering of two blocks, $A \mid B$, would be

Block A				Block B			
0	4	8	12	15	11	7	3
1	5	9	13	14	10	6	2
2	6	10	14	13	9	5	1
3	7	11	15	12	8	4	0

(4)

Further improvements are possible, but the coding efficiency with 4 × 4 reordering is low compared to the other reordering schemes.

Improvement in the coding efficiency is obtained by 8 × 2 reordering. This type of reordering slightly reduces the vertical redundancy between the runs, but that is largely compensated by the reduction in the number of runs. To exploit the maximum amount of correlation, two groups of four blocks of 4 × 4 pels must be considered. These blocks, named A, B, C, D, A', B', C', and D', are shown below:

A	B	C	D
A'	B'	C'	D'

. (5)

Table II—Reordered configuration of eq. (5)

A	0	2	4	6	8	10	12	14	15	13	11	9	7	5	3	1	*D*
	1	3	5	7	9	11	13	15	14	12	10	8	6	4	2	0	
B	1	3	5	7	9	11	13	15	14	12	10	8	6	4	2	0	*C*
	0	2	4	6	8	10	12	14	15	13	11	9	7	5	3	1	
B′	0	2	4	6	8	10	12	14	15	13	11	9	7	5	3	1	*C′*
	1	3	5	7	9	11	13	15	14	12	10	8	6	4	2	0	
A′	1	3	5	7	9	11	13	15	14	12	10	8	6	4	2	0	*D′*
	0	2	4	6	8	10	12	14	15	13	11	9	7	5	3	1	

Table II shows the reordered configuration.

Another type of reordering is the 16 × 1 reordering. Now the pels are arranged in order of increasing or decreasing threshold level. Eight 4 × 4 blocks shown below,

A	*B*	*C*	*D*	*E*	*F*	*G*	*H*

,

are transformed into:

$$\begin{array}{lllllllllll} A & 0 & 1 & \ldots & 14 & 15 & 15 & 14 & \ldots & 1 & 0 & E \\ B & 0 & 1 & \ldots & 14 & 15 & 15 & 14 & \ldots & 1 & 0 & F \\ C & 0 & 1 & \ldots & 14 & 15 & 15 & 14 & \ldots & 1 & 0 & G \\ D & 0 & 1 & \ldots & 14 & 15 & 15 & 14 & \ldots & 1 & 0 & H \end{array} \tag{6}$$

A slight improvement can be obtained by reversing every second group of four lines. Figures 4 and 5 show the picture "Karen" transformed by 8 × 2 and 16 × 1 reordering, respectively.

3.3 Implementation considerations

All the above reordering schemes are simple to implement. Except in the case of 2 × 8 reordering for one-dimensional coding, only four lines with a maximum of 16 pels per line are processed at a time. The basic operation is to address the bits in a different sequence. The reordering should add very little to the cost of the coder and decoder, certainly less than the dithering operation which itself is quite simple.

IV. COMPARISON OF PERFORMANCES

The coding length in bits per pel have been measured for the various reordering schemes and for the bit interleaving schemes of Judice.[3] The three test pictures of Fig. 1 were used. Only the one-dimensional and the two-dimensional codes recommended by the CCITT for standardization[1] are considered. The end of line codeword has been suppressed in the comparisons. In the two-dimensional case, all lines, except the first one, are coded with the two-dimensional code. Note that all the reordering schemes are two-dimensional, even when a one-dimensional code is used afterwards.

