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Statistical Behavior of Crosstalk Power Sum With Dominant Components

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The literature of digital transmission on wire-pair cables generally considers the probability distributions of both pair-to-pair crosstalk loss and its power sum to be normal on a dB scale. This paper presents extensive measured data of crosstalk among connectors and wire pairs on the backplane and associated stub cable of 466-type apparatus cases of the existing T1 system. The measured probability distribution of crosstalk power sum "bends" toward more severe crosstalk levels in the lower tail region, which is important for T1 system engineering. This bend is because of the effects of a few dominant components (i.e., within-slot or within-harness crosstalk) in the power sum. The simple normal model is too optimistic by 4 dB in estimating apparatus-case-crosstalk power sum at 0.1 percentile level. This paper shows that both the Monte Carlo and the lower bound methods for power sum calculations predict this bend in close agreement with the measured data. Although apparatus-case-crosstalk power sum is worse than previously assumed, the performance of T1 system has been adequately protected by the extra margin in the previous engineering rules to cover unknowns.

I. INTRODUCTION

Crosstalk interference is a prime limitation on the transmission capacity and the performance of digital transmission systems, such as T1,[1] T1C,[2,3] SLC-40,[4] T1D,[5] and SLC-96,[6] on twisted multipair cables. An important step in the design of digital systems and their associated engineering rules is the characterization of the power sum of pair-to-pair crosstalk loss. The crosstalk power sum is the total crosstalk interference which appears on a given pair as a result of coupling from all disturbers on other pairs.

The crosstalk power sum of a T-carrier system can be decomposed into two components: one component originates from crosstalk among wire pairs in the cable, and the other component originates from crosstalk among wire pairs on the backplane of the repeater apparatus case and the associated connectors and stub cable. At each repeater location of a T1 system, an apparatus case is used to house 50 regenerators of 50 one-way T1 systems. Figure 1 shows the 25-slot arrangement of the 466-type (without lightning protection device) apparatus case. Each slot holds two T1 regenerators. Figure 2 shows a portion of the wiring arrangement between the stub cable and the repeater connectors on the backplane of a 466-type apparatus case. The crosstalk originating from the wire pairs on the apparatus case backplane, connectors, and stub cable is known as apparatus-case-crosstalk (ACXT).

In new, 800-series, plastic apparatus cases, a carefully controlled wiring layout is used to minimize the ACXT to such an extent that ACXT can be neglected in the T1 system engineering rules. However, extensive laboratory and field measurements indicate that the old vintage apparatus cases, such as 466-type, are major contributors to T1 intersystem crosstalk. In this paper, we study the ACXT data of 466-type apparatus cases in detail because this is one type of apparatus case which has been widely deployed in the existing T1 plant. The statistics of ACXT are, therefore, important in characterizing the performance of the existing T1 systems and future higher bit rate systems proposed to be used in the existing T1 environment. All the ACXT data presented in this paper were measured at 0.772 MHz, the Nyquist frequency of the T1 bit rate. As explained in Section II of Ref. 7, the extreme tail region (i.e., 0.1 to 0.025 percent) of the probability distribution of the crosstalk power sum is important in the engineering of digital transmission systems in twisted pair cables.

Figure 3 shows the distribution of the power sum of ACXT of 466-type apparatus cases obtained from extensive laboratory[8] and field measurements.[9] In the simple normal model, the power sum data on Fig. 3 would be fitted by a straight line and the predicted 0.1 percentile would be 59 dB. However, the measured power sum distribution on Fig. 3 has a noticeable "bend" towards more severe crosstalk levels in

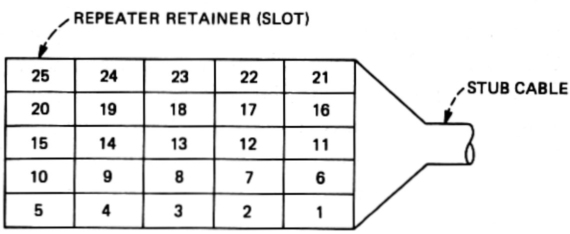

Fig. 1—The 466-type apparatus case with 25 repeater slots (i.e., retainers) for T1 repeaters.

