THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 60, No. 7, September 1981
Printed in U.S.A.

Digital Signal Processor:

Tone Generation

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(Manuscript received September 10, 1980)

Two programs have been written for the recently developed single-chip digital signal processor (DSP) integrated circuit that enable it to function as a tone generator in testing transmission systems. One program is based on the table look-up method and the other on the Maclaurin expansion method. A DSP tone generator based on the look-up method can generate up to 12 components and is suitable for all transmission testing applications. A generator based on the Maclaurin expansion method is limited to less than four components and is particularly applicable in two-tone testing.

I. INTRODUCTION

A tone generator is required for a number of transmission system tests. The tones required include *TOUCH-TONE*® signaling, multi-frequency (MF) signaling, a milliwatt source, centralized automatic reporting on trunks (CAROT) responses and CAROT test tones. In addition, a tone consisting of 21 components each having a settable phase and level is required for a fast Fourier transform (FFT)-based system. In all cases, the tone is to be transmitted on a 4-kHz digital channel, with a sample every 125 μs (Nyquist sampling rate of 8 kHz). Programs have been written for the DSP that enable it to function as a tone generator. The following two methods are used: (*i*) table look-up, and (*ii*) Maclaurin expansion.

The table look-up method consists of storing in read only memory (ROM) the trigometric values of $\sin(n\theta)$, where $0 \le n \le N$, and N is determined by both frequency granularity requirements and harmonic distortion considerations. At each sampling instant, the value of the sample is taken from the appropriate location of the ROM table and scaled for its desired level. If more than one tone is desired, each component is independently determined and then all component values added together to form the sample value.

The Maclaurin expansion,

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots,$$

can be used to determine successive sample values by direct calculation. The number of terms to be considered is a function of desired harmonic purity and the time allowed for the computation.

II. TABLE LOOK-UP METHOD

A program has been written for the DSP that uses the table look-up method to implement a tone generator. It produces up to six independently specified components in the 0- to 4-kHz range with an 8-bit, μ-255 encoded output. A diagnostic is included that checks most of the DSP features used by this routine. The program accepts successive 16-bit serial input instructions that:

(*i*) Specify tone generation or a diagnostic.

(*ii*) Specify the number of components, from 1 to 6, that comprise the tone. When the number of components specified is 0, quiet tone is generated. When 7 components are specified, milliwatt tone is generated.

(*iii*) Specify the frequency of a component, from 0 to 4 kHz, in 1/8-Hz steps.

(*iv*) Specify the phase of a component from 0 to 360° in 1/8° steps.

(*v*) Specify the level of a component as a fraction of full output, from 0 to 1 in steps as fine as 2^{-15}.

(*vi*) Steps 3, 4, and 5 are repeated for each component comprising the tone.

Tone generation starts with a reset, except when phase continuity is required. The output may be changed to a new frequency without loss of samples or phase continuity when the information about the new frequency is presented to the input buffer.

The diagnostic ends with the S0-bit set to 1, if all tests pass.

2.1 Algorithm

A full 2π sine table, with 512 16-bit entries is used. A $\pi/2$ sine table could be used but would require keeping track of quadrants and signs, which takes time and reduces the number of tones that could be generated.

The table entries are so arranged that the entry for 0° is at an address called &TABLE. Two hundred fifty-five successive locations above this contain the positive half of the sine wave trignometric entries. Two hundred and fifty-six entries below this address contain the negative going entries. Two additional entries, corresponding to the first nonzero positive entry, at location (&TABLE − 257) and 0 at (&TABLE + 256) are also included. Hence, the ROM table is as shown in Fig. 1.

Table accesses above and below the normal range may occur because of the effect of rounding. Electing truncation in the DSP in the initialization of this process would avoid the need for adding these memory locations, but their presence assures continuity of sample values.

The frequency of a component is determined by the number of table entries $\Delta\phi$ stepped per sample, if the samples being generated at a particular frequency are

$$y(nT) = A \sin(2\pi f n T),$$

where f is the desired frequency, T is the sampling interval, and n takes on integer values, then the phase increment between successive values is

$$2\pi f T = 2\pi \frac{f}{f_s},$$

where $f_s = \frac{1}{T} = 8000$ Hz is the sampling frequency.

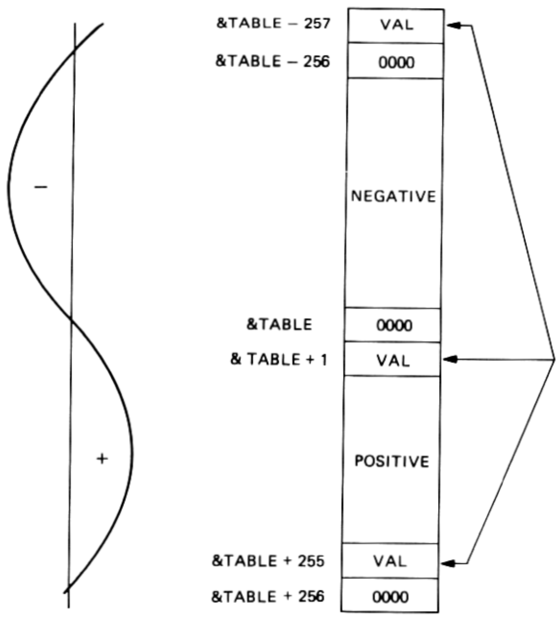

Fig. 1—Read only memory table.