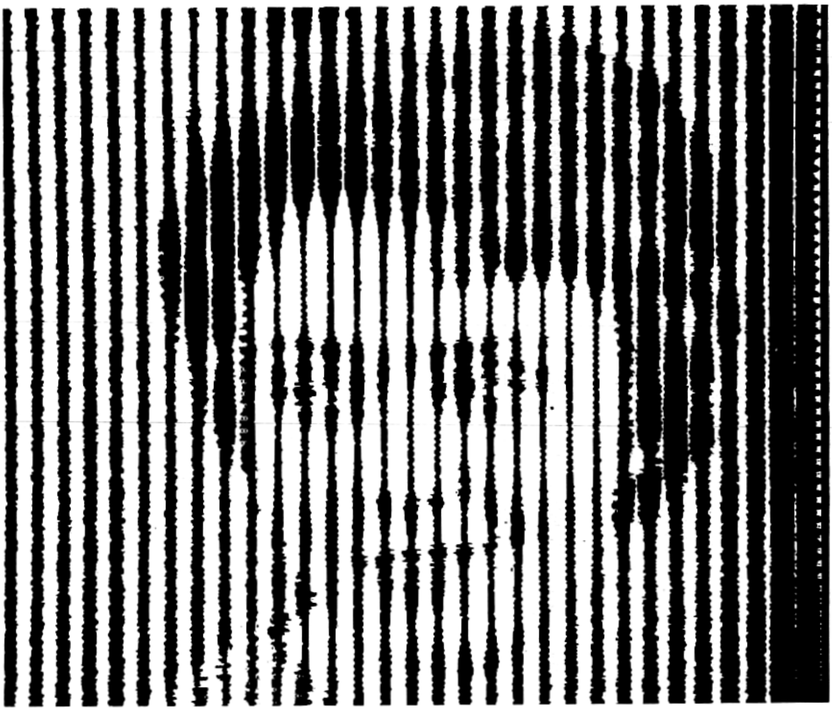

Fig. 4—Karen after 8 × 2 reordering for two-dimensional coding.

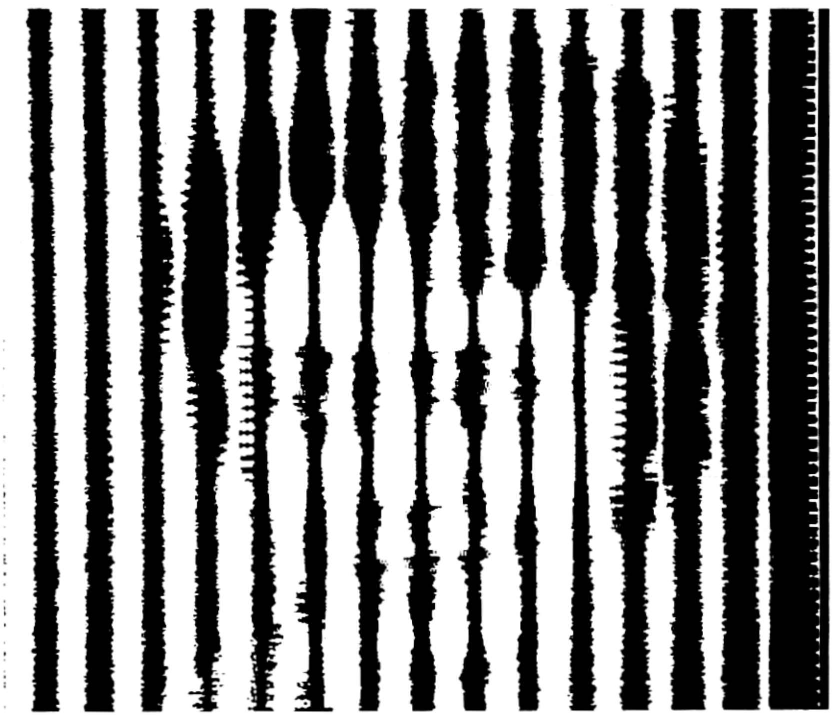

Fig. 5—Karen after 16 × 1 reordering for two-dimensional coding.

Table III—Coding length in bits per pel of the three test pictures when they are reordered and coded with the one-dimensional (1-D) and two-dimensional (2-D) codes proposed by the CCITT for standardization

	Karen		Engineering Drawing		House	
	1-D Code	2-D Code	1-D Code	2-D Code	1-D Code	2-D Code
1-D bit interleaving	0.433		0.607		0.365	
2-D bit interleaving	0.390	0.328	0.592	0.609	0.327	0.291
4 × 4 reordering for run-length coding	0.395		0.661		0.383	
2 × 8 reordering for run-length coding	0.381		0.633		0.372	
4 × 4 reordering for 2-D coding		0.514		0.701		0.475
8 × 2 reordering for 2-D coding		0.321		0.642		0.309
16 × 1 reordering for 2-D coding		0.306		0.621		0.301

Fig. 6a—Karen after 4 × 4 reordering for run-length coding.