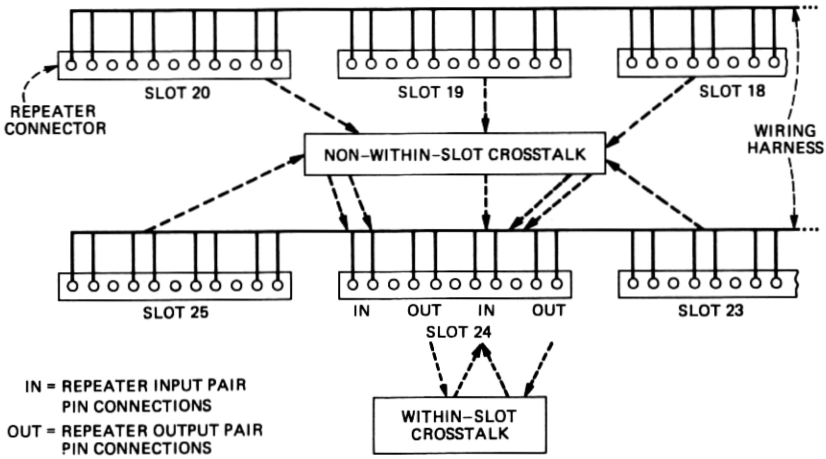

Fig. 2—Wiring diagram on the backplane of 466-type T1 apparatus case showing within-slot crosstalk and non-within-slot crosstalk for slot 24.

the lower tail region (≤5 percent). It cannot be described precisely by any simple model such as normal or gamma. This paper shows that the bend is because of the dominant effect of the within-slot pair-to-pair crosstalk which is, on the average, 15 dB worse than the non-within-slot pair-to-pair crosstalk. It is demonstrated that both the

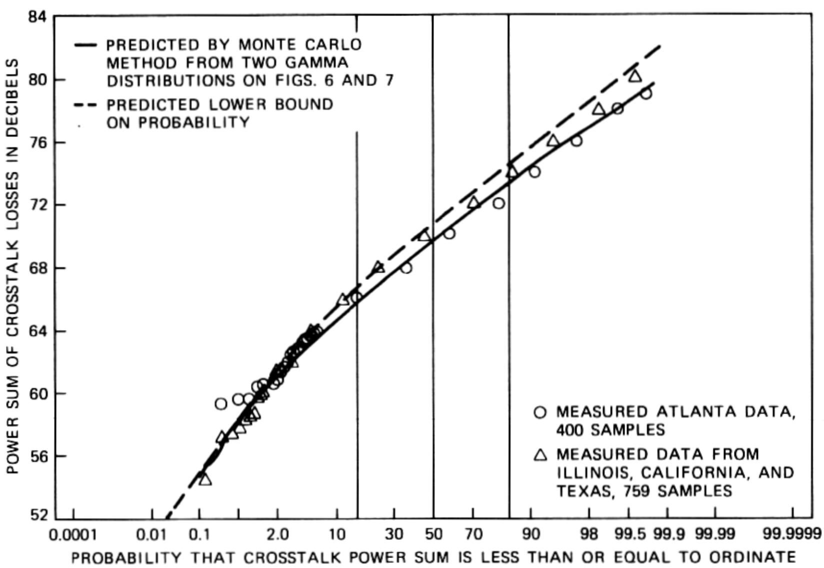

Fig. 3—Distribution of power sum of 50 pair-to-pair crosstalk losses of 466-type T1 apparatus case from laboratory measurements in Atlanta and field measurements in Illinois, California, and Texas.

Monte Carlo and the lower bound method for power sum calculations predict this bend. Both methods and the data indicate that the 0.1 percentile of the power sum distribution for 466-type cases is 55 dB which is 4 dB more pessimistic than the 59 dB predicted by the simple normal model. Therefore, the simple normal model may be too optimistic in estimating the T1 system margin. The previous engineering rules[10] for T1 system contain extra margin to cover "unknowns." The effect of ACXT on the bit-error-rate performance of T1 system has been adequately protected by the extra margin. The development of a more accurate ACXT model will reduce the unknowns and enable a greater exploitation of the system's capability by mitigating the need for large "uncertainty" margins. M. H. Meyers[11] has also investigated ACXT by a different approach.