Since a full 2π sine wave table is present in ROM and is represented by 512 entries, the phase difference between successive entries is $2\pi/512$. The number of entries, $\Delta\phi$, to be stepped per sample is, therefore,

$$\Delta\phi = \frac{2\pi f/8000}{2\pi/512} = 0.064f.$$

To minimize processing time, modulo 512 arithmetic is used so that no sign or table limit checking is required. Incrementing around the table is automatic. To accomplish this, the table is entered at &TABLE + N_ϕ, where N_ϕ is modulo 512 and is stored in register Y as shown in Fig. 2. As desired, N_ϕ is never larger than 255 and its sign alternates. Assuming an initial phase of 0, N_ϕ starts at zero and is then incremented by $\Delta\phi$. Thus, for p passes

$$N_\phi = \sum_{l=0}^{p-1} l\Delta\phi.$$

Given f, $\Delta\phi$ is obtained in modulo 512 by transferring as shown in Fig. 3.

To generate a frequency close to the desired value, it is important to have as much precision as possible in $\Delta\phi$. In this case, after W register truncation, it is 11 bits. The precision also has a marked influence on harmonic distortion.* The harmonic distortion products are shown to be 50 dB below the fundamental for this arrangement.

2.2 Phase continuity

To change the frequency, $\Delta\phi$ must be changed, but phase continuity can be preserved. In one sample interval, the new frequency is fetched and the new $\Delta\phi$, subsequently calculated, is placed into the appropriate random access memory (RAM) location. The calculations then proceed from the previously accumulated phase.

2.3 Flow chart

A flow chart of the DSP program is given in Fig. 4. In essence, the overall flow breaks down into the following areas shown in the chart:

(1a) Input a data control word, the frequencies, phases and levels, or

(1b) Input a diagnostic control word and run. The output is a pass or fail indication.

Because of the dynamic nature of the DSP RAM, it is necessary during the input data routine, to refresh the RAM locations.

Once the input data process is completed, one of three paths is possible:

(2a) Calculate sample values for $1 \leq n \leq 6$ tones.

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* W. N. Fabricius, unpublished work.

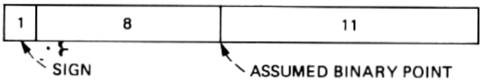


Fig. 2—Modulo 512 stored in Y register.

(2b) Calculate samples for quiet tone, $n = 0$.
(2c) Calculate samples for a 0 dBm, milliwatt, $n = 7$.
(3a) Continue calculating with phase continuity.
(3b) Continue with old input data.

2.4 Experimental results

The spectrum of the DSP output signal after decoding is given in Figs. 5, 6, and 7. Calculations, based on the work of Fabricius, indicate that the harmonic production is mainly because of the μ-255 format.

Some synthesized waveforms are represented by the waveforms in Figs. 8 and 9.

The measured phase jitter is 0.65° at 1000 Hz and 0.75° at 1004 Hz. The AM jitter experienced is less than 0.1 dB.

The linearity of the output signal fell well within the μ-255 format requirements. (For this experiment, a linear output was delivered by the program.)

2.5 Extensions

The program, without μ-255 encoding, has generated 13 components within a 125-μs sampling interval. A total of 21 components could be generated by synchronizing two DSPs, one producing 10 and the other

