THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 60, No. 5, May–June 1981
Printed in U.S.A.

Impact of Forecast Uncertainty on Feeder-Cable Sizing

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(Manuscript received September 8, 1980)

This paper estimates the forecast error distribution for outside plant using data from the central office forecast measurement plan. We then determine the impact of the forecast errors on feeder-cable sizing by using this distribution to estimate the conditional distribution of engineered cable size with respect to optimum cable size. The marginal distribution of optimum cable size is estimated from a growth rate distribution which in turn is estimated from cable shipment data. We then use the resulting joint distribution to weight the percentage cost penalty of each possible combination of optimum and forecast size. The impact analysis is done separately for each gauge. By weighting by the million conductor feet of each gauge shipped, we then obtain an estimate of overall sizing-error cost penalty. The resulting penalty estimate is about 0.5 percent of the annual feeder-cable-construction program.

I. INTRODUCTION

A feeder route is a major network of cables extending from the central office to within ½ mile or so of customers.[1] When a feeder route needs relief, a cable size is selected with the goal of minimizing the discounted sum of costs over time. Because of forecast errors, however, sometimes a cable that is larger or smaller than the optimum is placed. One often hears that the feeder-cable sizing curves are so flat that sizing decisions are relatively insensitive to forecast errors. On the other hand, a small percentage of a large construction program still represents a substantial amount of money. In this paper we attempt to quantify the impact of forecast deviations on the feeder network by first estimating the error distribution and then using it to examine the effect of forecast errors on feeder-cable sizing. While it is not presented here, a preliminary study indicates that the impact of forecast deviations on feeder-cable relief timing is at least as great as that on sizing.

II. FORECAST ERROR DISTRIBUTION

In this section, we derive an estimate for the distribution of forecast errors using data from the central office forecast measurement plan. It should be noted that all deviations between forecast and actual are included here under the forecast error category. Thus the forecast errors include some deviations caused by count errors and others caused by boundary changes that have not been reflected accurately in the records.

2.1 Nomenclature

The units of primary concern are available pairs, which include both working and idle pairs. The data available for estimating the forecast error distribution, however, are in terms of main stations (plus equivalent main stations). The distribution will therefore be derived first in terms of main stations plus equivalent main stations and then converted into available pairs.

The basic items of interest are defined here:

b = base in-service or total value (the actual on which the forecast was based),

t = forecast interval, in years,

f = forecast in-service value, and

a = actual in-service value for the date for which the forecast was made.

Several important variables are derived from the above basic ones:

$$\epsilon = f - a = \text{forecast deviation,}$$

$$g_f = \frac{f - b}{t} = \text{forecast average of annual growth rate, and}$$

$$g = \frac{a - b}{t} = \text{actual average of annual growth rate.}$$

These items are shown in Fig. 1.

2.2 Data description

Several years ago, the central office forecast measurement plan (COFMP) was established to collect forecast data from the Bell System operating companies. These data were collected for short-term wire center forecasts. The data used in this study were collected in the fourth quarter of 1978. For each of 1266 wire centers, we had the following:

(*i*) identification (company, area, and wire center),

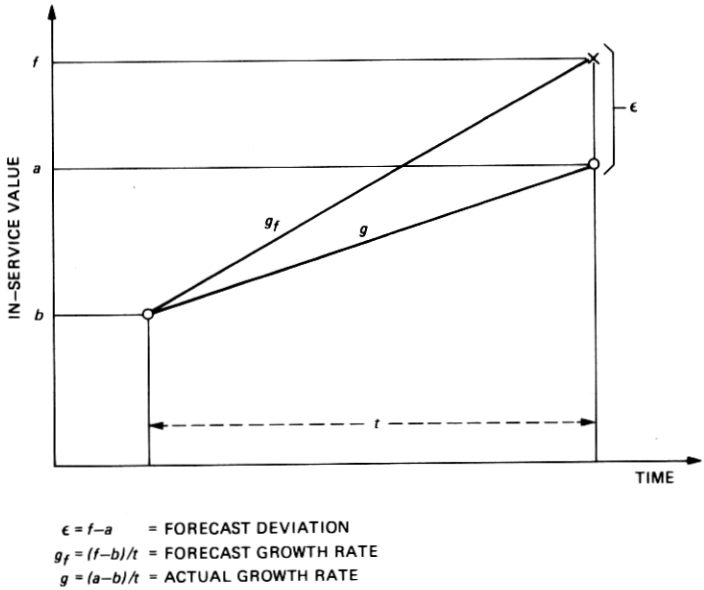

Fig. 1—Definitions of forecast variables.

(*ii*) b, f, and a, for $t = 1$, and

(*iii*) the number of main stations transferred from a wire center to another one during the forecast interval.

The values b, f, and a are in terms of main stations (plus equivalent main stations).

Table I gives examples of the above data items.

In addition to the above items, we had two items for each wire center that were not used in the study. For each wire center, the month of the end of the forecast period was available. Since in all cases, the month fell within a three-month period, we did not feel that the differences would be significant. A seasonal indicator was also available for each wire center. Several wire centers were flagged as

Table I—Examples of COFMP data

Company	Area	Wire Center	Main stations (plus equivalents)			
			Base	Forecast	Actual	Transfer
4	17	109	17785	18260	18400	+23
7	34	344	871	910	891	0
7	35	368	10277	10964	11021	0
10	40	430	3171	3240	3296	0
14	54	584	5715	5925	6013	0
14	56	596	2187	2365	2286	0
16	67	1141	40317	42205	42508	+374
17	68	1158	21618	24626	24378	0
18	73	1254	2054	2110	2100	0

having the annual maximum occur at some time other than the end of the year. This information was not needed for the study, since all forecast intervals were exactly one year.

Since it was desirable to model the forecast results in terms of growth rates, the forecast and actual values (f and a) were adjusted for any wire center that had a transfer by subtracting the signed value of the transfer from them. This allowed us to compare the forecast and actual growth rates for the original serving areas.

