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Adaptive Cancellation of Intersymbol Interference for Data Transmission

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In this paper, we analyze a technique for accurately detecting transmitted data symbols contained in a modulated signal that has been degraded by a linear dispersive channel and additive Gaussian noise. The approach uses an adaptive equalizer which provides preliminary decisions to an adaptive canceller. The canceller output is used to remove the interference from an adaptive matching filter, resulting in the desired signal. Channel equalization attempts to invert the channel transfer function, while avoiding excessive noise enhancement. However, cancellation (as used in echo cancellers), attempts to generate a replica of the interfering signal and subtract it from the actual received signal containing the sum of the desired signal and interference. The cancellation approach, unlike equalization, offers the possibility of removing interference without enhancing the level of noise already present in the received waveform. Simulation results for transmission over practical channels show significant improvement of linear cancellation over both linear forward and decision-feedback equalization.

I. INTRODUCTION

For the past twenty years, engineers have been seeking new techniques to combat the intersymbol interference (ISI) in data transmission over band-limited channels. Adaptive equalization with the mean-square algorithm has been the major technique that allowed a substantial increase in attainable transmission rate.[1] If the channel has only phase distortion, then the linear fractionally spaced equalizer can eliminate virtually all of the ISI without enhancing the noise level.[2,3] However, when amplitude distortion is present in the channel, any adaptive linear equalizer (LE) must compromise between inverting the channel transfer function and avoiding excessive noise enhancement. Inevitably, some noise enhancement occurs. Decision-feedback equalization can offer somewhat improved performance when amplitude distortion is present.[4,5] By using the Viterbi algorithm,[6] maximum-likelihood receivers, in principle, offer the best performance possible but depend on adaptive estimators of the channel and require an impractically high complexity when the channel impulse response is long, as in the case of the typical telephone channel.

If there were no ISI, the probability of error in detecting the transmitted data level (i.e. ± 1 or ± 3) would be the same as if only one such pulse were transmitted in isolation. In that case, the optimal receiver (for Gaussian noise) would be a matched filter and would yield a certain error probability, P_0. When pulses are sent sequentially, the effect of ISI cannot be totally eliminated. The maximum likelihood estimator of the entire sequence of transmitted symbols is known to result in an error probability that is somewhat larger than P_0.

In this paper, we describe a cancellation technique designed to achieve isolated-pulse, matched-filter performance. Extensive simulation results confirm that there is a significant improvement over linear or decision-feedback equalization for severe amplitude-distorting channels of practical interest.

II. BACKGROUND AND MOTIVATION

The idea of cancellation was used for the echo problem in two-wire telephony (see Ref. 7), where the received signal contains an interference component that is a filtered and delayed version of an "originating" signal. In that application, the originating signal is actually available at the same location. However, in data transmission the originating signal—the transmitted data stream—is not directly observable at the receiver. The idea of using preliminary decisions to generate an intermediate estimate of the transmitted data signal was independently proposed by various investigators. All of the proposals included adapting the coefficients of a filter that forms a replica of the ISI. Adaptation is, of course, needed since the appropriate filtering operation is not known in advance and can vary with time. Hirsch and Proakis proposed the cancellation scheme with the essential structure in Figure 1.[8,9] Here the canceller attempts to remove the ISI directly from the received line signal. This approach does not achieve improved performance over linear equalization if there is phase distortion. In this paper, we describe a linear canceller (LC) structure where a

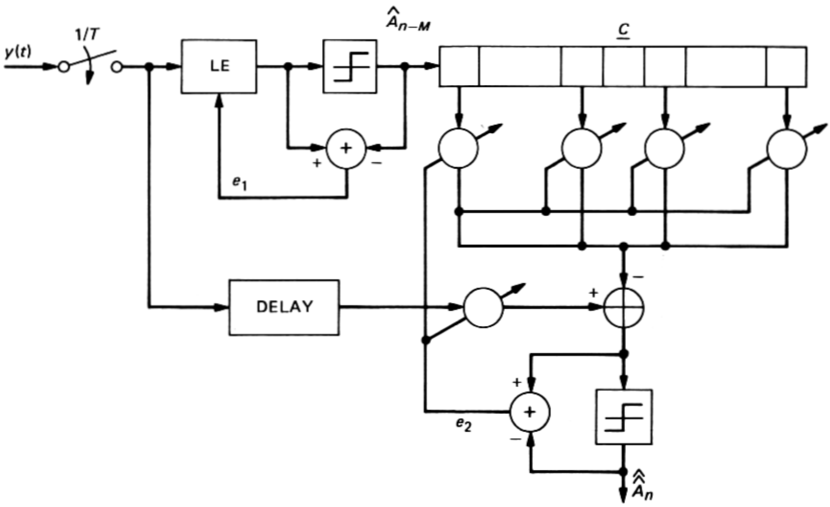


Fig. 1—First generation linear canceller.

