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Numerical Calculation of Optimum α for a Germania-Doped Silica Lightguide

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Using an exact numerical solution to Maxwell's equations, we determine the optimum refractive index profile parameter α for a germania-doped silica lightguide. Our results agree closely with the earlier work of Olshansky and Keck and with the work of Marcatili. However, differences exist that may be important in the manufacture of very high bandwidth lightguides.

I. INTRODUCTION

It is now well established that a lightguide with a near parabolic index profile can have single frequency bandwidths of 1 GHz·km or greater.[1-3] This is expected to happen if the profile is smooth, the central dip is narrow or small, the profile parameter is properly chosen, and modes that have a significant amount of their energy transported in the cladding are eliminated.[4-6] Our understanding of these factors has largely come from approximate solutions to Maxwell's equations for lightguides. The availability of computer software for exact numerical solutions[7] to Maxwell's equations allows us to reexamine some of these effects and to look at other factors that may influence lightguide bandwidths. In this paper, we determine optimum power law profile parameters (i.e., α parameters) as a function of wavelength for a germania-doped silica lightguide.

II. PROCEDURE

We study a lightguide with a 13.5 mole percent germania-doped silica core and a pure silica cladding. The core is assumed to be perfectly cylindrical with a diameter of 50 microns. The cladding is considered to be essentially infinite. The doping profile is such that the refractive index is a power law function of distance R. Thus,

$$N = N_1 + (N_2 - N_1)\cdot\left(\frac{R}{R_{cc}}\right)^{\alpha}, \qquad (1)$$

where N_1 is the refractive index at the center of the core, N_2 is the refractive index in the cladding, R_{cc} is the radius of the core, and α is the profile parameter. We assume that this functional form holds at all wavelengths of interest.

We have accurately determined the wavelength variation of the refractive index of silica and some doped silicas. Figure 1 gives the data for 13.5 mole percent germania-doped silica and pure silica. As is well known, the refractive index falls off slowly with wavelength for both of these glasses.[8] We express the data by the following modified Sellmeier formula:[9]

$$N_i = C_0 + C_1\lambda^2 + C_2\lambda^4 + C_3/(\lambda^2 - 0.035) + C_4/(\lambda^2 - 0.035)^2 + C_5/(\lambda^2 - 0.035)^3, \qquad (2)$$

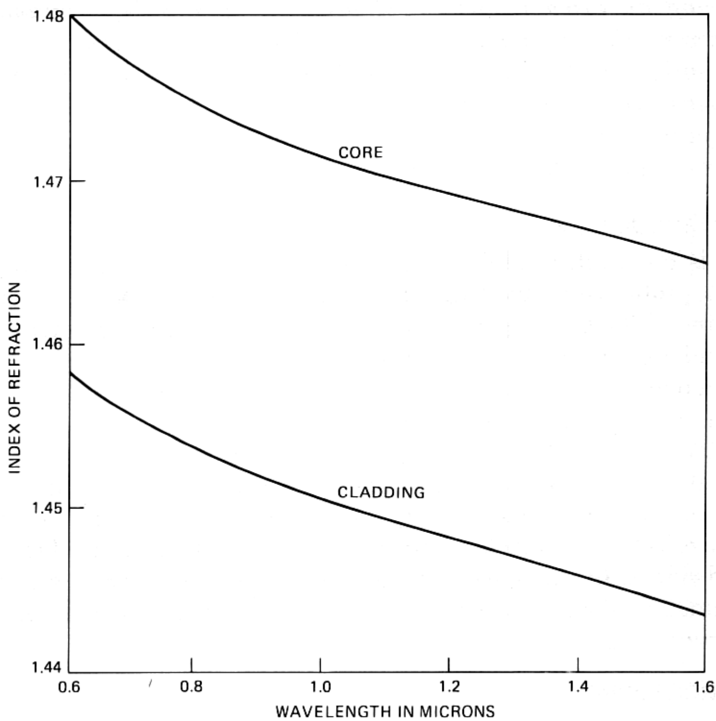

Fig. 1—The wavelength dependence of 13.5 mole percent germania-doped silica and pure silica.

where λ is in microns and $i = 1, 2$. Table I lists the coefficients for eq. (2) for both 13.5 mole percent germania-doped silica and silica.

Our numerical procedure to solve Maxwell's equations has been described in detail in an earlier paper.[7] Thus, we only give a very brief description here. The method is similar to that described by Vigants and Schlesinger,[10] and Vassel.[11] We write Maxwell's equations in cylindrical coordinates as

$$\frac{d\Gamma}{d\rho} = \rho^{-1}A(\rho)\Gamma(\rho), \tag{3}$$

where $\Gamma(\rho)$ is a column vector whose four elements are related to the tangential components of the electromagnetic field. They are as follows:

$$\begin{aligned} \Gamma_1 &= E_z, \\ \Gamma_2 &= -i\rho H_\phi Z_0, \\ \Gamma_3 &= \rho E_\phi, \\ \Gamma_4 &= -iH_z Z_0, \end{aligned} \tag{4}$$

where Z_0 is the wave impedance of free space and the variable ρ is defined as K_0R.