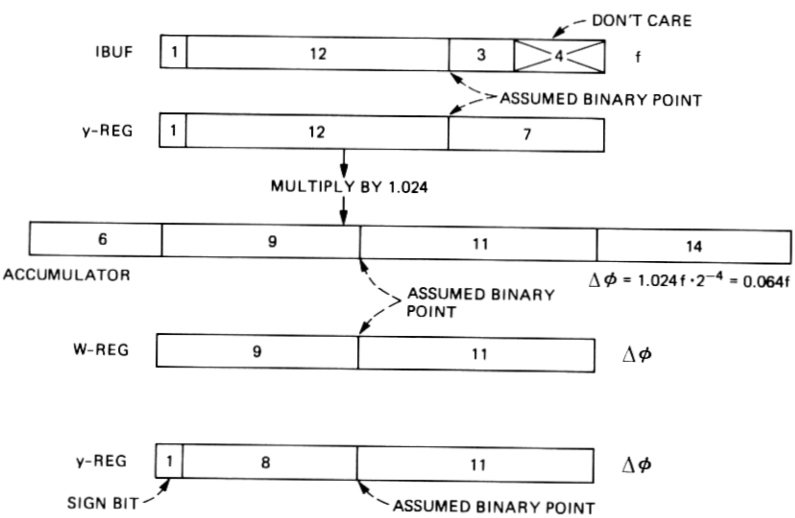
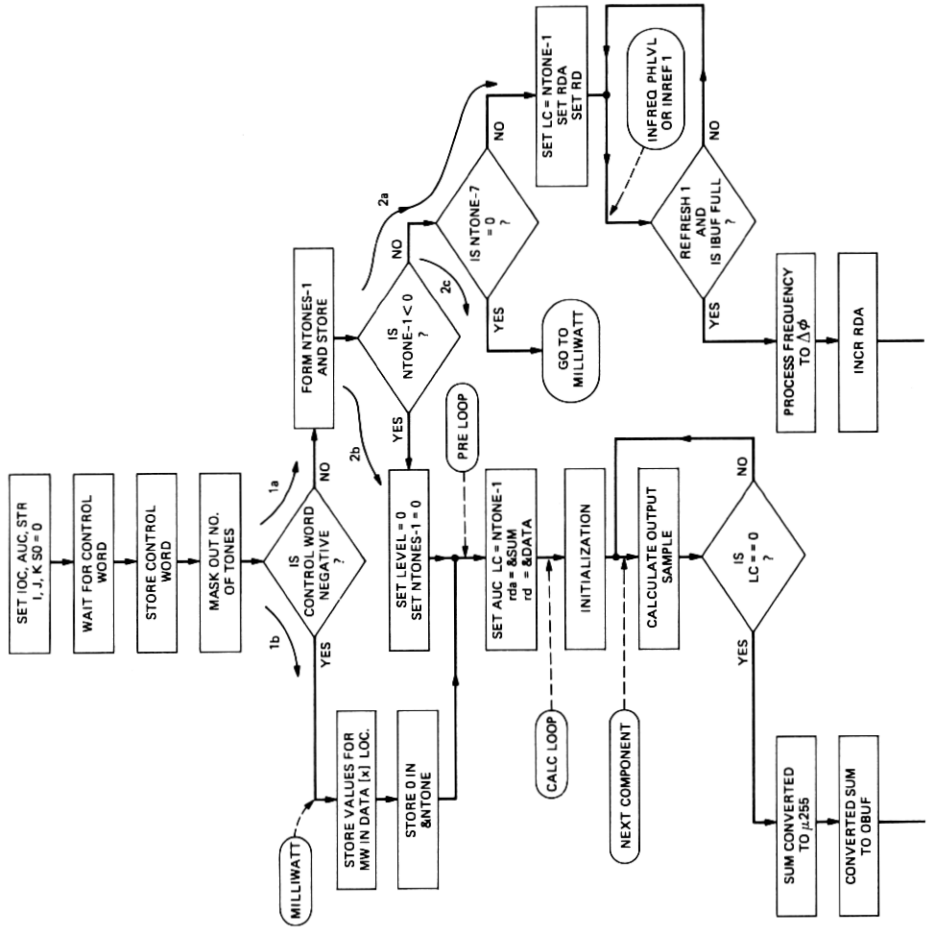


Fig. 3—Register manipulation for modulo 512.


SET IOC, AUC, STR
I, J, K S0 = 0
WAIT FOR CONTROL WORD
STORE CONTROL WORD
MASK OUT NO. OF TONES
IS CONTROL WORD NEGATIVE ?
YES
NO
1a
1b
MILLIWATT
STORE VALUES FOR MW IN DATA [x] LOC.
STORE 0 IN &NTONE
FORM NTONES-1 AND STORE
IS NTONE-1 < 0 ?
YES
NO
2a
2b
2c
SET LEVEL = 0
SET NTONES-1 = 0
PRE LOOP
IS NTONE-7 = 0 ?
YES
NO
GO TO MILLIWATT
SET LC = NTONE-1
SET RDA
SET RD
SET AUC LC = NTONE-1
rda = &SUM
rd = &DATA
CALC LOOP
INITIALIZATION
NEXT COMPONENT
CALCULATE OUTPUT SAMPLE
IS LC = = 0 ?
YES
NO
INFREQ PHLVL OR INREF 1
REFRESH 1 AND IS IBUF FULL ?
YES
NO
PROCESS FREQUENCY TO Δφ
INCR RDA
SUM CONVERTED TO μ255
CONVERTED SUM TO OBUF