transversal filter $\underline{W}$ is used instead of the delay in Fig. 1. Both $\underline{C}$ and $\underline{W}$ are adapted simultaneously with the error signal between the input to the final detector and the appropriate reference.

The motivation for this structure stems from the need to effectively detect high-speed data on channels that have both a high noise level and substantial amplitude (slope) and phase distortion. In certain conditions of practical interest, even if the equalizer were of infinite length, the noise enhancement of linear or decision feedback equalization makes it impossible to achieve the required error rate. Nevertheless, linear equalization is sufficient to obtain fairly moderate error rates so that the detected symbols may be adequate as preliminary decisions for a cancellation scheme.

In Section 3, we provide intuitive reasoning that the optimal choice for $\underline{W}$ is a matched filter, and the optimal $\underline{C}$ is a canceller whose tap weights are the samples of the channel autocorrelation function, except for the center tap, which has zero weight. We show in Section 4 that this is true under the assumption that the preliminary decisions $\hat{A}_k$ are correct. Section 5 covers adaptive operation and Section 6, simulation results.

III. FORMULATION

Let the transmitted data symbols be denoted A_1, A_2, A_3, $\cdots$ with each complex valued symbol having real and imaginary parts restricted to one of a finite set of values (i.e. ± 1, ± 3). A complex-valued pulse shape $p(t)$ is used to generate the baseband transmitted data signal

$$s(t) = \sum_k A_k p(t - kT). \tag{1}$$

The linear distortion of the channel results in the received waveform

$$X(t) = \sum_k A_k h(t - kT) + V(t), \tag{2}$$

where $V(t)$ is white noise and $h(t)$ is the overall channel impulse response.

Suppose initially that the receiver consists of a matched filter and a sampler (rate $1/T$) followed by a symbol detector as shown in Fig. 2. The matched filter has impulse response

$$W(t) = h^*(LT - t), \tag{3}$$

where * denotes complex conjugation and the integer L is chosen large enough so that the output is small for $t < 0$. The output of the sampler is then

$$U(mT) = \sum_k A_k r(mT - kT - LT) + V^1(mT), \tag{4}$$

where $V^1(t)$ is the colored noise at the matched filter output and $r(t)$ is the autocorrelation function of the pulse $h(t)$; that is, $r(t) = \int_{-\infty}^{\infty} h(s)g(s + t)ds$. Equation (4) can be written as

$$U(mT) = A_{m-L}r(0) + V^1(mT) + I_{m-L}, \tag{5}$$

where

$$I_{m-L} = \sum_{k \neq m-L} A_k r(mT - kT - LT). \tag{6}$$

Suppose that at time $t = mT$ the receiver must detect the currently observable symbol A_{m-L} and it knows all prior symbols A_{m-L-1}, $A_{m-L-2}, \ldots$ and all subsequent symbols $A_{m-L+1}, A_{m-L+2}, \ldots, A_m$ that determine the value I_{m-L} of the total ISI. In this case, I_{m-L} is a known constant and can be subtracted from $U(mT)$. What remains is exactly the output value that the matched filter would produce if the transmitter sent a single isolated pulse $A_{m-L}p[t - (m - L)T]$. Hence, the ideal performance, with error probability P_0, would be achieved.

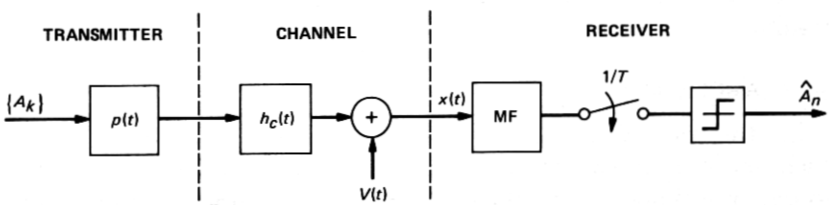


Fig. 2—Model of transmitter, channel and matched filter (MF).