An initial examination of the data showed that one company had a much larger percentage of its wire centers represented than did any other company. Of the 1266 wire centers, 507 were from that company. The COFMP was intended to collect data only for wire centers that have at least 500 main and equivalent-main telephones and that have a traffic order prepared while the forecast is in effect. Eliminating wire centers that are less than 500 in size, and arbitrarily retaining every third wire center of size 500 or greater reduced the representation from that company to the point where it was similar to that of the other companies. At this point, we retained 863 of the 1266 wire centers, and felt that they provided a representative cross section of the Bell System. In Fig. 2, the forecast growth rate is plotted versus the actual growth rate for these 863 wire centers.

2.3 Model development

From Fig. 2, it is obvious that the variance is increasing with g. We tried both square root and log transformations and found that the log transformation, with a shift of 50 for both g_f and g, did an excellent job of stabilizing the variance.

To be able to use a shift of 50, we had to drop 12 data points with g_f and/or g less than or equal to -50. Six of these points were of relatively little interest, for both g_f and g were negative, and we expect the model to be used for cases where cable is placed to accommodate growth ($g_f > 0$) or where it should be placed ($g > 0$). The other six points had either g_f or g less than -60, with the other value positive (all except one were greater than $+120$). These six points would have been outliers and should have received small weights for any reasonable model. Therefore, their loss is not serious.

The remaining 851 data points are shown in Fig. 3, where $\ln(g_f + 50)$ is plotted against $\ln(g + 50)$. At a glance, it appears that the variance is now decreasing with g. That this is not the case will be shown later. Basically, the illusion is due to more points existing at the lower g values.

We used robust regression to estimate the relationship between g_f and g. After an initial ordinary least-squares step, we used a Huber iteration to downweight points with residuals more than 1.5 standard

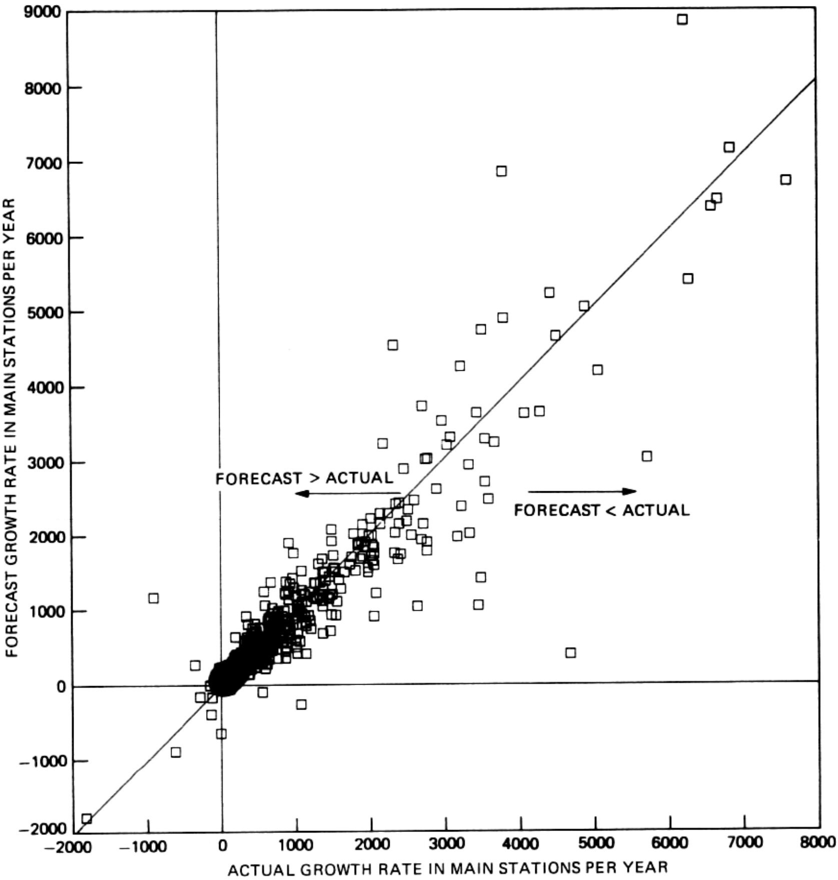

Fig. 2—Forecast vs actual growth rates.

deviations away,[2] using the estimate of the standard deviation obtained from the first step. We followed the Huber step by a biweight iteration,[3] using a dispersion value six times the Huber estimate of the standard deviation.

The growth rate model is

$$\ln(g_f + 50) = \alpha + \beta \ln(g + 50) + \nu, \tag{1}$$

where α and β are parameters to be estimated and ν is a residual noise term with mean 0 and a variance σ^2 to be estimated. The estimates resulting from each step of the regression are given in Table II.

Figures 4 through 7 show residual plots for the residuals from the final step. The plots against the dependent and independent variables shown in Figs. 4 and 5 indicate reasonably well-behaved residuals. Although it would have been difficult to make use of any relationship

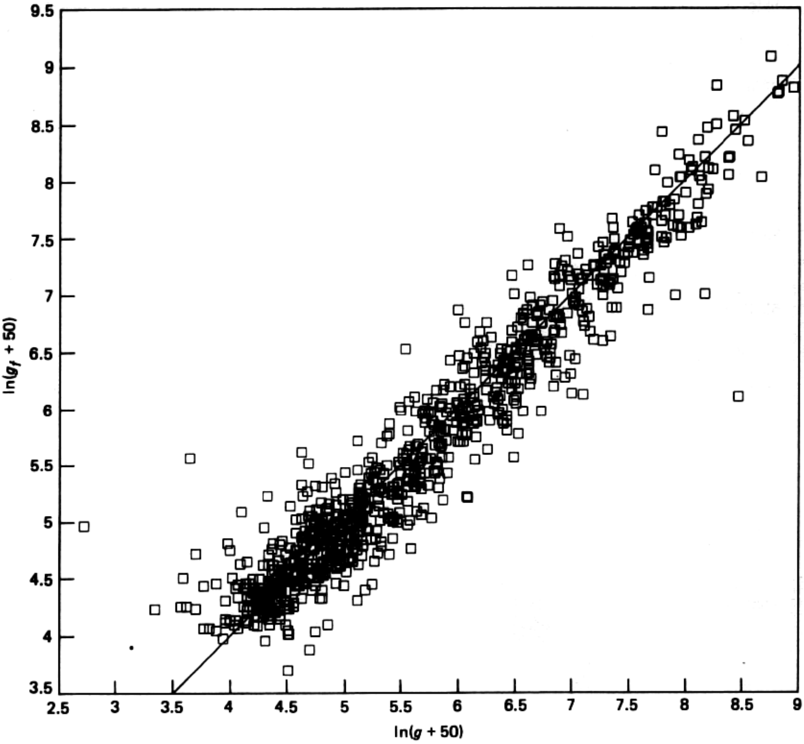

Fig. 3—ln(g_f + 50) vs ln(g + 50).

involving the size of an area for which a forecast is produced, the residuals were plotted versus the base size in Fig. 6. Figure 6 indicates that there is no structure involving the base size that needs to be included in the model. Finally, the residuals are shown for each of the 19 Bell System operating companies in Fig. 7. Here, too, there is no obvious need to include a company effect in the model. The larger extreme residuals generally occur for those companies with a larger number of data points.