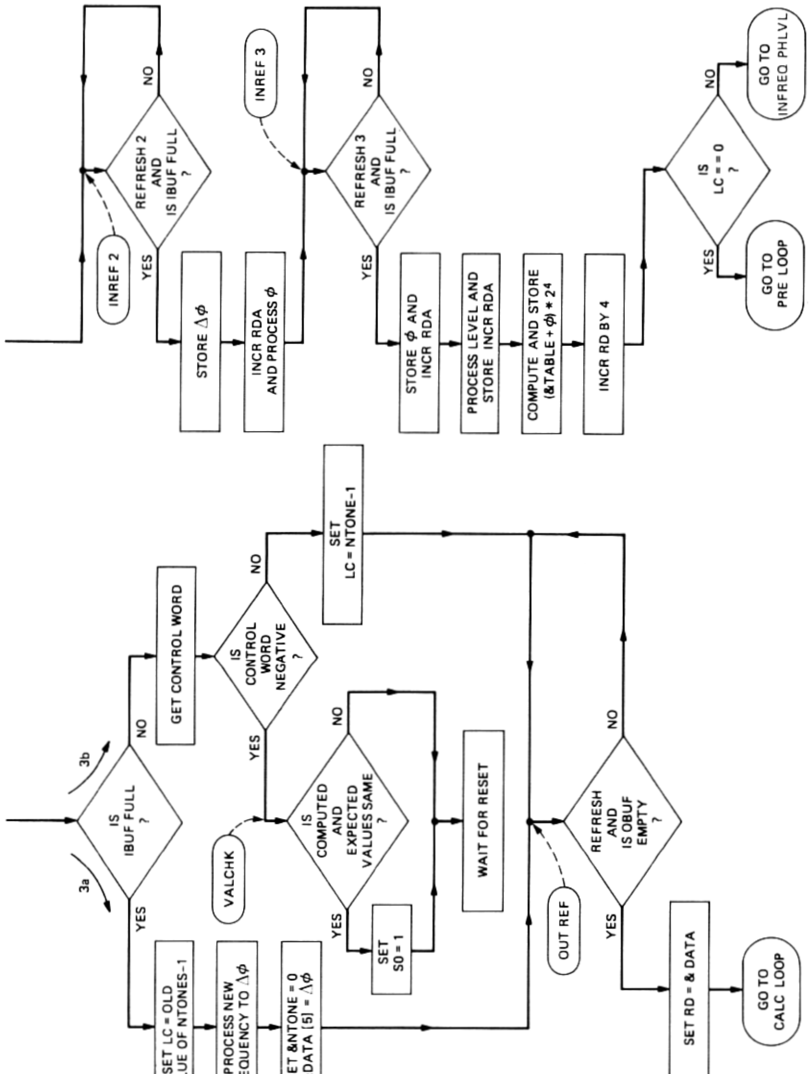


Fig. 4—Flow diagram for DSP tone-generator program.

11. The first DSP would serially transmit its 20-bit sample value, after generation, to the second DSP. This would take 8 μs at a 2.5-Mb/s rate, and could be accomplished during the time the second DSP was computing its 11th component. The second DSP would then take this value in its input buffer and add it to its sample value, convert the result to μ-255, and transmit it.

III. MACLAURIN EXPANSION

A program has been written for the DSP that can evaluate a truncated Maclaurin expansion in less than 30 μs. Frequency resolution is not limited by quantizing as in the table look-up method, but by the accumulator and the product registers in the DSP.

3.1 Algorithm

Each component is generated by a truncated Maclaurin series expansion. The choice of the number of terms included in the approxi-

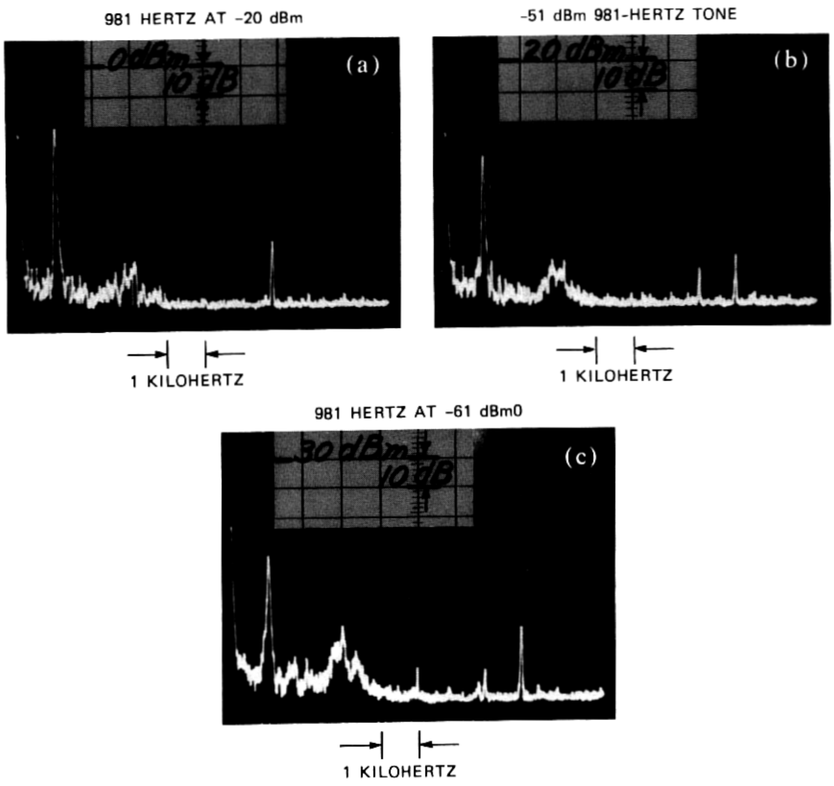


Fig. 5—Tone spectrum—low level (981 Hertz).

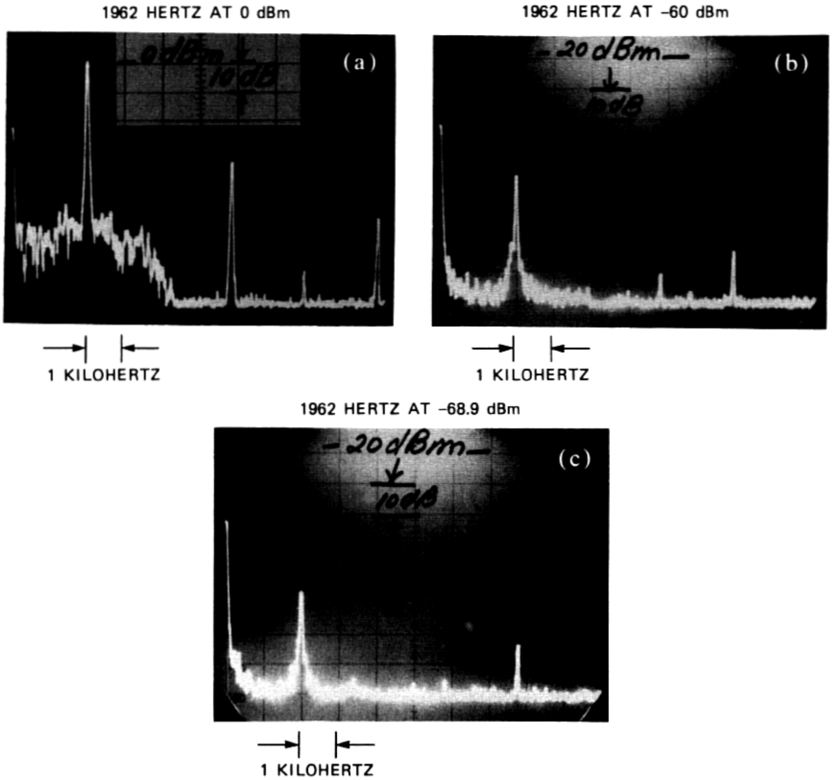


Fig. 6—Tone spectrum at 1962 Hertz.

mation and the point about which the series is expanded all affect the harmonic distortion.