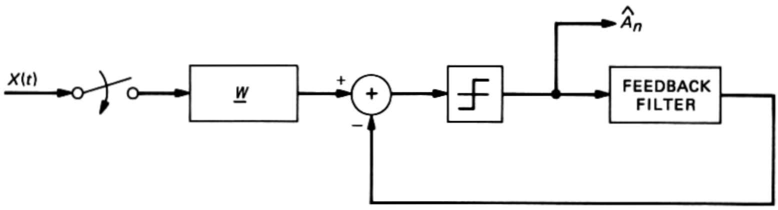


Fig. 3—Decision feedback equalizer.

The above reasoning suggests that we could approach the ideal—isolated symbol—performance with each symbol decision if we could generate a good estimate of the total ISI, I_{m-L}, at each sampling instant, $t = mT$. The decision feedback approach can be viewed as a partial step in this direction. This approach is based on the idea that we can estimate the prior symbols called precursors by storing and processing the outputs $\hat{A}_{m-L-1}, \hat{A}_{m-L-2}, \ldots$ already produced by the detector. The part of I_{m-L} determined by the precursors can then be constructed. By applying the decision $\hat{A}_i$ to a feedback filter, the output is subtracted from $U(mT)$ and the resultant signal is applied to the detector. The decision-feedback equalizer (DFE) is shown in Fig. 3. Since we have not removed all of the ISI, the resulting performance will be inferior to that of the isolated-pulse case. This discussion shows that the DFE technique can be regarded as a partial step towards the goal of totally removing ISI. We next examine how we go beyond the stage of postcursor cancellation to include precursor cancellation.

Suppose at time mT we could also eliminate the subsequent, postcursor, symbols $A_{m-L+1}, A_{m-L+2}, \ldots, A_m$ that also contribute to the total interference at time mT. Then, using eq. (6), an estimate of the total ISI, I_{m-L}, would be available at time instant mT. This is not possible using the output of the detector in Fig. 2. However, suppose a separate equalizer operates on the received data signal $y(t)$ as shown in (a) of Fig. 4. If optimally designed, it will have a modest error rate, and we can use its decision $\hat{A}_n$, as preliminary or tentative decisions for the purpose of constructing our estimate of I_{m-L}. Now there is no problem in obtaining both precursor and postcursor estimates needed to form I_{m-L}. By introducing a fixed time delay, D, to the received signal prior to the matched filter as shown in (a), the LE has a head-start in estimating data symbols. A practical implementation of an adaptive passband canceller would take the form shown in (b) of Fig. 4.

The delay D can actually be incorporated into the matched filter by choosing L suitably large. The cancellation filter $\underline{C}$ produces the estimate $\hat{I}_{m-L}$ of the actual interference