It should be pointed out that Figs. 4 through 7 show only 847 of the 851 points. The other four points are shown in Table III. Three of these are outliers that received zero weight in the biweight iteration and one lies just beyond the range that was plotted.

Table II—Results of each step of the regression

Step	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\sigma}$	R^2
Ordinary Least Squares	0.363	0.929	0.322	0.920
Huber	0.277	0.943	0.285	0.937
Biweight	0.250	0.948	0.274	0.942

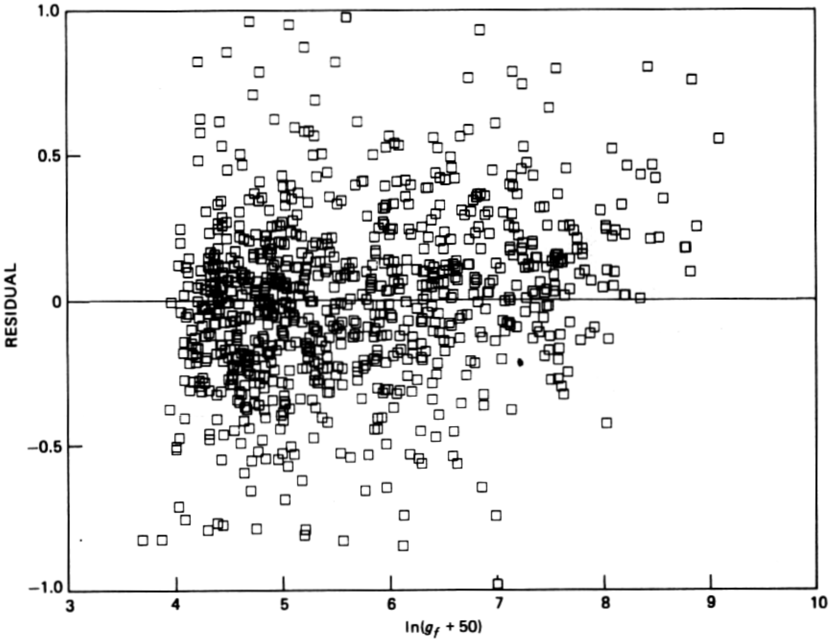

Fig. 4—Residual vs ln(g_f + 50).

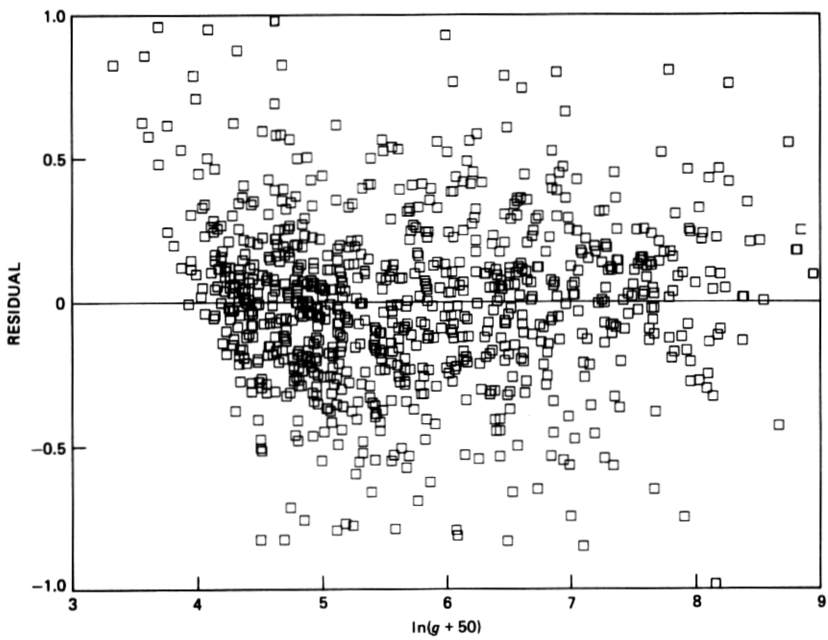

Fig. 5—Residual vs ln(g + 50).

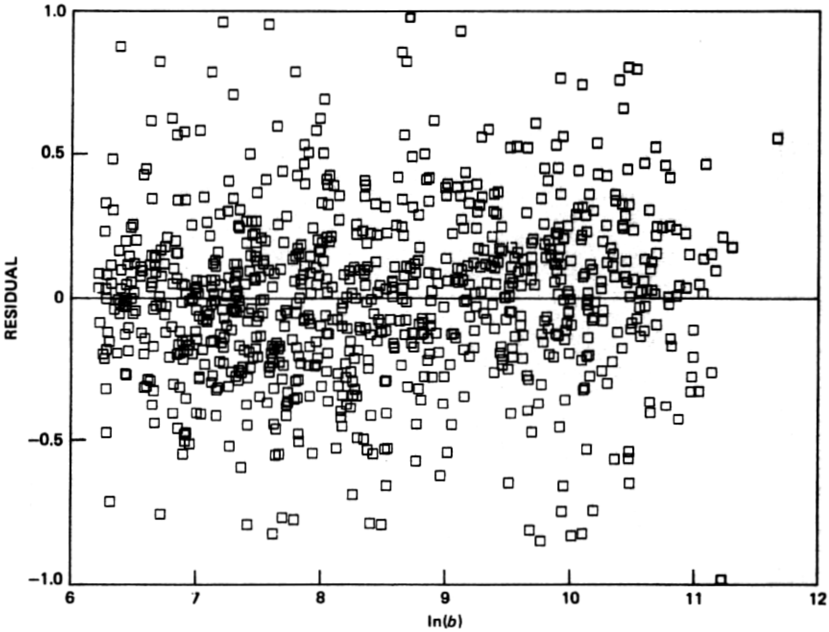

Fig. 6—Residual vs ln(b).