Consider the following Maclaurin series expansions around zero,

$$\sin(\theta) = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \cdots$$

$$\cos(\theta) = 1 - \frac{\theta^2}{2!} - \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \cdots.$$

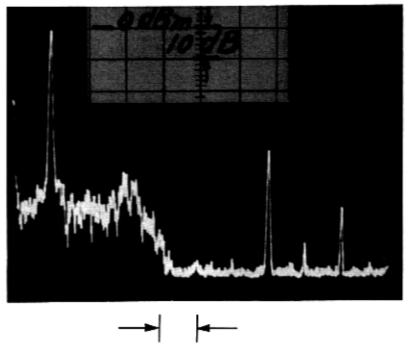
The accuracy of the approximation, of course, becomes better when more and more terms of the expansion are used. The harmonic distortion because of series truncation of the cosine series is examined in the Appendix. Let $P(\theta)$ represent the portion of the series that is retained, and $R(\theta)$ represent the remaining terms so that

$$\cos\theta = P(\theta) + R(\theta),$$

where both of these functions are considered defined *only* in the interval

$$-\pi/2 \leq \theta \leq \pi/2.$$

981 HERTZ AT 0 dBm



1 KILOHERTZ

Fig. 7—Tone spectrum—high level (981 Hertz).

They are replicated, with sign reversals, in successive intervals of π forming a wave periodic in the interval 2π. An upper bound on the harmonic distortion, because of truncation after the nth term of a Maclaurin expansion, is

$$\text{Distortion} \leq = 20 \log \left[\frac{(2n)!}{(\pi/2)^{2n}} \right].$$

100-HERTZ SQUARE WAVE (6-TONE COMPONENTS)

981-HERTZ WAVEFORM (0 dBm)

3-TONE WAVEFORM

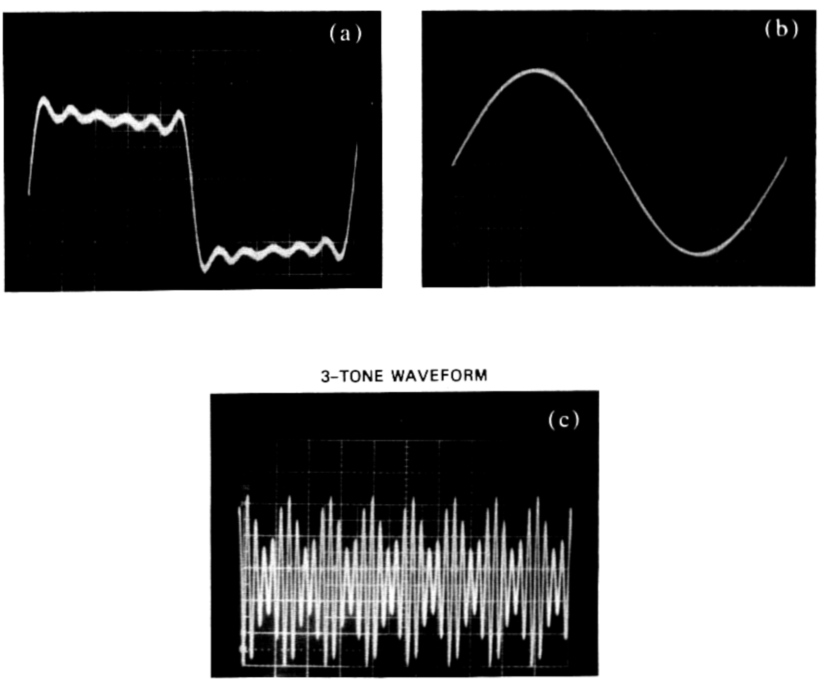


Fig. 8—Time domain waveforms.