II. ATLANTA DATA AND THE SIMPLE NORMAL AND GAMMA MODELS

The ACXT data of eight 466-type apparatus cases were measured in Bell Laboratories in Atlanta by using a computer operated transmission measurement set.[8,12,13]

The laboratory data are shown as circles on Figs. 4 and 5 for pair-to-pair crosstalk loss and the power sum, respectively. The solid line and the dashed line on Fig. 4 represent the gamma and normal approximation, respectively, to the pair-to-pair distribution. The power sum distribution is strongly controlled by the behavior of pair-to-pair distribution in the tail region of low crosstalk loss.[7] The gamma

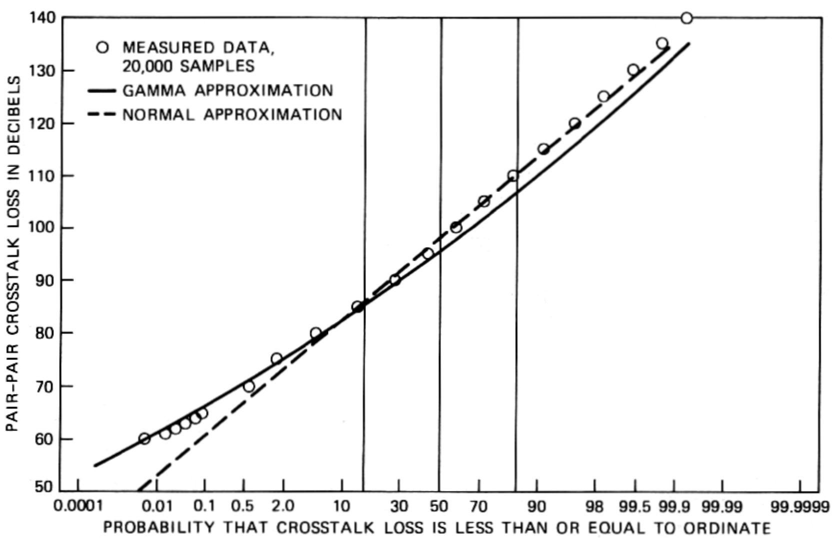

Fig. 4—Distribution of pair-to-pair crosstalk loss of 466-type T1 apparatus case. Data measured from eight apparatus cases in Bell Laboratories, Atlanta.

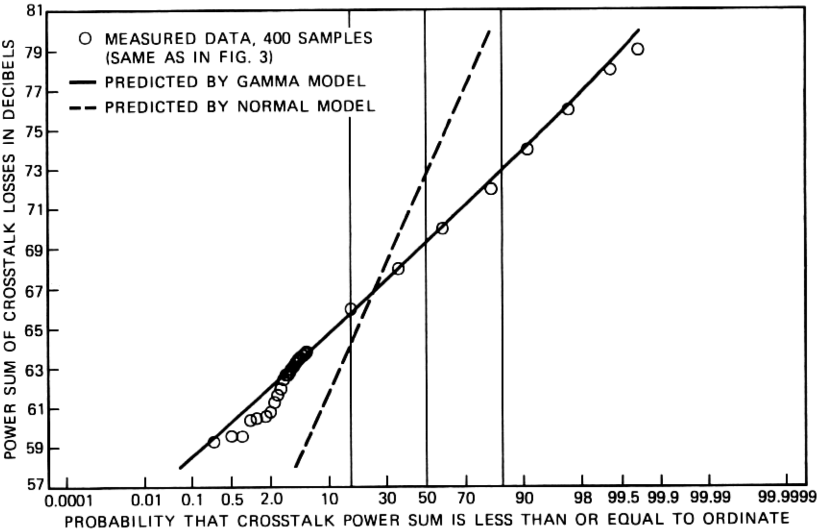

Fig. 5—Distribution of power sum of 50 pair-to-pair crosstalk losses of 466-type T1 apparatus case. Data measured from eight apparatus cases in Bell Laboratories, Atlanta.

approximation fits the pair-to-pair data very closely in the critical region of low crosstalk loss, whereas the normal approximation is too pessimistic in this important tail region.