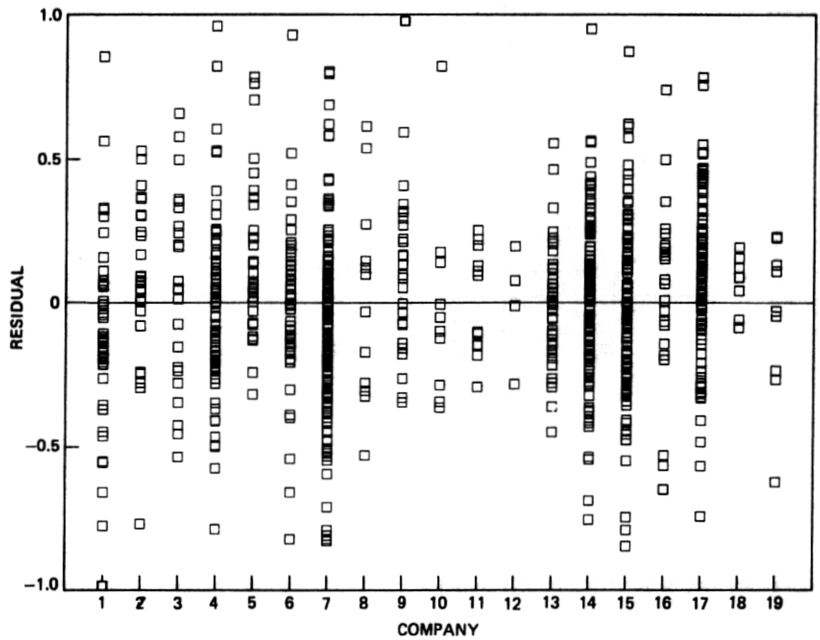

Fig. 7—Residual vs company.

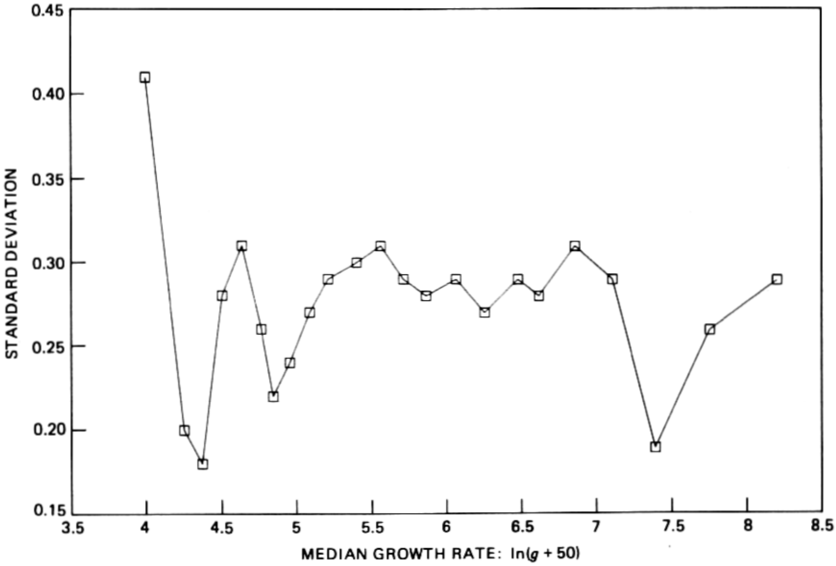

Fig. 8—Standard deviation vs $\ln(g + 50)$.

To determine if the variance of the residuals is sufficiently constant with respect to g, the 851 data points were ranked by g and divided into 23 groups of 37 points each. For each group, an unbiased estimate of the standard deviation was calculated, using the weights resulting from the biweight step in the regression. The resulting values are shown in Fig. 8, plotted versus the median values of $\ln(g + 50)$. No overall trend is obvious in Fig. 8 and regression confirms that it is reasonable to assume a constant variance.

A standardized deviate was found for each data point by subtracting the value predicted by the regression model (1) and dividing by the regression standard deviation,

$$d \equiv \frac{\ln(g_f + 50) - [0.250 + 0.948 \ln(g + 50)]}{0.274}. \tag{2}$$

A Q-Q plot of these deviates against the standard unit normal showed an excellent fit for the bulk of the data but with tails larger than given

Table III—Residuals for the four points not shown in Figs. 4 to 7

Wire Center	Residual	$\ln(g_f + 50)$	$\ln(g + 50)$	b	Company	Biweight Weight
128	2.14	4.96	2.71	46,328	4	0
138	1.86	5.56	3.64	15,232	4	0
153	−2.17	6.10	8.46	15,587	4	0
467	1.03	6.52	5.53	17,255	13	0.410

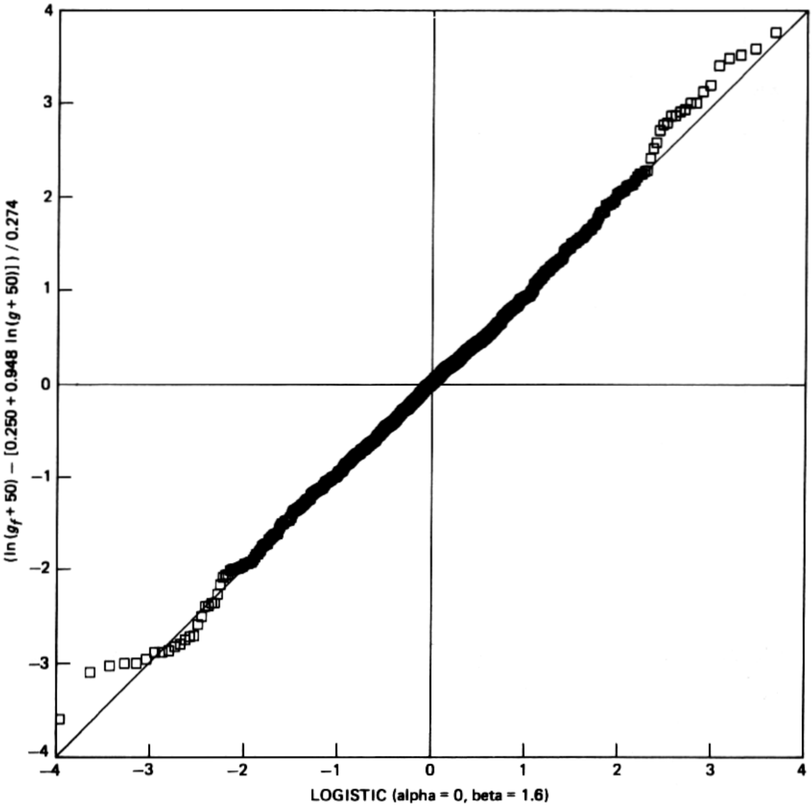

Fig. 9—Q–Q plot of empirical distribution vs logistic.