The 4×4 matrix $A(\rho)$ can be written as

$$\begin{vmatrix} 0 & (N_e^2/\kappa) - 1 & 0 & -MN_e/\kappa \\ \rho^2\kappa - M^2 & 0 & MN_e & 0 \\ 0 & MN_e/\kappa & 0 & \rho^2 - (M^2/\kappa) \\ -MN_e & 0 & N_e^2 - \kappa & 0 \end{vmatrix} \tag{5}$$

This matrix describes the properties of the media.

We solve eq. (3) by an optimized fourth-order Runge–Kutta procedure.[12] This is a higher-accuracy formula than used in our earlier work. The most important computational results are the effective indices N_e for the various modes in the lightguide.

The group indices N_g are calculated from the effective indices by the following formula:

$$N_g = N_e - \lambda dN_e/d\lambda. \tag{6}$$

Table I—Coefficients for eq. (2)

13.5 Mole Percent Germania-Doped Core	Silica Cladding
$C_0 =$ 1.4706868	1.4508554
$C_1 =$ −0.0026870	−0.0031268
$C_2 =$ −0.0000356	−0.0000381
$C_3 =$ 0.0035756	0.0030270
$C_4 =$ −0.0000828	−0.0000779
$C_5 =$ 0.0000018	0.0000018

The derivative is obtained from numerical calculations of N_e at three closely spaced wavelengths.

For a lightguide with the materials and profile described earlier, a simple approximate relationship exists between group index N_g and effective index N_e for certain modes. In particular, for a given profile parameter α, angular mode number M, wavelength λ, and for modes far from cutoff, the effective indices are nearly linearly related to the group indices. Evidence for this can be obtained from Fig. 2, curve (a). In this case, we have $\lambda = 0.6328$ micron, $M = 0$, and $\alpha = 2$. The solid line is the linear least-squares fit to the data when the last two modes

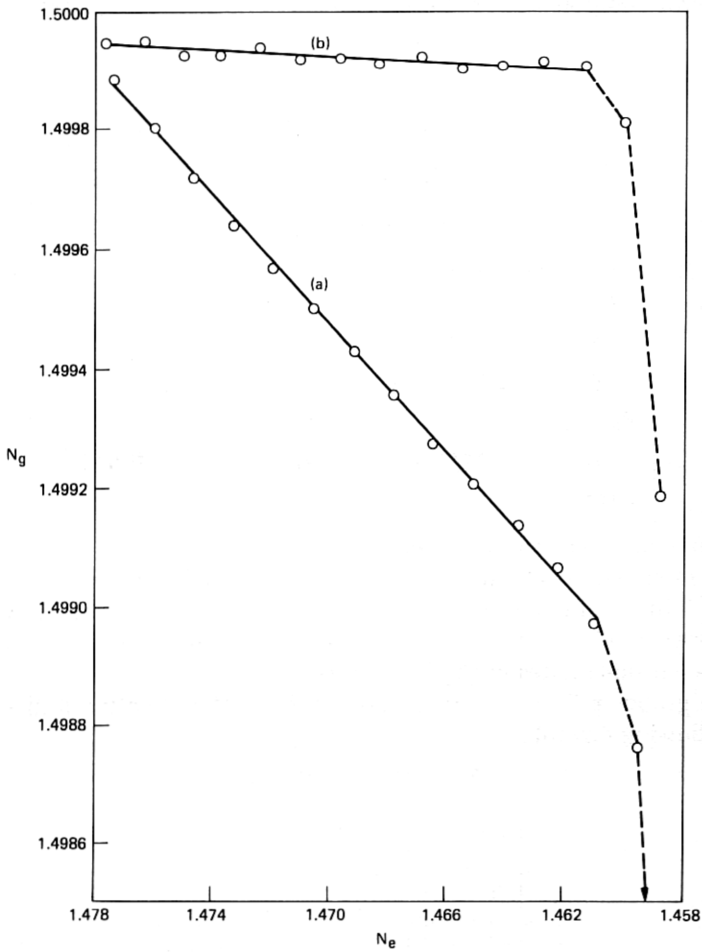

Fig. 2—Plot of group index (N_g) versus effective index (N_e) at a wavelength (λ) of 0.6328 micron with $M = 0$. In (a) the profile parameter α is 2.0 while in (b) α is 2.2. Note that a good linear relationship exists between N_g and N_e if the last few modes are neglected.