The worst value of 60 dB on Fig. 4 does *not* imply that the pair-to-pair distribution is truncated at the 60-dB level. A finite sample measurement of a random variate (e.g., crosstalk loss) always yields a finite worst value even if the parent distribution of the variate is untruncated. The sample worst value varies randomly from one set of measurement (e.g., from one cable) to another. The probability distribution of the worst value (i.e., the extreme value) is the subject of extreme value statistics which have been studied extensively.[14,15,16] Therefore, the existence of a finite worst value from a finite sample measurement of cable crosstalk loss does not necessarily imply that the parent distribution of crosstalk loss is truncated.

Figure 5 shows that the power sum distribution predicted by the gamma model (solid line) agrees reasonably well with the measured data over a large portion of the distribution, but the discrepancy in the lower tail region is noticeable. On the other hand, the prediction by the untruncated normal model (dashed line) differs substantially from the data. The equations and the calculation procedure of the gamma model have been described in Ref. 7. The estimated statistical parameters of apparatus-case crosstalk based on the simple gamma model are listed in Table I.

Many authors have used the Wilkinson's method[17] to calculate the power sum distribution from the pair-to-pair distribution. The obvious

Table I—Statistical parameters of apparatus-case crosstalk
(Eight 466-type apparatus cases measured in Atlanta)

Gamma Model† (For all data)		
Pair-to-pair	$\bar{y}$	3.30×10^{-8}
	σ_y	2.23×10^{-8}
	υ	80.0
	β	0.833
	M_x(dB)	96.0*
	σ_x(dB)	10.7*
Power sum of 50 pair-to-pair crosstalk losses	$\bar{s}$	1.65×10^{-7}
	σ_s	1.58×10^{-7}
	μ	568.00
	α	6.58
	M_Q(dB)	69.30
	σ_Q(dB)	3.62

* These values are estimated by gamma model and are slightly different from those of normal model.

† The definitions of terms and equations related to gamma model are given in Ref. 7.

discrepancy between the measured data and the dashed line predicted by the untruncated normal model in Fig. 5 has prompted some authors to abandon the Wilkinson's method[17] entirely and to simply fit the measured power sum distribution on Figs. 3 and 5 by a normal distribution. As will be shown later, this approach is too optimistic by 4 dB at the critical 0.1 percent point. Thus, the normal model faces a dilemma of being too pessimistic (see Fig. 5), if Wilkinson's method of power sum calculation is used, and being too optimistic at the 0.1 percent point, if Wilkinson's method is by-passed (i.e., simply fit the measured power sum distribution by a normal distribution). The use of truncated normal model for pair-to-pair distribution suffers a drawback of uncertain truncation point as discussed in Ref. 7 and several dBs of error at the 0.1 percent point just mentioned.

In engineering applications, the behavior of the power sum distribution in the lower tail region is most important because the engineering objective of T1 systems is set at the 0.1 percent point for 50-section metropolitan applications. Unfortunately, Figs. 3 and 5 show that the measured data deviate substantially from the predictions by gamma and normal models in the important lower tail region. These discrepancies are predictable by both the Monte Carlo and the lower bound method as discussed in the next section.

Reference 7 and this paper indicate that an accurate prediction of crosstalk power sum distribution from pair-to-pair distribution is often difficult. One is tempted to abandon the pair-to-pair crosstalk statistics entirely and to rely solely on the measured power sum distribution. However, the studies of pair-to-pair statistics and other decompositions, such as within-slot versus non-within-slot crosstalk, and within-harness versus non-within-harness crosstalk, are necessary to provide insights for understanding and for proper modeling of non-Gaussian power sum distribution. The technique for predicting power sum distribution from pair-to-pair distribution is also necessary in characterizing some practical situations where the apparatus cases are only partially filled.