$$\hat{I}_{m-L} = \sum_{k \neq m-L} \tilde{A}_k r(mT - kT - LT), \tag{7}$$

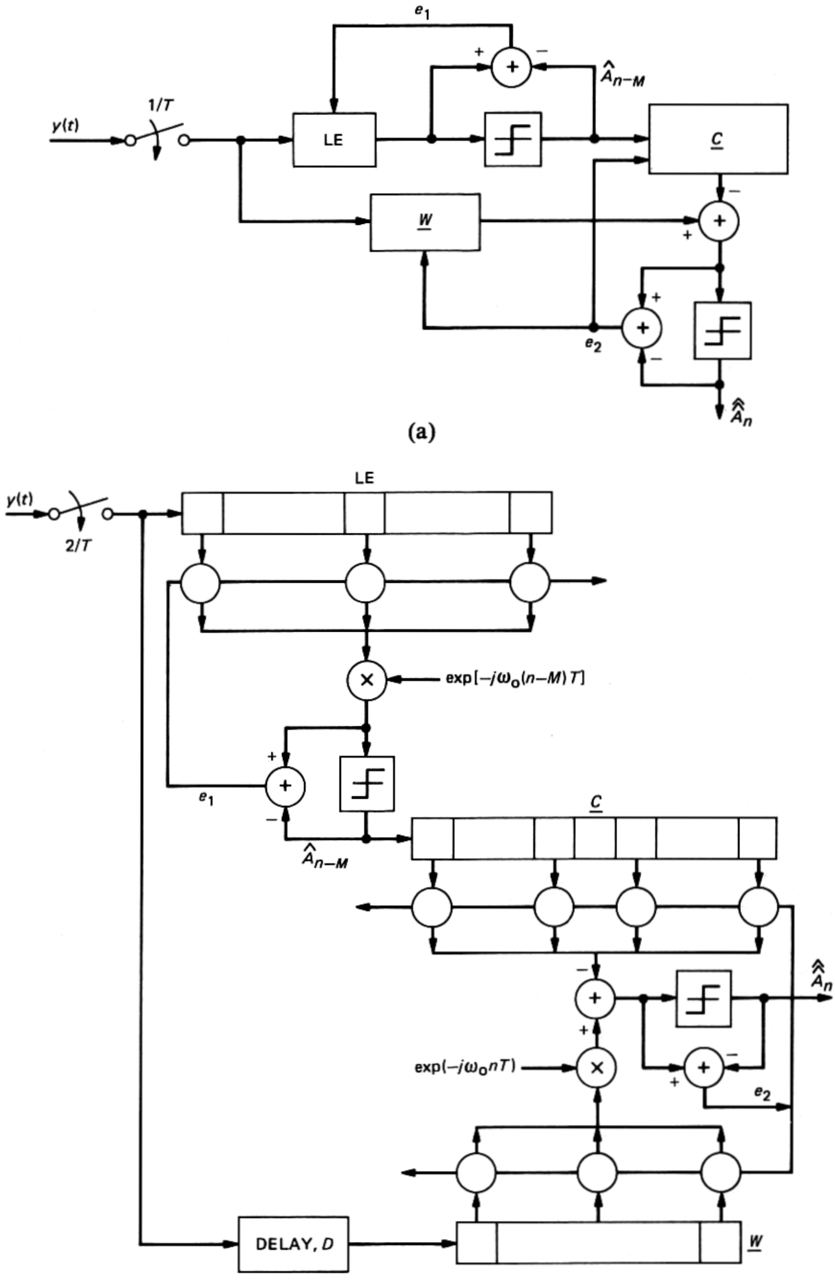


Fig. 4—(a) Linear canceller. (b) Passband linear canceller structure.

where $\tilde{A}_k$ is the sequence of preliminary decisions. Note that the transversal filter $\underline{C}$, which takes as input $\hat{A}_m$ and produces the output $\hat{I}_{m-L}$ at time mT, has an impulse response

$$c_i = \begin{cases} r[i - (m - L)T] & i \neq m - L \\ 0 & i = m - L \end{cases}. \qquad (8)$$

We shall see in Section IV that, under the assumption of perfect preliminary decisions, this is indeed the optimum impulse response in the mean-square sense.

IV. OPTIMAL CANCELLATION

4.1 Derivation of optimal filter coefficients

To determine the optimal pair of filters $\underline{W}$ and $\underline{C}$ for the cancellation scheme, we make the simplifying assumption that the preliminary decisions available from the LE are correct. We focus on the structure shown in Fig. 5, where the filter $\underline{W}$, called the matching filter, is a $T/2$-spaced infinite length transversal filter preceded by a sampler operating at rate $2/T$ samples per second. The filter $\underline{C}$, called the canceller, is a T-shaped infinite length transversal filter.

The matching filter $\underline{W}$ has input samples, $y_l = y(lT/2)$, where $y(t)$ is the received line signal, and the output, U_n, of $\underline{W}$ is taken at time instants, $t = kT$, as indicated in Fig. 5 by the $1/T$ rate sampler. The input to canceller $\underline{C}$ is the true data sequence $\{A_n\}$ since we have assumed the tentative decisions are correct. Thus, Fig. 5 shows $\underline{C}$ being fed directly by the transmitted data symbols. It is not necessary to explicitly consider the time delay D since we are allowing infinite length filters. The output of the canceller V_n is subtracted from the matching-filter output. This difference producing $U_n - V_n$ to be applied to a slicer may be viewed as a linear estimator of the data symbol A_n. The goal is to determine the filters $\underline{W}$ and $\underline{C}$ that minimize

$$E_n = E(|\epsilon_n|^2), \qquad (9)$$

the mean-square error (mse), where $\epsilon_n = U_n - V_n - A_n$.