by the normal. Substituting for the normal, a logistic distribution with parameters $\alpha = 0$ and $\beta = 1.6$ (mean 0 and variance 1.29) resulted in the satisfactory Q–Q plot shown in Fig. 9. All but three of the 851 points are shown in Fig. 9. The other three are given in Table IV.

We conclude that it is reasonable to assume that the density and distribution functions of d are given by

$$f(d) = \frac{1.6\, e^{-1.6d}}{(1 + e^{-1.6d})^2} \qquad \text{and} \qquad F(d) = \frac{1}{1 + e^{-1.6d}}. \tag{3}$$

The modeling was done using COFMP data expressed in terms of main stations plus equivalent main stations, so all growth rates used so far have been in terms of main stations plus equivalent main stations per year. To study the impact of forecast errors on the feeder cable network, we need growth rates in terms of available pairs per year. If we now define $g^{(p)}$ to be a growth rate in terms of available pairs per year, and similarly define $g^{(m)}$ to be the corresponding growth rate in terms of main stations plus equivalent main stations per year, the relationship can be estimated as follows:

$$g^{(m)} = 0.65\, g^{(p)}. \tag{4}$$

The 0.65 ratio in the above expression is the 1977 Bell System average main frame fill. Since the actual fill is generally somewhat lower at cross sections out on the route than it is at the main frame, this ratio tends to slightly understate the effect of forecast errors on available pairs. Substituting (4) in (2) for both g_f and g and dropping the superscripts, gives

$$d = \frac{\ln(0.65 g_f + 50) - [0.250 + 0.948 \ln(0.65 g + 50)]}{0.274}, \tag{5}$$

where the growth rates are now in terms of available pairs per year.

From (3) and (5), we find that the conditional density and distribution functions of g_f with respect to g are

$$f(g_f | g) = \frac{3.8\, e^{-5.84y}}{(0.65 g_f + 50)\,(1 + e^{-5.84y})^2},$$

and

$$F(g_f | g) = \frac{1}{1 + e^{-5.84y}}, \tag{6}$$

where

$$y = \ln(0.65 g_f + 50) - [0.250 + 0.948 \ln(0.65 g + 50)],$$

and where g_f and g are given in terms of available pairs per year. Expression (6) is used in Section III to estimate the impact of forecast errors on feeder-cable sizing.

First, however, three additional aspects of the model derivation need to be discussed. The COFMP data are all for a forecast interval of one year. How well does (6) work for other values of t? One might intuitively expect that forecast errors, even when normalized by divid-

Table IV—Three points omitted in Fig. 9

Wire Center	Empirical Value	Logistic Value
153	−4.65	−7.92
138	3.96	6.79
128	4.65	7.81

ing by the forecast interval as is done when dealing with growth rates, would be larger for longer forecast intervals. Indeed this tended to be the case for 22 wire centers from one company that had data on wire centers forecasts for 1-, 2-, 3-, and 4-year intervals. On the other hand, similar data from another company shows that the relative error tends to decrease for the longer intervals. An experienced forecaster was not surprised at this decrease and explained it as follows. It is often easier to determine the potential development for an area than to determine when an area will achieve that development. In view of the scant and conflicting evidence, we decided that (6) should be used for the forecast intervals encountered in feeder-cable sizing.

The forecast error distribution was derived from wire center data and is to be used in the feeder-cable network. Wire center forecasts are based in part on time series data that often do not exist for portions of a feeder route. Thus one would expect that the forecast deviations for outside plant forecasts may be somewhat larger than indicated by expression (6). In the absence of specific data, however, (6) is used as the estimate for outside plant forecast errors.

Also, if the main frame fill becomes lower than 0.65, the estimated forecast deviations in terms of available pairs will be somewhat greater than given by (6).

III. FORECAST ERROR IMPACT ON FEEDER-CABLE SIZING

We use the forecast error distribution derived in Section II to estimate for each gauge the conditional distribution of discrete-engineered cable sizes (with size based on forecast) with respect to each possible discrete optimum cable size based on actual growth. Important assumptions are that growth is linear and that there are no structure congestion problems or opportunities to use pair gain systems. If inflation is not considered, it would be appropriate to use a 12 percent discounting rate. A 6 percent inflation rate for underground cable leads to a 6 percent discounting rate when inflation is considered.

The marginal distribution of optimum cable sizes for each gauge is determined from the distribution of growth rate in that gauge, and that is, in turn, estimated from cable-shipment data. The conditional and marginal cable-size distributions give the joint distribution for each gauge of optimum and forecast (i.e., engineered) cable sizes.

The percentage cost penalty of each possible combination of optimum and forecast size is determined and multiplied by the probability of that combination occurring. These values are summed to give an overall sizing-error cost penalty for each gauge. When weighted by MCF (million conductor feet) of each gauge shipped, they provide an estimate of the cost impact of forecast errors on feeder-cable sizing.

The following parts of this section describe the steps in detail.

3.1 Growth rate distribution

An estimate of the marginal probability distribution of the optimum cable size is needed for each gauge. Were it not for the discounting rates used in the past by some companies to size cables, it would be possible to estimate these distributions directly from cable shipment data. This section estimates the growth rate distribution for each gauge, using cable shipment data and an economic sizing relationship that relates growth rate to cable size and gauge under the discounting rates used. In Section 3.2.1, we use the growth rate distribution derived here to estimate, for each gauge, the marginal probability distribution of the optimum cable size under the discounting rate that considers inflation.