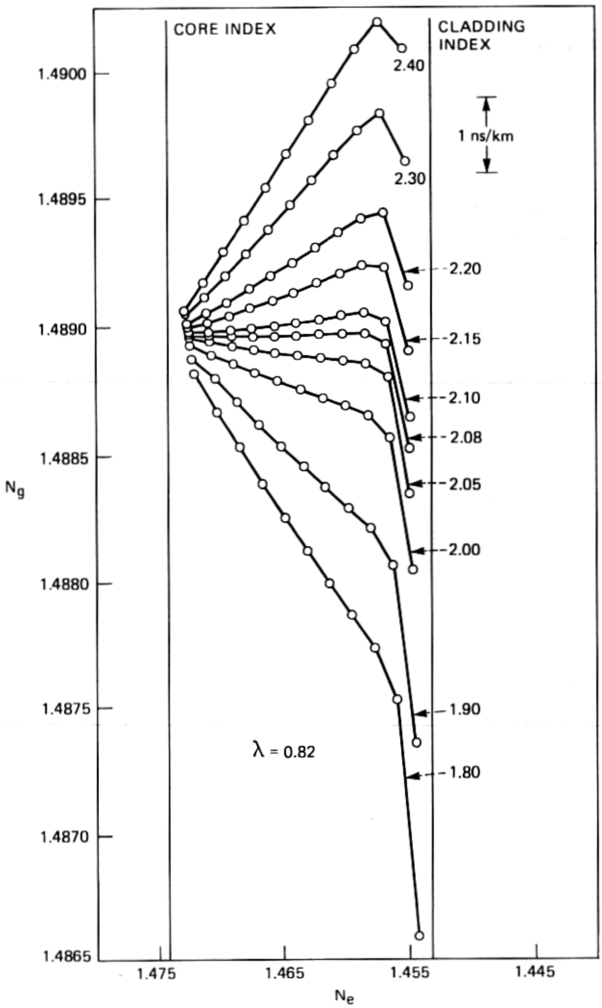

Fig. 3—Plot of group index versus effective index for $\lambda = 0.82$ micron and $M = 0$. When $\alpha = 2.08$ the group indices are all nearly the same except for those modes being influenced by the cladding.

are deleted. (The last mode is too far off scale to be plotted.) Obviously a nearly linear relationship exists between N_g and N_e. In addition, there is a very slight systematic deviation from the least-squares line. This suggests that additional terms are needed to completely describe the data. However, this will be of no consequence to the analysis to be described here.

The last two modes (those that deviate strongly from the line) have a significant fraction of their energy being transported in the cladding.

Thus, they are not tightly bound to the core and normally are not considered in an optimum α calculation.

An optimum profile parameter α, for a particular family of modes, is one that minimizes the spread in group indices for that family at a particular wavelength. For example, we might consider all the modes associated with a specified M value. In this case, the optimum α would be the one that gives the smallest slope to the least-squares line relating N_e and N_g.