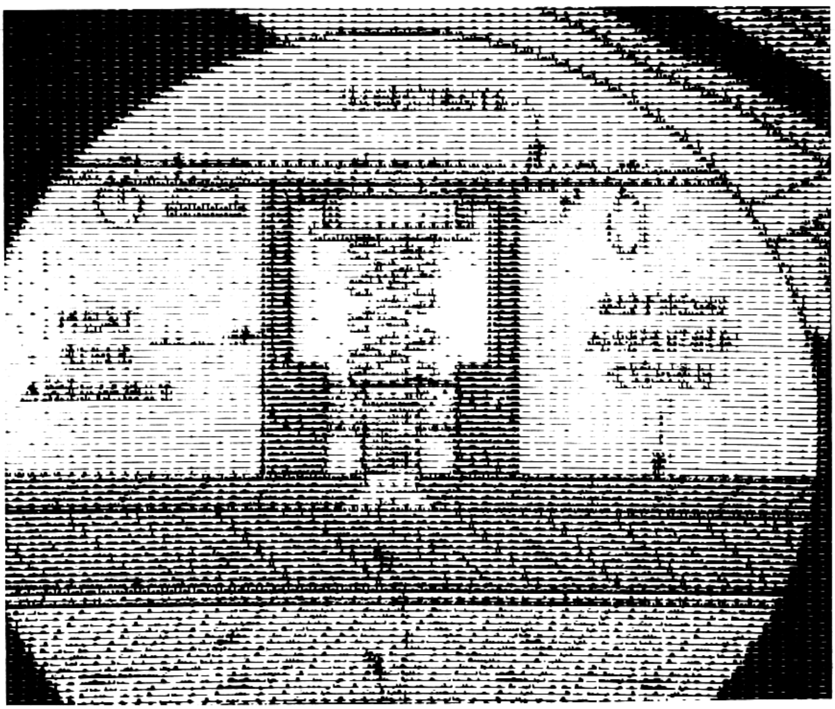

Fig. 6b—Engineering drawing after 4 × 4 reordering for run-length coding.

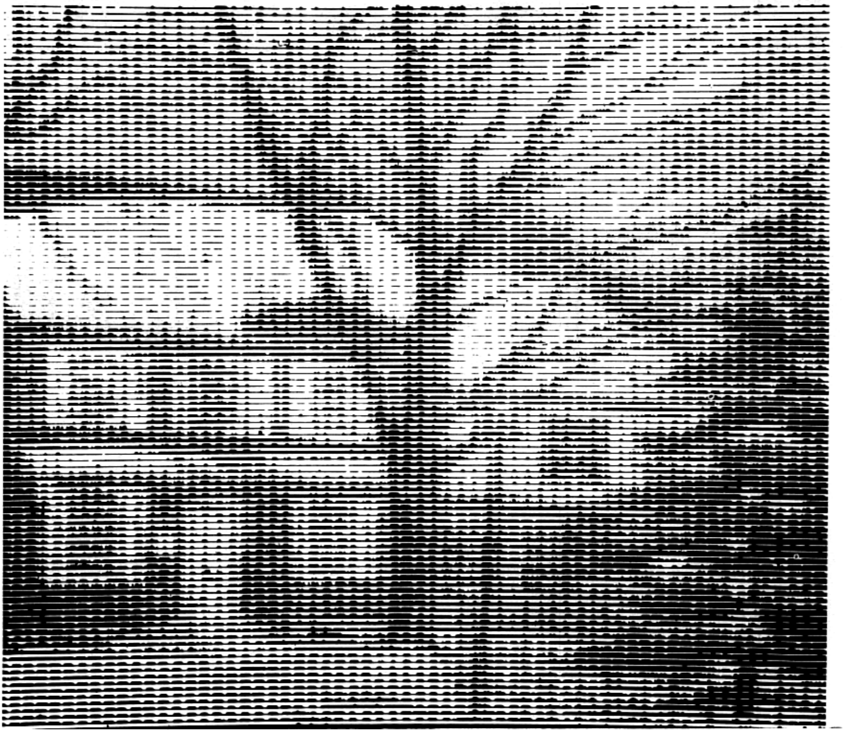

Fig. 6c—House after 4 × 4 reordering for run-length coding.

Table III shows the coding lengths. In the case of the one-dimensional CCITT code, the compressions with the 4 × 4 and 2 × 8 reordering are slightly lower than with the two-dimensional bit-interleaving scheme of Judice. Interest in the 2 × 8 reordering is small because, compared to the simpler 4 × 4 reordering, the decrease in coding length is less than 5 percent.