1977 shipments of pulp-insulated exchange cable provide the base for estimating the growth-rate distributions of feeder cable. The cable shipment data give the MCF of each size and gauge shipped. Let $F(x_i)$ be the probability that an MCF of pulp cable of the gauge being considered is of size less than or equal to x_i.

The economic sizing relationship is used to relate points on these cable-size cumulative distribution functions to points on the growth-rate cumulative distribution functions. Assuming linear growth, the present worth cost of using a cable size, x, to meet a growth rate, g, is[4]

$$PW(x, g) = \frac{(a + bx)/r}{1 - e^{-rx/g}}, \tag{7}$$

where

r = the discounting rate,

a = the cable intercept cost (\$/year/sheath foot), and

b = the cable incremental cost (\$/year/pair foot).

The values of a and b used are the annual charge values for underground cables, since most feeder cables are placed in ducts:

gauge	a	b
26	0.38	0.0011
24	0.36	0.0014
22	0.33	0.0020
19	0.30	0.0036

(8)

It has been estimated that about one third of the companies considered inflation for sizing cables that were shipped in 1977. Thus we used a weighted average of the two discounting rates to estimate the growth rate distribution,

$$r = 0.10. \tag{9}$$

Let $g(x_i)$ be the growth rate such that the optimum discrete cable size is x_i for g just less than $g(x_i)$ and x_{i+1} for g just greater than $g(x_i)$. It can be found by iteratively solving $PW(x_i, g) = PW(x_{i+1}, g)$ for g, where x_{i+1} is the next-larger discrete cable size. Substituting from (7), $g(x_i)$ is the value that yields the following equality:

$$\frac{(a + bx_i)/r}{1 - \exp[-rx_i/g(x_i)]} = \frac{(a + bx_{i+1})/r}{1 - \exp[-rx_{i+1}/g(x_i)]}. \tag{10}$$

Because some cable shipments were affected by structure congestion and other factors, the raw growth rate distribution found by substituting (8) and (9) in (10) is not as smooth as one would expect the actual distribution must be. Therefore, a specific form was assumed for the distribution, and parameters were determined by fitting the data. We found that it is reasonable to assume that $[g(x)]^{1/2}$ is normally distributed. Figure 10 shows the data for all four gauges plotted on normal probability paper. Using ordinary least-square fits of $[g(x)]^{1/2}$ versus unit normal standard deviations corresponding to $F(x)$, gives the following growth rates:

$$\sqrt{g} \sim N(\mu_g, \sigma_g^2), \tag{11}$$

gauge	μ_g	σ_g
26	27.23	8.38
24	24.08	6.49
22	19.42	4.77
19	12.12	3.18

The above distributions are shown as the straight lines on Fig. 10; Fig. 11 shows the density functions for 26, 24, and 22 gauge. As one would expect, the growth rates for the finer gauge demand are generally larger than for coarser gauge demand.

One could argue that the cable shipment data more properly lead to an estimate of the forecast growth rate distribution, instead of the actual growth rate distribution as assumed here. It is not expected that this would result in a significant change in the sizing penalty estimate, but it could be studied by iteratively assuming distributions for the actual growth rate and solving through the conditional cable-size distributions until the resulting marginal distribution for the forecast cables agreed sufficiently closely with cable-shipment data.

3.2 Cable-size errors

For each gauge, the joint distribution of optimum and forecast cable sizes is found. Let

x^* = an optimum cable size,

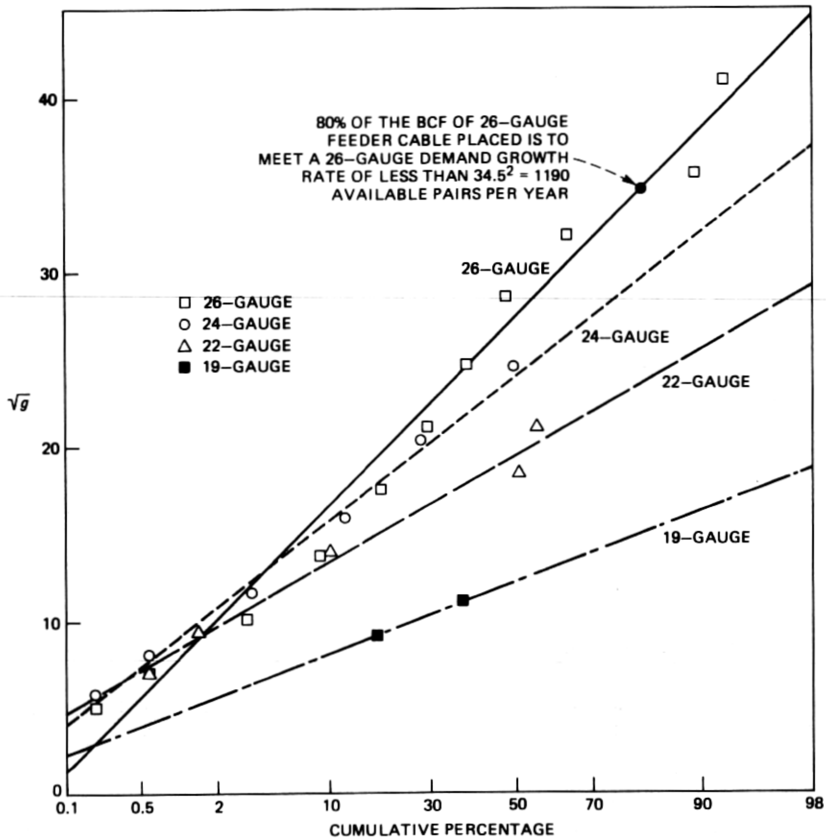

Fig. 10—Growth rate distribution. Normal probability plot of the square root of the growth rate is based on 1977 Western Electric shipments and 10 percent discounting rate.

x_f = a forecast cable size (i.e., an engineered size based on a forecast),

$p(x_f | x^*)$ = conditional probability of x_f given x^*,

$p_{x^*}(x^*)$ = marginal probability of x^*, and

$p(x^*, x_f)$ = joint probability of x^* and x_f.