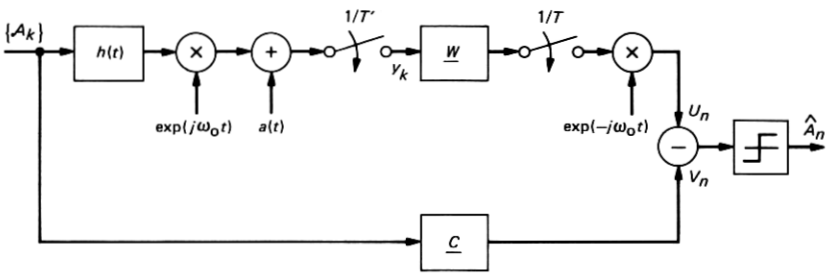


Fig. 5—Model of a linear canceller.

Let $\alpha(t)$ denote the additive receiver noise as shown in Fig. 5. Then, the input to the matching filter is given by

$$y_l = \exp(j\omega_0 lT/2) \sum_v A_v h(lT/2 - vT) + \alpha(lT/2)$$

$$= \exp(j\omega_0 lT/2) \left[\sum_v A_v h(lT/2 - vT) + \alpha_1(lT/2) \right], \tag{10}$$

where $h(t)$ is the complex impulse response of the channel and includes the effect of the transmitter shaping filter $p(t)$ as treated in Section III. The term $\alpha_1(\cdot)$ is the complex baseband noise sample. All summations are over the integers from $-\infty$ to ∞, unless otherwise indicated. The output U_n of the matching filter is

$$U_n = \exp(-j\omega_0 nT) \sum_k W_k^* y_{2n-k}, \tag{11}$$

where W_i denotes the tap weights of $\underline{W}$. Let $h_l = h(lT/2)$ and define

$$b_l = \sum_v W_v^* h_{l-v}. \tag{12}$$

Then U_n may be written in the form

$$U_n = \sum_s A_s b_{2n-2s} + N_n, \tag{13}$$

where

$$N_n = \sum_s W_s^* \alpha_{2n-s} \tag{14}$$

is the noise at the output of the matching filter.

We shall let C_i denote the ith tap weight of the cancellation filter for each integer i, except $i = 0$. We make the constraint that the *center tap weight of the cancellation filter is zero.* This restricts the role of the canceller to removing ISI and prevents the canceller from making use of the current data symbol which must be estimated by the output signal from the matching filter. The canceller output V_n is given by

$$V_n = \sum_{i \neq 0} C_i^* A_{n-i}, \tag{15}$$

and we assume that

$$E|A_n|^2 = 1, \tag{16}$$

which is a convenient normalization of the data symbol power level.

To minimize the mse in eq. (9), we differentiate E_n with respect to the complex tap weights $\{C_k\}$ and $\{W_k\}$ and set the derivatives to zero. Using eq. (15), it can be shown to yield

$$E(\epsilon_n^* A_{n-m}) = 0 \qquad m \neq 0, \tag{17}$$

and using eq. (11),

$$\exp(-j\omega_0 nT) E(\epsilon_n^* y_{2n-m}) = 0 \qquad \text{all } m. \tag{18}$$

Thus, these optimality conditions require that the error signal ϵ be orthogonal to the observable inputs to the $\underline{C}$ and $\underline{W}$ filters, namely, $\{A_n\}$ and $\{y_n\}$.

We discuss two cases of interest. In Case 1, the channel is wideband. We assume that the $\underline{W}$ filter processes the line signal directly from the channel without prior band-limiting so that even when sampled at a rate of $2/T$, the noise samples are uncorrelated. In Case 2, which is more relevant to our situation of a channel band-limited to the voice frequency range, the noise samples at $T/2$ spacing are correlated.

Case 1. Uncorrelated noise samples

In Case 1,

$$E[\alpha(lT/2)\alpha^*(kT/2)] = \sigma^2 \delta_{lk},$$

where σ^2 is the noise variance. We also define a new term,

$$R_h(l) = \sum_j h^*(jT/2) h(jT/2 + lT/2), \tag{19}$$

which is the autocorrelation function of the $T/2$-sampled impulse response, $h(jT/2)$. Then, with

$$E_h = \sum_j \left| h\left(j\frac{T}{2}\right) \right|^2 = R_h(0), \tag{20}$$

we show in the Appendix that the matching filter has $T/2$-spaced tap weights

$$W_m = \exp(-j\omega_0 mT/2) \frac{h(-mT/2)}{E_h + \sigma^2}, \qquad \text{all } m, \tag{21}$$

which is clearly proportional to a matched-filter impulse response. The $\underline{C}$ taps are shown in the Appendix to be

$$C_m = \frac{1}{(E_h + \sigma^2)} R_h(2m), \qquad m \neq 0. \tag{22}$$

Thus, the canceller impulse response, for $m \neq 0$, is that of the overall T-spaced impulse response of the channel and matching filter.