III. POWER SUM CALCULATIONS BY MONTE CARLO AND LOWER BOUND METHODS

The extensive laboratory and field measurements indicate that the distribution of the power sum of apparatus-case crosstalk has a noticeable bend towards more severe crosstalk levels in the lower tail region as shown in Fig. 3. The lower tail region of the measured data on Fig. 3 has an effective standard deviation (i.e., slope) of 6 dB on the normal probability coordinates. This slope agrees very well with the slope of the measured distribution of T1 repeater section margins in the lower tail region.[9] Thus, the laboratory measurements and field measurements consistently indicate that the distribution of the power sum of ACXT cannot be described precisely by a simple model, such as the normal or the gamma distributions.

With such understanding, we will avoid assuming any simple model for the total power sum distribution and will use more sophisticated techniques, such as the Monte Carlo method or the lower bound technique to obtain the correct power sum distribution.

Previous studies by Marlow[18] and Janos[19] indicate that the power sum distributions will have a bend if there are strong, dominant components whose mean or standard deviation differs substantially from those of the other components of the power sum. Under such circumstances, the lower tail of the power sum distribution will behave like that of the dominant components and, hence, show a bend. With this hint, we naturally look for the possible existence of dominant components in the power sum of apparatus-case crosstalk disturbers.

Figures 1 and 2 show the repeater slot and wiring arrangements of T1 apparatus case. Each T1 system in an apparatus case suffers from two within-slot disturbers* and 48 non-within-slot disturbers assuming the case is 100 percent filled. The Atlanta laboratory data show that the mean value of the within-slot crosstalk is 15 dB worse than that of the non-within-slot crosstalk. Such a large difference means that the within-slot crosstalk and the non-within-slot crosstalk must be treated separately in the power sum calculations.

The circles on Fig. 6 show the distribution of power sum of the 48 non-within-slot crosstalk disturbers measured in Atlanta Laboratory.

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*Each apparatus case slot holds two T1 regenerators.

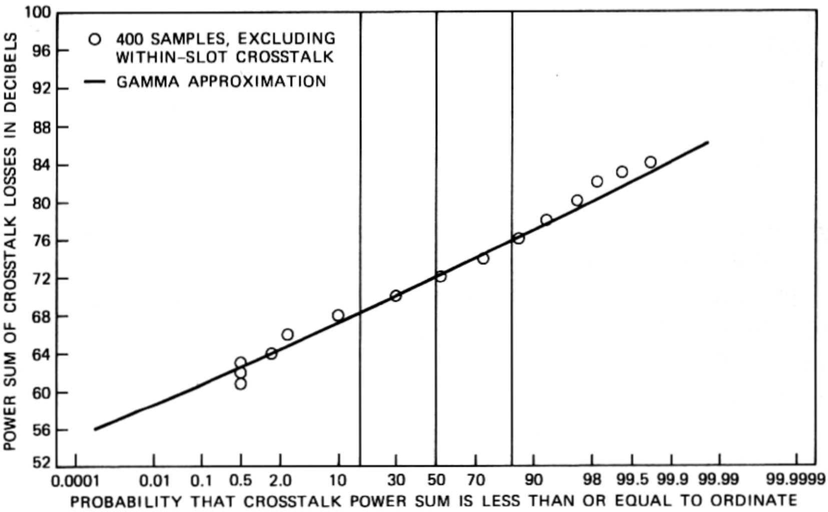

Fig. 6—Distribution of power sum of 48, pair-to-pair, non-within-slot, crosstalk losses of 466-type T1 apparatus case. Data measured from eight apparatus cases in Bell Laboratories, Atlanta. Data represents between-slot crosstalk component of data in Fig. 5.