3.2.1 Marginal probability distribution of x^*

The distribution of x^* is found from the growth rate distribution of Section 3.1. The discounting rate that considers inflation, 6 percent, was used to determine the growth rate intervals for which each discrete cable size is optimum. That is,

$$r = 0.06. \tag{12}$$

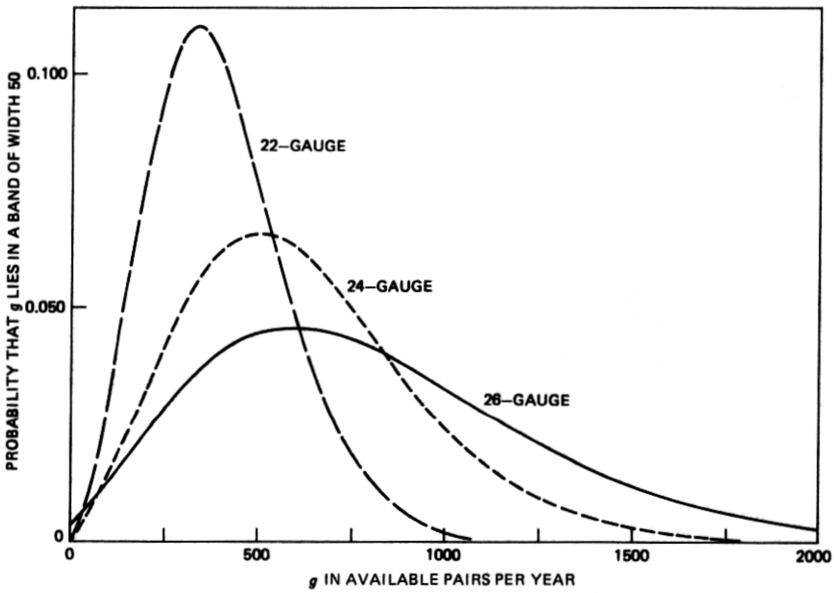

Fig. 11—Growth rate density function. Distribution of actual growth rates is based on 1977 Western Electric shipments and 10 percent discounting rate.

The growth rate intervals were determined by substituting (8) and (12) in (10). The probability of the growth rate being in each of the intervals is then found from (11). Table V gives 26-gauge values. Actually the interval for the smallest cable size should start at a growth rate of 0, but since the probability that $g < 0$ is so small, it was included with that for the smallest cable.

3.2.2 *Conditional probability distribution of x_f given x^**

The distribution of x_f given x^* is found for each gauge from the forecast error distribution of Section II. Again, using a 6 percent

Table V—Twenty-six gauge values

x^*	g	$P_{x^*}(x^*)$
300	$-\infty$ – 16.65	0.003
400	16.65 – 29.97	0.002
600	29.97 – 61.05	0.005
900	61.05 – 114.16	0.014
1200	114.16 – 182.96	0.027
1500	182.96 – 267.47	0.046
1800	267.47 – 367.67	0.071
2100	367.67 – 483.58	0.098
2400	483.58 – 615.19	0.120
2700	615.19 – 762.51	0.132
3000	762.51 – 1007.01	0.186
3600	1007.01 – ∞	0.296
		1.000

discounting rate to consider inflation, the forecast growth rate intervals that correspond to each discrete value of x_f being called for are those found in Section 3.2.1. This assumes that each cable is sized to minimize the present worth cost based on the forecast growth rate.

The probability that x_f is selected when x^* is the optimum size is then found from the forecast error distribution (6). In that expression, g is the value for which x^* minimizes (7) when x^* is considered as a continuous variable. Taking the derivative of (7) with respect to x, setting it to zero, and replacing x with x^* gives

$$e^{rx^*/g} - \frac{rx^*}{g} - 1 = \frac{ar}{bg}.$$

A value for g is found for each gauge and x^* such that the above equality holds when (8) and (12) are substituted for a, b, and r. For a 26-gauge example, let $x^* = 1200$. The corresponding growth rate, g, is 148.6 available pairs per year. Figure 12 shows the distribution of g_f, given this value of g, and the intervals corresponding to each cable size being selected. Figure 13 then shows the distribution of x_f, given $x^* = 1200$.

Since one would expect the cost penalty for not placing a cable when one should have been placed to be at least as large as the penalty due to placing the smallest cable available, the probability that g_f is negative is added to that for the smallest x_f.

3.2.3 Joint probability distribution of x^* and x_f

The joint probability is the product of the marginal probability of Section 3.2.1 and the conditional probability of Section 3.2.2. That is,

$$p(x^*, x_f) = p_{x^*}(x^*)\, p(x_f | x^*). \qquad (13)$$

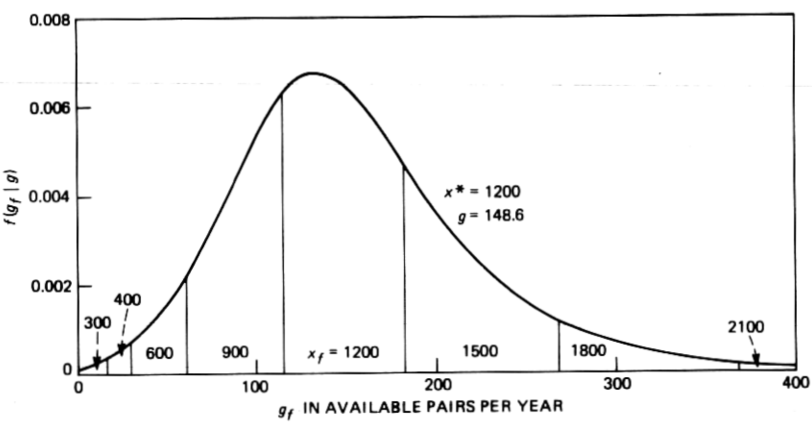

Fig. 12—Conditional density of g_f given g.