Case 2. Correlated noise samples

As described earlier, Case 2 corresponds to the voiceband telephone channel, where the noise has approximately the same bandwidth as the signal so that noise samples at $T/2$ spacing are correlated.

The noise correlation is

$$E[(\alpha kT/2)\alpha^*(lT/2)] = R_n(k - l).$$

We define $W(\omega)$ as the Fourier transform of the $\underline{W}$ tap weights, $S_n(\omega)$ is the sampled noise spectrum, and $\tilde{H}(\omega)$ is the Fourier transform of the channel-sampled impulse response. Then, with

$$\xi = \frac{T}{4\pi}\int_{\frac{-2\pi}{T}}^{\frac{2\pi}{T}} \frac{|\tilde{H}(-\omega - \omega_0)|^2}{S_n(-\omega)} d\omega, \tag{23}$$

we show in the Appendix that

$$W(\omega) = \frac{\tilde{H}(-\omega - \omega_0)}{(1 + \xi)S_n(-\omega)}. \tag{24}$$

The corresponding Fourier transform of the canceller is

$$C(\omega) = \frac{1}{1 + \xi} \frac{|\tilde{H}(-\omega)|^2}{S_n(-\omega + \omega_0)}. \tag{25}$$

Note that since

$$R_h^*(-2m) = R_h(2m),$$

and $S_n(-\omega + \omega_0)$ is real, we see from eqs. (22) and (25) that the optimum, infinitely long canceller sampled impulse response is Hermitian symmetric about the center, i.e.

$$C_m = C_{-m}^*. \tag{26}$$

4.2 Derivation of mse

We now derive the mse achieved for Case 1, under the assumption that $\underline{W}$ and $\underline{C}$ have the optimal impulse responses given by eqs. (21) and (22), respectively. The matching filter output is given by

$$U_n = \frac{1}{E_h + \sigma^2} \sum_s A_s R_h(2n - 2s) + N_n. \tag{27}$$

The noise, N_n, is the result of applying white noise with variance σ^2 to the matching filter eq. (65). Hence, using eqs. (66) and (67), we find that

$$E|N_n|^2 = \frac{\sigma^2 E_h}{(E_h + \sigma^2)^2}. \tag{28}$$

Also, the canceller output is given by

$$V_n = \frac{1}{E_h + \sigma^2} \sum_{s \neq n} A_s R_h(2n - 2s). \tag{29}$$

Hence, the error signal, with $R_h(0) = E_h$, is

$$\epsilon_n = U_n - V_n - A_n = \frac{E_h}{E_h + \sigma_n^2} A_n + N_n - A_n$$

$$= -\left(\frac{\sigma^2}{E_h + \sigma^2}\right) A_n + N_n, \tag{30}$$

so that, using eq. (30), the mse is

$$E_n = E|\epsilon_n|^2 = \frac{\sigma^2}{E_h + \sigma^2}. \tag{31}$$

The s/n, ρ, at the output of the cancellation system is defined as the ratio

$$\rho = \frac{E|U_n - V_n - N_n|^2}{E|N_n|^2}. \tag{32}$$

Hence, we find that

$$\rho = \frac{E_h}{\sigma^2}. \tag{33}$$

4.3 On the property of invariance of mse to timing phase

The mse expression in eq. (31) can be written in terms of the channel-sampled power spectrum. If $H(\omega)$ is the Fourier transform of the overall impulse response $h(t)$, then the transform of $h(jT/2)$ is

$$\tilde{H}(\omega) = \frac{2}{T} \sum_k H\left[\omega + \frac{4\pi k}{T}\right], \ |\omega| < \frac{2\pi}{T}, \tag{34}$$

and

$$E_h = \sum_j \left| h\left(j\frac{T}{2}\right)\right|^2$$

$$= \frac{T}{4\pi} \int_{-\frac{2\pi}{T}}^{\frac{2\pi}{T}} |\tilde{H}(\omega)|^2 d\omega. \tag{35}$$