The circles, triangles, and crosses on Fig. 7 show the distributions of power sum of the two within-slot-crosstalk disturbers measured in Atlanta, Illinois, and California, respectively. The solid lines on Figs. 6 and 7 are the corresponding gamma approximations with the param-

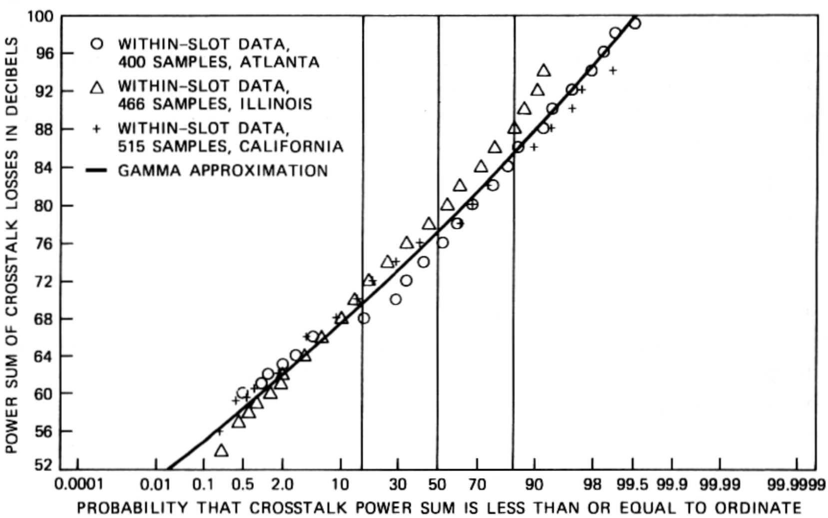

Fig. 7—Distributions of power sum of two, pair-to-pair, within-slot, crosstalk losses of 466-type T1 apparatus case from laboratory measurements in Atlanta and field measurements in Illinois and California. Data represents within-slot crosstalk component of data in Figs. 3 and 5.

Table II—Statistical parameters of non-within-slot crosstalk of apparatus case
(Eight 466-type apparatus cases measured in Atlanta)

Gamma model for power sum of 48 non-within-slot crosstalk (Fig. 6)	
$\bar{s}$	8.99×10^{-8}
σ_s	9.10×10^{-8}
μ	551.00
α	6.20
M_Q(dB)	72.20
σ_Q(dB)	3.79

eters listed in Tables II and III, respectively. The total power sum of the 50 pair-to-pair crosstalk disturbers is equal to the power sum of the two gamma distributed random variables on Figs. 6 and 7. Tables II and III show that the mean values of these two gamma variates differ by only 7 percent (i.e., 72.2 vs. 77.5 dB), whereas the standard deviations differ by 100 percent (i.e., 3.8 versus 8.0 dB). A bend on the distribution of their power sum is, therefore, expected.

The Monte Carlo method for power sum calculation has been used by many authors.[17,18] The lower bound technique is described in the Appendix. Figure 3 shows that both the Monte Carlo and the lower bound methods predict the bend in the total power sum distribution in close agreement with the measured data. The Monte Carlo result* agrees very well with the measured data over the entire range. The predicted lower bound (in probability) is practically identical to the Monte Carlo result in the lower tail region ($\leq$2 percent) and is applicable to T1 system engineering. The lower bound method has the advantages of being simple computationally and of providing some physical insights into the power sum behavior in the tail region as discussed below.

Let x denote the power sum resulting from within-slot crosstalk (Fig. 7) and let y denote the power sum due to non-within-slot crosstalk (Fig. 6). Furthermore, let z denote the power sum of x and y, the total power sum of 50 pair-to-pair crosstalk disturbers. The Appendix shows that a lower bound, $P_{LB}(z \leq b)$, of the probability distribution of z is:

$$P_{LB}(z \leq b) = P(x \leq b) + P(y \leq b) - P(x \leq b) \cdot P(y \leq b), \qquad (1)$$

where b represents the crosstalk level at which the probabilities are of interest. The data in Figs. 6 and 7 show that

$$P(y \leq b) \ll P(x \leq b) \quad \text{for} \quad b \leq 62 \text{ dB}, \qquad (2)$$

*A sample size of 5000 is used in obtaining the Monte Carlo result (the solid line) in Fig. 3.