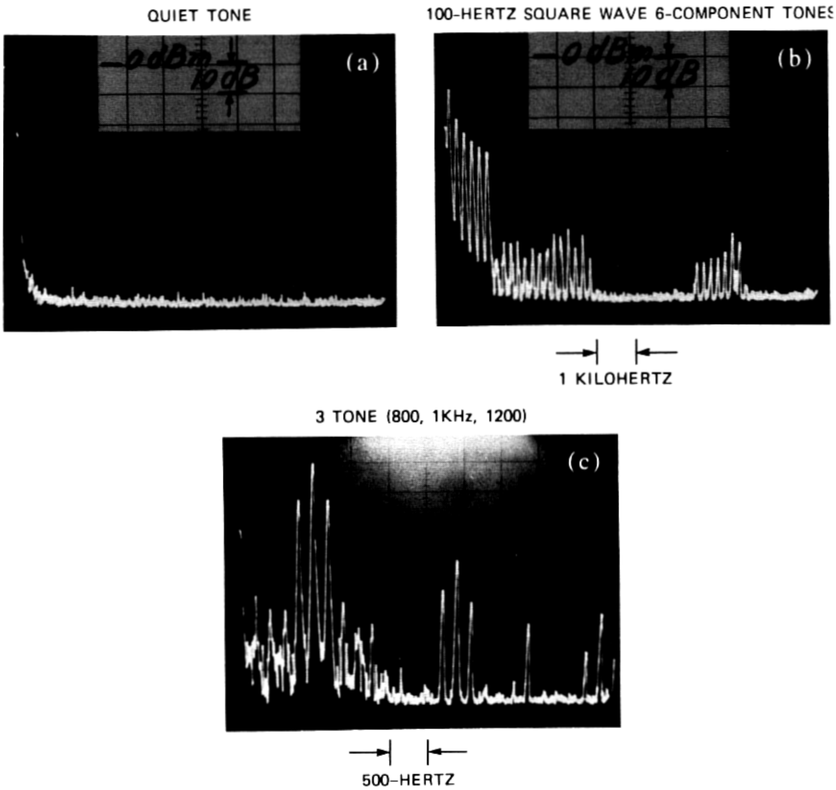


Fig. 9—Spectrum of time waveforms.

Table I lists this upper bound for values of n. This is a mathematical upper bound and ignores the effects of quantizing distortion.

The harmonics generated due to the μ-255 format generally are in the −42 dB range. The total signal to distortion is in the 36-dB range. Thus, $n = 4$ should be used for the Maclaurin expansion since it produces distortion components less than those generated by the μ-255 format.

By a similar analysis, it can be shown that the sine series should also be terminated after four terms so that the harmonic distortion upper bound is −78.9 dB. The Maclaurin cosine series is chosen instead

Table I—Harmonic distortion

n	Upper Bound
3	−33.61 dB
4	−60.73 dB
5	−91.97 dB

of the sine series, since its highest term contains θ^6 instead of θ^7 and, thus, less programming is required.

The program starts at $-\pi/2$ and increments in steps of $\phi = 2\pi f\text{T}$, where f is the desired frequency and T is the interval between samples. The computed function should remain positive until after n samples, when the accrued phase θ_n is such that $\theta_n = -\pi/2 + n\phi$ and exceeds $\pi/2$. At this time, π is subtracted from the accrued phase yielding a new value θ_n such that $-\pi/2 < \theta_n < 0$ and the process starts again. The computed value is now made negative. The negative sign prefixes each sample until, again, the accrued phase exceeds $+\pi/2$. Once again, π is subtracted from the accrued phase. The sign of each computed point becomes positive, and the algorithm cycle is complete.

An expansion around $\pi/4$ would result in less error in the resulting approximation at the end points, but it would require more programming to fix the sign. The errors for an expansion around zero are less than those attributable to the μ-255 operation and, thus, it is not important to reduce them.

3.2 Flow diagram

A simplified diagram, Fig. 10, covers the generation of one component. Note that the loop counter (LC) serves as a flip-flop for determining the sign of the result. Suppose LC = 0 and $\pi/2$ has been exceeded. The right-hand branch decision (LC≠0) point in Fig. 8 causes LC to be decremented to −1. During the next pass when the left-hand decision (LC≠0) is reached, the sign of the result is changed. This is maintained until the accumulated phase would again exceed $+\pi/2$, when traversing the right-hand decision point forces LC = 0. Now, the computed result is positive and the process repeats.

3.3 Experimental results

Figure 11 shows the spectrum of the DSP output after decoding. The harmonics are approximately 42 dB below the fundamental as predicted for a μ-255 decoder. Note, however, the differences in the spectrums for frequencies of 500 and 502 Hz. Since 500 Hz is a rational submultiple of 8 kHz, all samples per sine wave cycle are repeated in subsequent cycles. There is essentially no quantizing noise, i.e., the sample values are completely periodic. The pronounced spectral lines demonstrate this purity. For the 502-Hz case, there is no periodicity in the samples for successive cycles and, hence, quantizing noise exists. The peak value of the spectral harmonics are reduced and the noise floor is raised.

IV. CONCLUSIONS

A DSP tone generator using the table look-up method is able to generate up to 12 components (or 21 components for 2-coupled gen-