With the two-dimensional CCITT code, the two-dimensional bit interleaving and the 16 × 1 reordering lead to similar coding lengths. The reordering scheme seems preferable since it is simpler. The two-dimensional bit-interleaving scheme requires either the storage of the entire picture or several scans of the picture. The storage can be reduced to four lines without sacrificing the coding performance, but this requires modifying the CCITT code so that the fourth previous line is used as a reference line rather than the previous line.

One advantage of the 4 × 4 reordering for run-length coding is that the reordered picture is similar to the original dithered picture. Figure 6 shows the reordered versions of the three test pictures. Except for the "Engineering drawing," the other two pictures have reasonable

Table IV—Entropies in bits per pel for the three test pictures when they are reordered

	Karen		Engineering Drawing		House	
	Run-length Entropy	2-D Entropy	Run-length Entropy	2-D Entropy	Run-length Entropy	2-D Entropy
1-D bit interleaving	0.313		0.429		0.276	
2-D bit interleaving	0.293	0.272	0.420	0.478	0.256	0.245
4 × 4 reordering for run-length coding	0.302		0.468		0.293	
2 × 8 reordering for run-length coding	0.266		0.433		0.262	
4 × 4 reordering for 2-D coding		0.436		0.608		0.388
8 × 2 reordering for 2-D coding		0.271		0.526		0.265
16 × 1 reordering for 2-D coding		0.247		0.500		0.243

quality. Thus, a facsimile receiver without the dither postprocessor is able to reproduce a recognizable approximation of the dithered picture.

Table IV gives entropy measurements for comparison purposes. The two-dimensional run-length entropy is obtained from the distribution of the symbols used in the two-dimensional CCITT code. The entropies are about 15 to 20 percent lower than the corresponding coding lengths of Table III. Improved codes leading to coding lengths only 10 percent higher than the entropy can be devised, but are outside the scope of this study. Entropy comparisons with Ref. 8 show that higher compression can be obtained by more sophisticated techniques.

V. CONCLUSION

We have shown that dithered pictures can be transmitted efficiently with facsimile apparatus that uses either the one-dimensional or the two-dimensional codes proposed by the CCITT for standardization. The addition of a simple reversible preprocessor at the transmitter and a postprocessor at the receiver is required. The number of bits transmitted is 1.5 to 3.5 times lower than without coding. This compares favorably with compression ratios obtained by other schemes. The preprocessor consists of local reordering of pels. One of the pel reordering schemes has the advantage that a recognizable version of the original dithered picture is obtained with facsimile receivers not containing the postprocessor. This study has been made using only three small dithered pictures and a 4 × 4 dither matrix. Additional studies using a larger set of pictures and a larger dither matrix are necessary before implementation.

VI. ACKNOWLEDGMENT

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REFERENCES

1. R. Hunter and A. H. Robinson, "International Digital Facsimile Standards," Proc. IEEE, *68,* No. 7 (July 1980), pp. 854–67.
2. J. O. Limb, "Design of Dither Waveforms for Quantized Visual Signals," B.S.T.J., *48,* No. 7 (September 1969), pp. 2555–82.
3. B. Lippel and M. Kurland, "The Effect of Dither on Luminance Quantization of Pictures," IEEE Trans. Commun. Technol., *COM-19,* No. 6 (December 1971), pp. 879–88.
4. C. N. Judice, J. F. Jarvis, and W. H. Ninke, "Using Ordered Dither to Display Continuous Tone Pictures on an AC Plasma Panel," Proc. Soc. Inform. Display, *15/4* (Fourth Quarter 1974), pp. 161–4.
5. C. N. Judice, "Data Reduction of Dither Coded Images by Bit Interleaving," Proc. Soc. Inform. Display, *17,* No. 2 (1976), pp. 91–9.
6. J. F. Jarvis, C. N. Judice, and W. H. Ninke, "A Survey of Techniques for the Display of Continuous Tone Pictures on Two-Level Displays," Comput. Graph. Image Process., *5,* No. 1 (March 1976), pp. 13–40.
7. A. N. Netravali, F. W. Mounts, and J. D. Beyer, "Techniques for Coding Dithered Two-Level Pictures," B.S.T.J., *56,* No. 5 (May–June 1977), pp. 809–19.
8. O. Johnsen, "A New Code for Transmission of Ordered Dithered Pictures," B.S.T.J., preceding article in this issue.