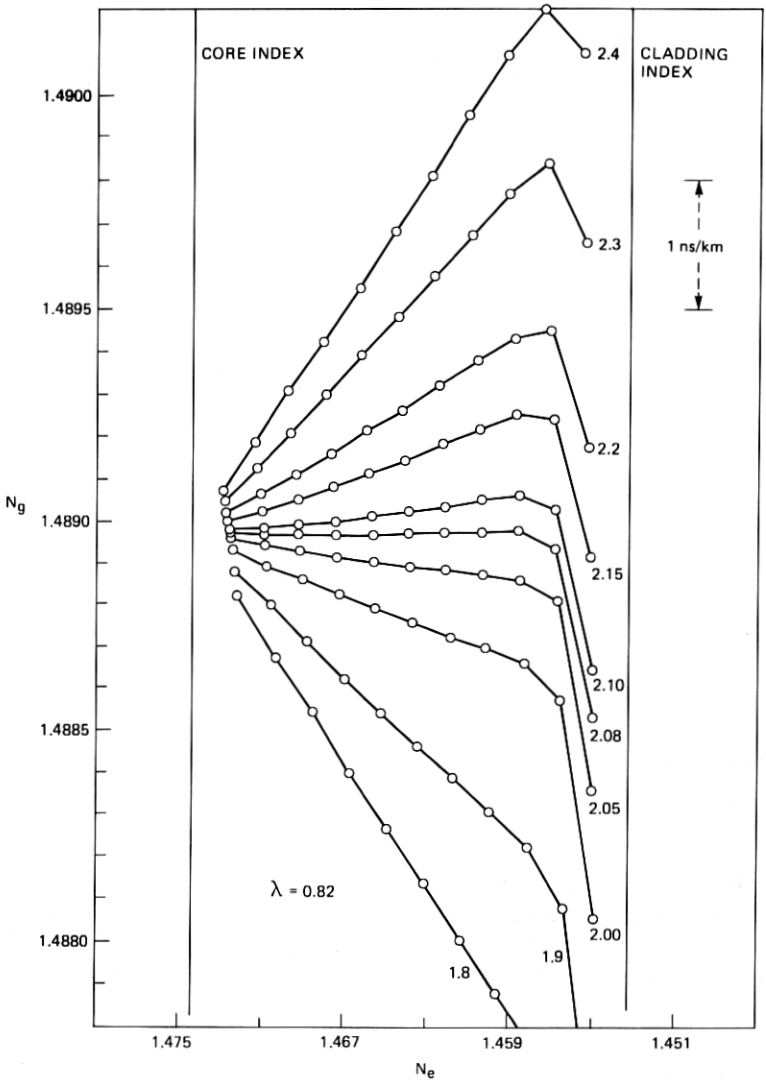

Fig. 4—Part of the data of Fig. 3 on an expanded scale.

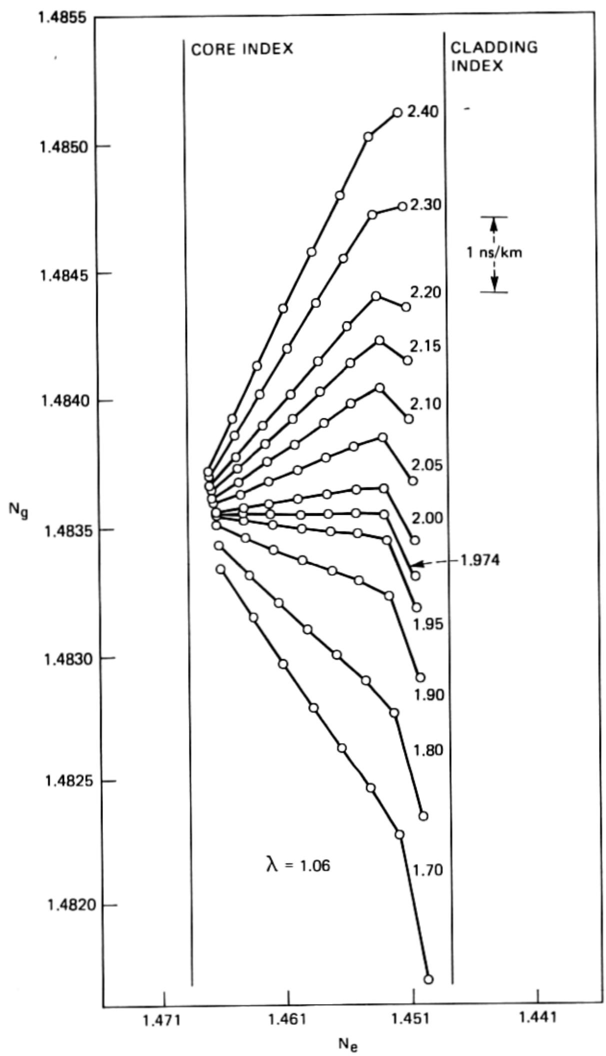

Fig. 5—Plot of group index versus effective index for $\lambda = 1.06$ microns and $M = 0$.

Figure 2, curve (b) shows a near optimum value of $\alpha = 2.2$ for $M = 0$ and $\lambda = 0.6328$ micron. We see that the least-squares line is now nearly horizontal and that evidence exists for the necessity of higher-order terms to exactly fit the data. The strong drop-off of the last few modes is still evident.

A very good overall optimum α for all M values will be nearly identical to the optimum α for $M = 0$. As will be shown shortly, this results in excellent equalization of the lower-order modes and a good equalization of the higher-order modes. For most engineering applications, optimum α's in the wavelength range of 0.63 to 1.55 microns are needed.

III. RESULTS

Figures 3 through 7 show plots of N_g versus N_e for various α parameters and wavelengths for $M = 0$. Figure 4 shows part of the