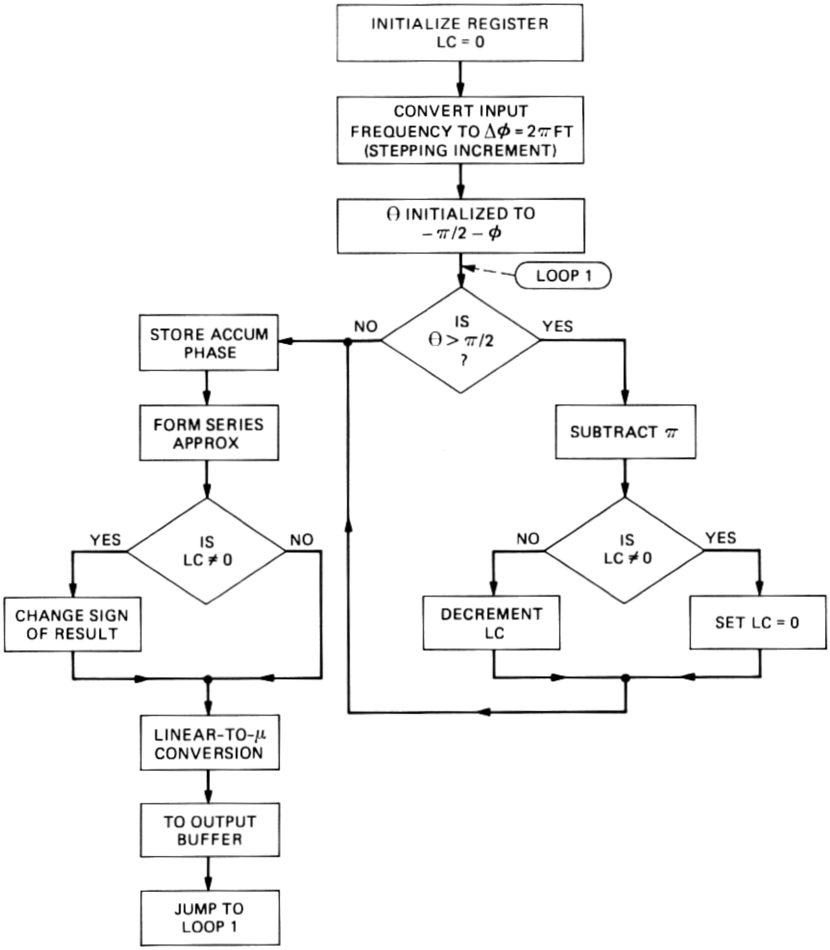


Fig. 10—Maclaurin expansion flow diagram.

erators) assuming a 4-kHz, μ-255 channel with an output sample every 125 μs. Harmonic distortion is substantially below that inherent in the channel. Such a versatile generator is suitable for all transmission system testing applications.

Given a suitable decoder, a DSP tone generator can also be used to create any analog wave representable by 13 spectral lines restricted to the 4-kHz band. A sine wave component can be generated every 17.75 μs and a 26-kHz sine wave can be generated. By suitable program changes, it seems possible to produce a 50-kHz sine wave.

A DSP tone generator using the Maclaurin expansion method is limited to less than four components. It requires less ROM than the table look-up method and, hence, is applicable where such a program

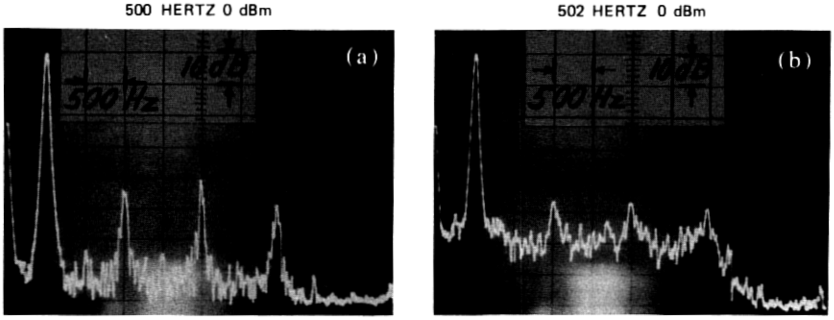


Fig. 11—Maclaurin spectrums.

would be co-resident with other measurement routines in a single DSP. Where 2-tone testing is required, e.g., envelope delay distortion, slope-sag intermodulation distortion, etc., such a generator is especially applicable.

V. ACKNOWLEDGMENTS

Grateful thanks to E. J. Angelo and J. R. Boddie for their general assistance; to D. P. Yorkgitis for providing the diagnostic and refresh routines for the table look-up method; to Basil Papatrefon for testing the programs and deriving all experimental results; and to Jack Salz for offering his suggestions concerning the mathematics bounding the harmonic distortion for the Maclaurin expansion approach.

APPENDIX

This analysis is directed to specifying an upper bound on the harmonic content of a truncated Maclaurin expansion.

Given the Maclaurin expansion

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \frac{\theta^8}{8!} - + \frac{\theta^{2(n-1)}}{[2(n-1)]!} \cdots, \tag{1}$$

consider it as being made up of a finite polynomial $P(\theta)$ consisting of the first n terms of the above expansion and the remaining portion of the series $R(\theta)$, i.e.,

$$\cos\theta = P(\theta) + R(\theta).$$

Consider now that a periodic function be generated such that

$$P_N(\theta) = P(\theta) \quad \text{for} \quad -\frac{\pi}{2} \le \theta < \frac{\pi}{2},$$

and

$$P_N(\theta) = 0 \quad \text{elsewhere.} \tag{2}$$

Similarly,

$$R_N(\theta) = R(\theta) \quad \text{for} \quad -\frac{\pi}{2} \leq \theta < \frac{\pi}{2},$$

and

$$R_N(\theta) = 0 \quad \text{elsewhere.} \tag{2a}$$

By a replicating procedure, then

$$\cos\theta = \sum_{n=-\infty}^{\infty} (-1)^n P_N(\theta - n\pi) + \sum_{n=-\infty}^{\infty} (-1)^n R_n(\theta - n\pi). \tag{3}$$

The portion of eq. 3,

$$\cos\theta \approx \sum_{n=-\infty}^{\infty} (-1)^n P_N(\theta - n\pi),$$

is the approximation used for sample generation in the Maclaurin series program given in this paper.