Table III—Statistical parameters of within-slot crosstalk of apparatus case
(466-Type apparatus cases measured in Atlanta, Illinois, and California)

Gamma model for power sum of two within-slot crosstalk (Fig. 7).	
$\bar{s}$	8.45×10^{-8}
σ_s	2.75×10^{-7}
μ	101.00
α	1.258
M_Q(dB)	77.50
σ_Q(dB)	8.00*

* Since a gamma distribution is not a straight line on a normal probability coordinates, the slope of the gamma distribution in the lower tail region is not the same as that (i.e., 8 dB of σ) in the middle range.

which implies that:

$$P_{LB}(z \leq b) \simeq P(x \leq b) \quad \text{for} \quad b \leq 62 \text{ dB}. \tag{3}$$

The data in Figs. 3 and 7 indeed support this simple approximation. Therefore, the lower bound method demonstrates through eq. (3) that the total power sum distribution behaves like that of the dominant component x in the lower tail region (i.e., for those situations where ACXT is worse than 62 dB). Notice that the dominant component x represents the power sum of the two within-slot-crosstalk losses.

VI. CONCLUSION

The extensive data on T1 apparatus-case crosstalk for 466-type cases from laboratory measurement in Atlanta and field measurements in Illinois, California, and Texas consistently indicate that the power sum distribution has a bend towards more severe crosstalk levels in the lower tail region. It is shown that this bend is because of the dominant effect of within-slot crosstalk. Both the Monte Carlo and the lower bound methods of power sum calculations predict this bend if the power sum contains a dominant component which differs substantially from other components.

The 0.1 percentile of the distribution of power sum of ACXT is about 55 dB for 466-type apparatus case. This is about 4 dB worse than that predicted by the conventional normal model which ignores the bend in the tail region.

VII. ACKNOWLEDGMENT

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APPENDIX

A Lower Bound of Power Sum Distribution

This appendix describes a technique to obtain a lower bound of the probability distribution of crosstalk power sum. In the low probability tail region, this lower bound is fairly tight and provides a simple approximation to the power sum distribution. This approach is inspired by the work of Marlow and Farley.[18,20]

Let $P(x \leq b)$ and $P(y \leq b)$ be the cumulative distributions of two, positive, independent, random variables x and y, respectively, and let

$$z = -10 \log_{10}[10^{\frac{-x}{10}} + 10^{\frac{-y}{10}}] \tag{4}$$

be the power sum of x and y. This definition implies that

$$z \leq \min(x, y), \tag{5}$$

and

$$P(z \geq b) \leq P(\min(x, y) \geq b) \tag{6}$$

$$= P(x \geq b, y \geq b), \tag{7}$$

where $\min(x, y)$ denotes the minimum of x and y, and $P(x \geq b, y \geq b)$ denotes the probability that both x and y exceed b. The independence of x and y implies that

$$P(x \geq b, y \geq b) = P(x \geq b) \cdot P(y \geq b). \tag{8}$$

By definition:

$$\begin{aligned} P(z < b) &\equiv 1 - P(z \geq b) \\ P(x < b) &\equiv 1 - P(x \geq b) \\ P(y < b) &= 1 - P(y \geq b) \quad . \end{aligned} \tag{9}$$

Combining eqs. (7), (8), and (9) yields

$$\begin{aligned} P(z < b) &\geq 1 - P(x \geq b) \cdot P(y \geq b) \\ &= 1 - [1 - P(x < b)] \cdot [1 - P(y < b)] \\ &= P(x < b) + P(y < b) - P(x < b) \cdot P(y < b). \end{aligned} \tag{10}$$

Therefore, the right-hand side of eq. (10) represents a lower bound (in probability) for the distribution of the power sum z. Notice that this lower bound can easily be calculated when the distributions $P(x \leq b)$ and $P(y \leq b)$ of the components x and y are known.

In the lower tail region where both $P(x \leq b)$ and $P(y \leq b)$ are small, the probability of both x and y being less than b simultaneously is extremely small. Therefore, the inequality eqs. (5) and (10) asymtotically approach equalities in the lower tail region (i.e., when b is small). This means that in the low probability tail region, the lower bound eq. (10) is fairly tight and provides a simple but accurate approximation to the power sum distribution.

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