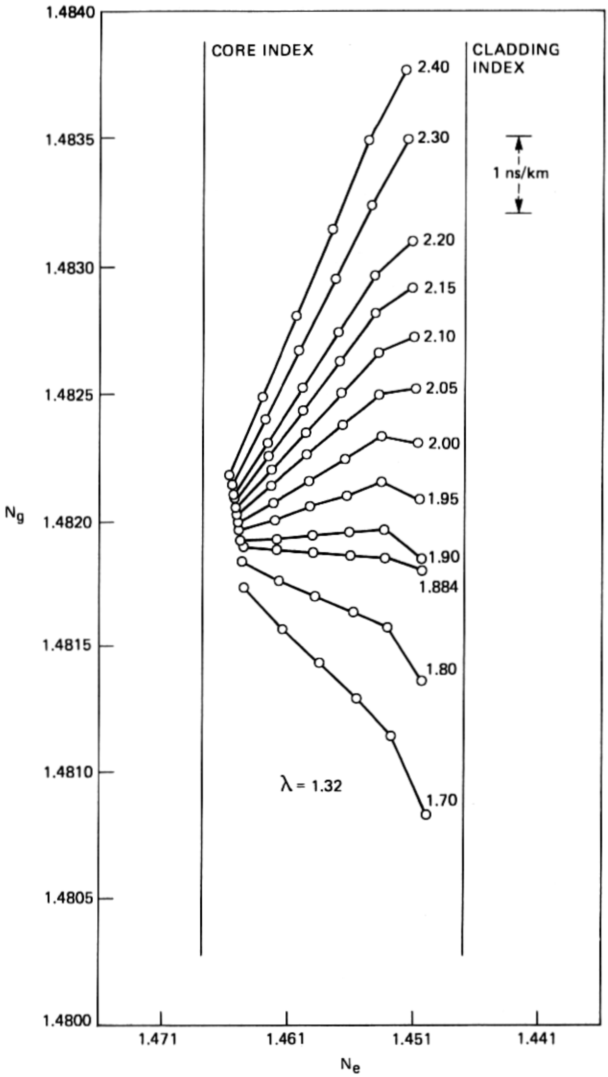

Fig. 6—Plot of group index versus effective index for $\lambda = 1.32$ microns and $M = 0$.

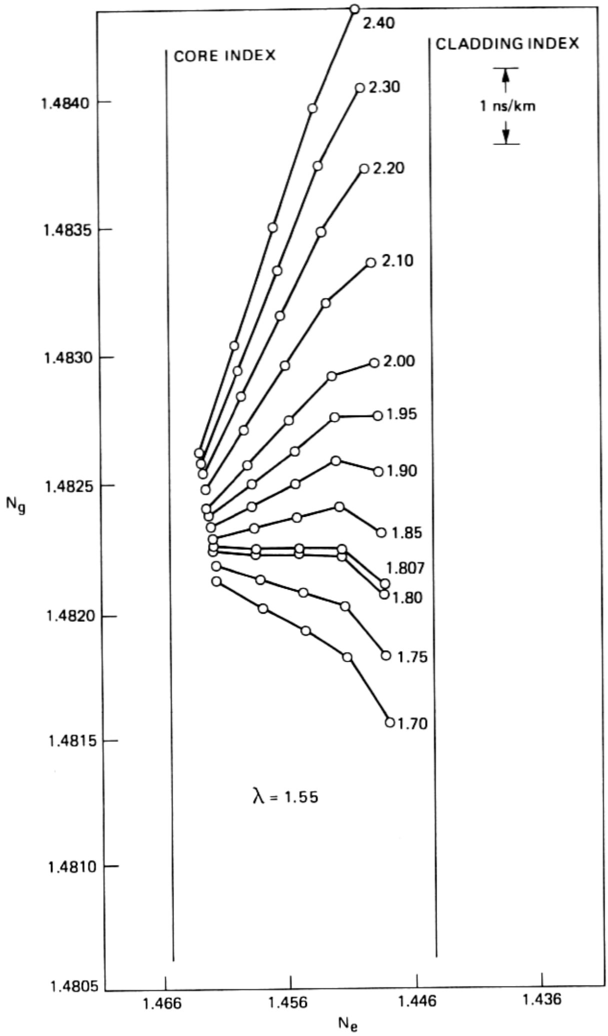

Fig. 7—Plot of group index versus effective index for λ = 1.55 microns and $M = 0$.

data of Fig. 3 on an expanded scale. Note that in all a nearly linear relationship exists between N_g and N_e, except for the last couple of modes. As mentioned before, these modes are nearing cutoff and are usually not considered in an optimum α determination.

At each wavelength, we see that there is an α value, α_{opt}, that makes the slope of the line nearly zero. Table II lists α_{opt} values for wavelengths of engineering importance and also for a few other wavelengths as well.

The data of Figs. 3–7 pertaining to the infrared wavelengths are

Table II—Optimum alpha values

λ (microns)	α_{opt}
0.6328	2.211
0.73	2.130
0.82	2.081
0.90	2.037
0.983	2.000
1.06	1.974
1.20	1.925
1.32	1.884
1.40	1.856
1.55	1.807

summarized in Figs. 8 and 9. Figure 8 shows the slope of the line relating N_g to N_e as a function of wavelength for various α values. We see that most infrared wavelengths of interest require an α value in the range from 1.8 to 2.1. Figure 9 shows the same data, but this time as a function of α for various wavelengths. The same conclusion is obvious.