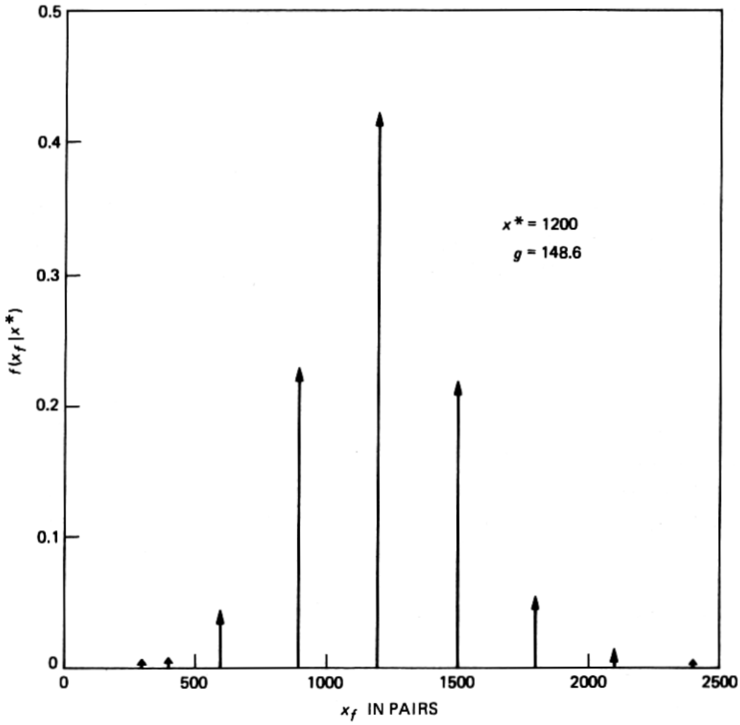

Fig. 13—Conditional density of x_f given x^*.

3.3 Penalties for size errors

Let $C(x^*, x_f)$ denote the percentage cost penalty when x_f is selected and x^* is optimum. Using the present worth notation of (7),

$$C(x^*, x_f) = 100 \frac{PW(x_f, g) - PW(x^*, g)}{PW(x^*, g)}. \tag{14}$$

As in Section 3.2.2, $g = g(x^*)$ is the value for which x^* minimizes $PW(x^*, g)$, for each discrete value of x^*. Expression (14) is evaluated by substituting (7) for $PW(x, g)$, (8), and (12) for a, b, and r.

3.4 Overall cost of size errors

For each gauge, the overall penalty is the sum of the cost penalties (14), weighted by the probabilities (13). That is, the expected penalty for one gauge is

$$\sum_{x^*} \sum_{x_f} p(x^*, x_f) C(x^*, x_f).$$

The overall penalty is the sum of the above penalties weighted by the 1977 Western Electric shipments of pulp-insulated exchange cable of

Table VI—Penalty for each gauge and overall weighted penalty

Gauge	Percent Sizing Error Cost Penalty	Cable Shipments (%)	Contribution to Overall Sizing Error Cost Penalty	
26	0.534	44.6	0.238	
24	0.462	37.0	0.171	
22	0.502	18.2	0.091	
19	0.542	0.2	0.001	
		100.0	0.501%	Expected overall cost penalty

each gauge. Table VI gives the penalty for each gauge and the overall weighted penalty.

Figure 14 shows graphically the main factors that contribute to the sizing-error cost penalty of 26-gauge feeder cable. The solid curves give contours of equal cost penalty, $C(x^*, x_f)$. The dashed curves give the 1, 10, 90, and 99 percent points on the cumulative conditional distri-

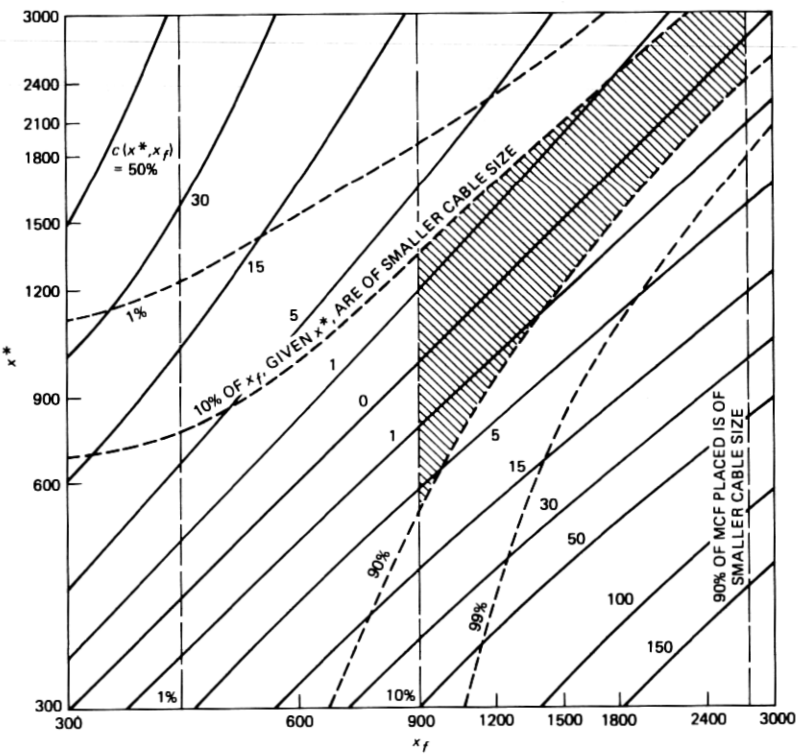

Fig. 14—Factors contributing to sizing-error cost penalty of 26-gauge feeder cable (6-percent discounting rate).

bution of $p(x_f | x^*)$. Vertical lines show the 1, 10, and 90 percent points on the cumulative MCF distribution of cable shipments of 26-gauge pulp cable. The cross hatches emphasize the area of greatest interest, where 80 percent of the shipments occur and there is an 80 percent chance of finding x_f, given x^*. The cost penalty is less than 1 percent in most of this region.

IV. SUMMARY

We have derived an estimate for the outside plant forecast error distribution. We give this distribution, expression (6), as the conditional distribution of the forecast growth rate with respect to the actual growth rate. We then used it with cable-shipment data to estimate that the feeder-cable sizing penalty due to forecast errors is about 0.5 percent of the annual feeder-cable-construction program. This represents a substantial amount of money, even though it is a small percentage, because of the large construction program.

V. ACKNOWLEDGMENTS

Thanks are due to N. B. Robbins, F. M. Stumpf, J. H. Irven, and R. Sherman for their helpful comments and suggestions. Thanks are also due to C. L. Mallows, B. Kleiner, and A. E. Freeny for their assistance with the forecast error distribution modeling.

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