Because of the definitions in eqs. (2) and (2a), each sum specified in eq. 3 is periodic with period 2π and, hence, can be expanded into a Fourier series. Let p_k and r_k be the Fourier series coefficients defined by

$$\sum_{n=-\infty}^{\infty} (-1)^n P_N(\theta - n\pi) = \sum_{k=-\infty}^{\infty} p_k e^{ik\theta}, \tag{4}$$

$$\sum_{n=-\infty}^{\infty} (-1)^n R_N(\theta - n\pi) = \sum_{k=-\infty}^{\infty} r_k e^{ik\theta} \tag{5}$$

and

$$\cos\theta = \sum_{k=-\infty}^{\infty} p_k e^{ik\theta} + \sum_{k=-\infty}^{\infty} r_k e^{ik\theta}, \tag{6}$$

where

$$p_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} (-1)^n P_N(\theta - n\pi) e^{-ik\theta} d\theta \tag{7}$$

and

$$r_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} \sum_{n=-\infty}^{\infty} (-1)^n R_N(\theta - n\pi) e^{-ik\theta} d\theta. \tag{8}$$

Interchanging the order of integration and summation in eq. 7, then

$$p_k = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} (-1)^n \int_{-\pi}^{\pi} P_N(\theta - n\pi) e^{-ik\theta} d\theta. \tag{9}$$

Changing the variable of integration $y = \theta - n\pi$ modifies this equation to be

$$p_k = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} (-1)^{n+kn} \int_{-\pi(n+1)}^{-\pi(n-1)} P_N(y) e^{-iky} dy, \tag{10}$$

and further simplifies to

$$p_k = \frac{1}{2\pi} \sum_{n=-\infty}^{\infty} (-1)^{n(1+k)} \int_{-\pi(n+1)}^{-\pi(n-1)} P_N(y) e^{-iky} dy. \tag{11}$$

Because of the definition of $P_N(y)$, the integral only exists for $n = 1$, -1, 0;

$$p_k = \frac{1}{2\pi} \left[\int_{-\pi/2}^{\pi/2} P_N(y) e^{-iky} dy + (-1)^{(1+k)} \int_{-\pi/2}^{0} P_N(y) e^{-iky} dy \right.$$
$$\left. + (-1)^{-(1+k)} \int_{0}^{\pi/2} P_N(y) e^{-iky} dy \right]. \tag{12}$$

Combining these terms results in

$$p_k = \frac{1}{2\pi} [1 + (-1)^{(1+k)}] \int_{-\pi/2}^{\pi/2} P_N(y) e^{-iky} dy. \tag{13}$$

When k is even, $p_k = 0$, therefore, only odd harmonics exist in the periodic function given by eq. 4. Since $P_N(\theta)$ is an even function, it follows that

$$p_k = \frac{2}{\pi} \int_{0}^{\pi/2} P_N(\theta) \cos k\theta d\theta \qquad k \text{ odd}. \tag{14}$$

By a completely similar development for r_k, and substituting into eq. 6,

$$\cos \theta = 2 \left[\sum_{\substack{k=1 \\ k\text{odd}}}^{\infty} p_k \cos k\theta + \sum_{\substack{k=1 \\ k\text{odd}}}^{\infty} r_k \cos k\theta \right]. \tag{15}$$

From this equation, it is apparent that

$$p_k = -r_k \quad \text{and} \quad k \neq 1 \tag{16}$$

since only the fundamental exists on the left-hand side of the equation.

Given the properties of $R(\theta)$ such that each term is positive and smaller than the preceding term, then one can state that the magnitude of the first term of $R(\theta)$ is greater than $|R(\theta)|$. Hence, if $P(\theta)$ contains

n terms then

$$\frac{\theta^{2n}}{2n!} \geq |R(\theta)|. \tag{17}$$

The stipulation that the terms are successively smaller implies for the m^{th} and $m^{\text{th}} + 1$ term that

$$1 \leq [\theta^{2(m-1)}/(2(m-1))!]/[\theta^{2m}/(2m!)] = \frac{2m(2m-1)}{\theta^2}.$$

Since this must hold for all $-\pi/2 \leq \theta \leq \pi/2$, then

$$2m(2m-1) \geq \left(\frac{\pi}{2}\right)^2$$

and, hence, for this inequality to hold

$$m > 1 \quad \text{applies.}$$

Equation 17, therefore, applies for maximum θ, after the second term, and certainly then

$$[(\pi/2)^{2n}/(2n)!] \geq |R(\theta)| \quad \text{for} \quad n \geq 2. \tag{18}$$

Parsevals theorem states that

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\left[\sum_{n=-1}^{\infty}(-1)^n R_N(\theta - n\pi)\right]^2 d\theta = \sum_{\substack{k=1 \\ k\text{odd}}}^{\infty} r_K^2. \tag{19}$$

Since $R(\theta)$, and hence, $R_N(\theta)$, are bounded, as has been stated in eq. (18), then certainly

$$[(\pi/2)^{2n}/(2n)!]^2 \geq \sum_{\substack{k=3 \\ k\text{odd}}} r_k^2 = \sum_{\substack{k=3 \\ k\text{odd}}} p_k^2. \tag{20}$$

Therefore, the first term of $R(\theta)$, evaluated at $\pi/2$, represents an upper bound on any harmonic component of the replicated $P_N(\theta)$ i.e., the truncated Maclaurin expansion. The harmonics of

$$(\text{Replicated} \qquad P_N(\theta)) < 20 \log [(2n)!/(\pi/2)^{2n}]. \tag{21}$$