Figure 10 displays a calculation of the group indices for all the bound

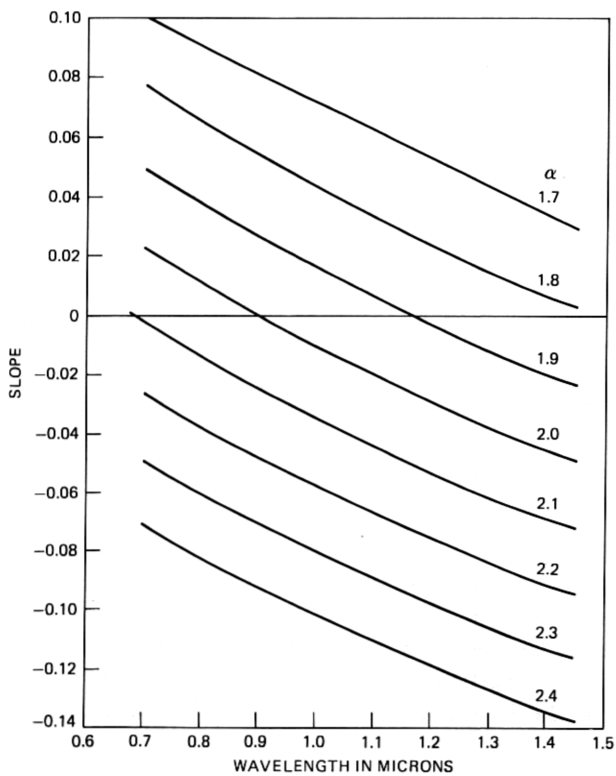

Fig. 8—Plot of the slope of N_g versus N_e as a function of λ for various α values.

modes in the fiber for $\lambda = 0.82$ micron and $\alpha_{opt} = 2.081$. This plot gives a vivid representation of the fiber characteristics. We see that indeed $\alpha = 2.081$ gives a good equalization for all the modes if those being influenced by the cladding are ignored. A delay-time calibration corresponding to 1 ns/km is also included in the plot.

IV. DISCUSSION

Within the framework of the WKB approximation, Marcatili[13] has obtained an exact result for α_{opt} that minimizes pulse widths. His formula is as follows:

$$\alpha_{opt} = (2 - p)(1 + \sqrt{1 - 2\Delta}) - 2, \tag{7}$$

where

$$p = \frac{N_1}{N_g^0} \cdot \frac{\lambda}{\Delta} \cdot \frac{d\Delta}{d\lambda}, \tag{8}$$

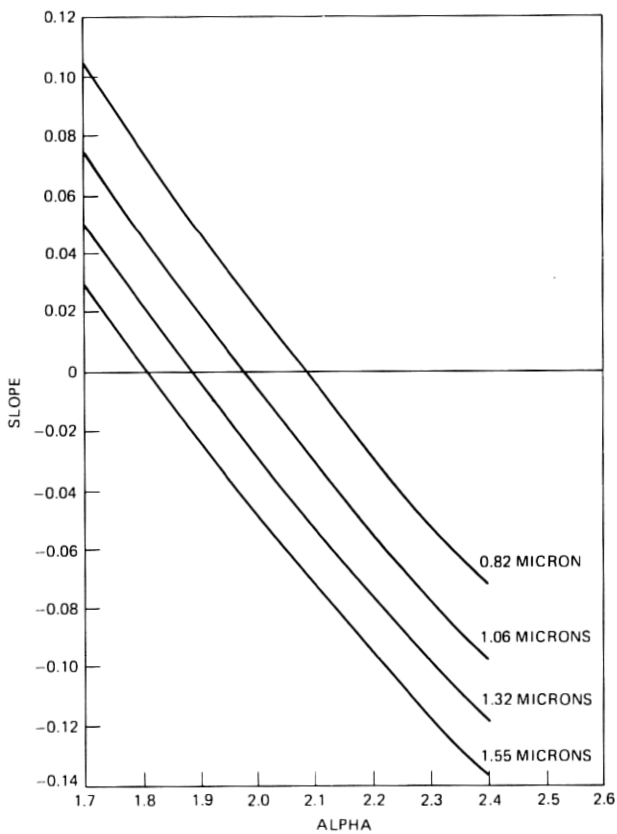

Fig. 9—Plot of the slope of N_g versus N_e as a function of α for various λ values.

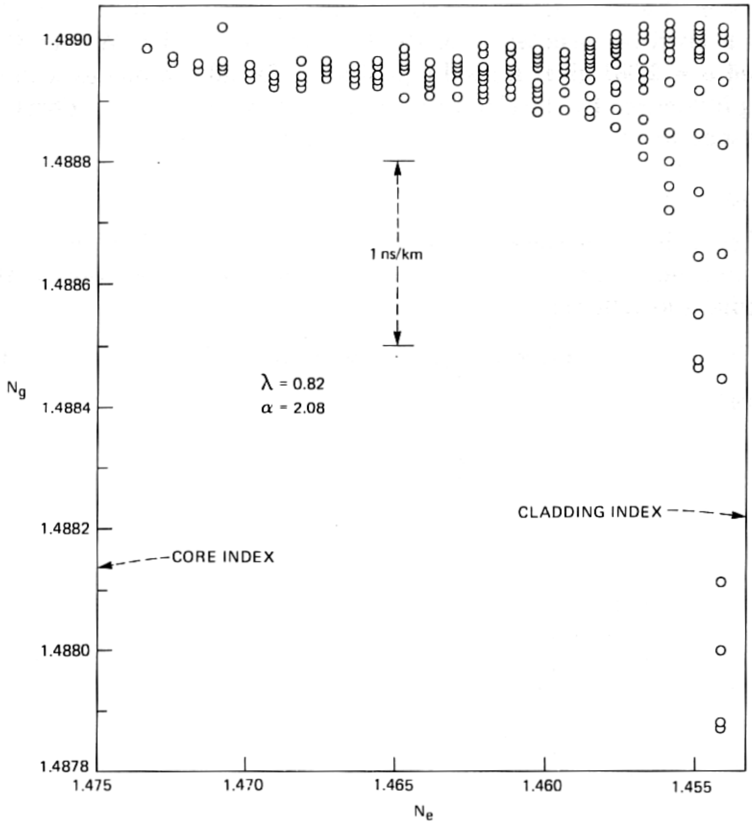

Fig. 10—Plot of group index versus effective index for all the bound modes in the lightguide. We see that for λ = 0.82 micron and α = 2.081 there is a good equalization of all the group indices.

$$N_g^0 \text{ is the group index on axis, and } \Delta = \frac{N_1 - N_2}{N_1}. \tag{9}$$

Also, within the same WKB framework, Olshansky and Keck[14] have obtained an approximate formula for α_{opt} that minimizes the rms width of a pulse. Approximations that coincide with Olshansky and Keck's can also be obtained from the exact result of Marcatili.

It is clear that α_{opt} depends upon the type and number of approximations employed, how the various modes are weighted, and whether we wish to minimize pulse width, rms width, or some other parameter. Another possibility would be to define an optimum α that would include the effects of those modes having significant energy transported in the cladding. This is likely to lead to an α quite different from that obtained in this work.

Figure 11 shows the results of Olshansky, Marcatili, and ourselves. There is great similarity in the shape of the curves and in the actual numbers. The deviation between our results and the WKB results is largest at the longer wavelengths. It is obvious that the agreement between all results is good and, at the present state of the manufacturing art, can be considered the same. However, this may not necessarily be the case in the future. To illustrate (see Fig. 12), we calculate N_g as a function of N_e at 1.55 microns for $M = 0$ and $\alpha_{opt} = 1.775$ (Olshansky) and $\alpha_{opt} = 1.807$ (this work). There is an obvious difference in slope; this difference would be significant in the design of high bandwidth lightguides.

A good representation of our optimum α values is given by the following formulas:

$$0.63 \text{ micron} \leq \lambda \leq 1.00 \text{ micron},$$

$$\alpha_{opt} = 2.9970857 - 1.6647237\lambda + 0.6629181\lambda^2, \tag{10}$$

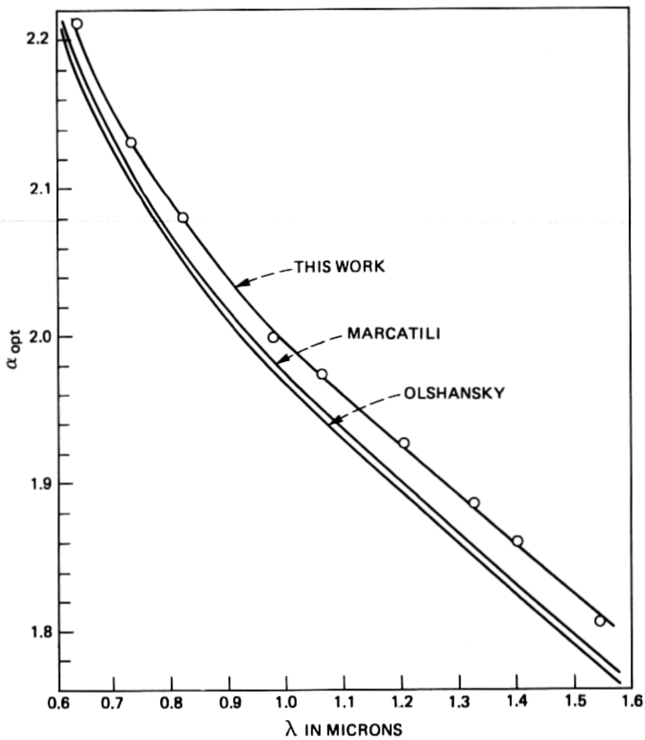

Fig. 11—Plots of optimum profile parameter α versus wavelength for a 13.5 mole percent germania-doped silica lightguide. The index dispersion data employed is from Fleming's work. A clear similarity exists among the results of Marcatili, Olshansky, and this work.

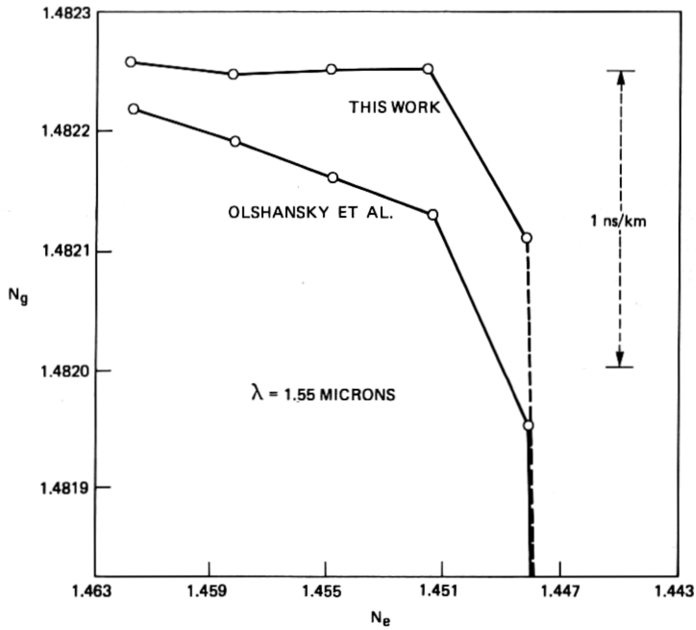

Fig. 12—A plot of N_g versus N_e for $\lambda = 1.55$ microns. $\alpha_{opt} = 1.775$ is calculated from the formula of Olshansky et al., and $\alpha_{opt} = 1.807$ is from this work.

$$1.00 \text{ micron} \leq \lambda \leq 1.55 \text{ microns},$$

$$\alpha_{opt} = 2.3835734 - 0.4170708\lambda + 0.0290252\lambda^2. \tag{11}$$

It is reasonable to assume that by linear interpolation or extrapolation we can generate the refractive index dispersion curves for germania-doped silica in the range from 0 to 16 mole percent. Using this data, we can then calculate the dependence of N_g on N_e for a number of levels of doping. Figure 13 shows a family of curves for $M = 0$, $\alpha = 1.884$, and $\lambda = 1.32$ microns. There is a good equalization of N_g for all the compositions. We also see that, as expected, the number of modes decreases as the doping level drops. Thus, at the 1 percent doping level for $M = 0$ only one N_e exists. As usual, the last couple of modes for each doping level show a substantial cladding effect.

Finally, the data displayed in Figs. 8 and 9 may have some implications to lightguide engineering. From Fig. 8, we see that the various α curves pass through zero with about the same slope. This suggests that the range of operating wavelengths will be the same for all design wavelengths. Likewise from Fig. 9, we see that the various λ curves

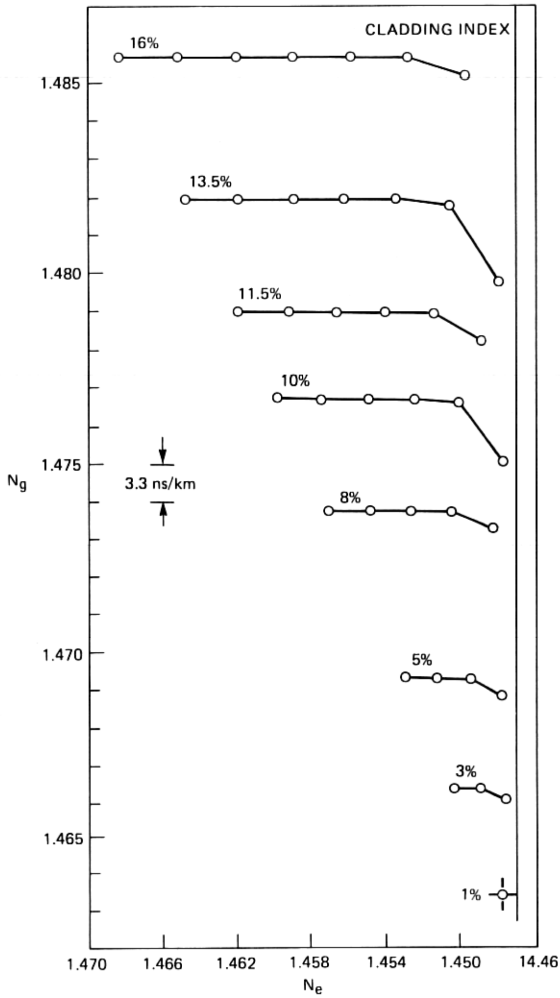

Fig. 13—A family of plots of N_g versus N_e for various doping percentages of germania. The α value is 1.884 and $M = 0$.

pass through zero with similar slopes. This suggests that the allowable error in α will be about the same for all design α's.

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