Therefore, from eq. (31) the mse of the optimum LC, normalized to unit signal power, is

$$E_{LC} = 1 \Bigg/ \left[\frac{T}{4\pi\sigma^2} \int_{-\frac{2\pi}{T}}^{\frac{2\pi}{T}} |\tilde{H}(\omega)|^2 d\omega + 1\right] \tag{36}$$

If the matching filter were T-spaced, then we have

$$\tilde{H}(\omega) = \frac{1}{T} \sum_k H\left(\omega + \frac{2\pi k}{T}\right), \tag{37}$$

and

$$E_h = \frac{T}{2\pi} \int_{\frac{-\pi}{T}}^{\frac{\pi}{T}} |\tilde{H}(\omega)|^2 d\omega. \tag{38}$$

We can compare the results with similar expressions for the optimum LE and DFE in Ref. 5 which assume matched filters preceding the equalizers. They are

$$E_{\mathrm{LE}} = \frac{T}{2\pi} \int_{\frac{-\pi}{T}}^{\frac{\pi}{T}} \frac{1}{Y(\omega) + 1} d\omega, \tag{39}$$

and

$$E_{\mathrm{DFE}} = \exp\left\{-\frac{T}{2\pi} \int_{\frac{-\pi}{T}}^{\frac{\pi}{T}} \ln[Y(\omega) + 1] d\omega\right\}, \tag{40}$$

where

$$Y(\omega) = \frac{1}{\sigma^2} \sum_k \left| H\left(\omega + \frac{2\pi k}{T}\right)\right|^2. \tag{41}$$

The expression in eq. (39) is the same as that for an infinitely long fractionally spaced LE,[3] whereas the result for a symbol-spaced equalizer is

$$E_{\mathrm{LE}} = \frac{T}{2\pi} \int_{\frac{-\pi}{T}}^{\frac{\pi}{T}} \left[1 \Big/ \frac{1}{\sigma^2} \left| \sum_k H\left(\omega + \frac{2\pi k}{T}\right)\right|^2 + 1\right] d\omega. \tag{42}$$

Finally, we note that if the overall channel-impulse response has less than 100 percent rolloff and the matching filter is $T/2$-spaced, then

$$\tilde{H}(\omega) = \frac{1}{T} H(\omega),$$

and

$$E_h = \frac{T}{4\pi} \int_{\frac{-2\pi}{T}}^{\frac{2\pi}{T}} |H(\omega)|^2 d\omega.$$

Clearly, E_{LC} is independent of any phase characteristics and, hence, the canceller performance is insensitive to timing phase. This property is shared by the fractionally spaced LE[2,3] and the symbol-spaced LE and DFE which are preceded by matched filters, as exhibited by eqs. (39) and (40). On the other hand, by itself, the symbol-spaced LE is sensitive to timing phase, as seen in eq. (42), because the integrand is a function of the folded spectrum of the channel. This is because a symbol-spaced filter cannot properly synthesize a proper matched filter. Likewise, the LC with a T-spaced matching filter is sensitive to timing phase, although some simulations have indicated that the effect of a bad timing phase seems to be less on the LC than on the LE.

V. ADAPTIVE CANCELLER STRUCTURE

In practice, we have finite length filters and the ensemble averages described above are not available to the receiver. As in the case of the adaptive LE, we adjust the complex $\{C_k\}$ taps of the canceller and $\{W_k\}$ taps of the matching filter to minimize the instantaneous squared error,

$$|\epsilon_n|^2 = |U_n - V_n - A_n|^2, \tag{43}$$

at each time instant nT. As shown in (b) of Fig. 4, U_n and V_n are the outputs of the matching filter and canceller, respectively, where

$$U_n = \exp[-j(\omega_0 nT + \hat{\theta}_n)] \sum_{-L}^{L-1} W_k^* y_{2n-k}, \tag{44}$$

and

$$V_n = \sum_{\substack{k=-M \\ \neq 0}}^{M} C_k^* \hat{A}_{n-k}, \tag{45}$$

where we have $2L$ complex $\underline{W}$ taps at $T/2$ spacing and $2M$ complex $\underline{C}$ taps at T spacing. Unlike the ideal case in Fig. 5, $\hat{A}_{n-k}$ are the tentative decisions obtained from the LE, and $\hat{\theta}O_n$ is the estimate of any phase jitter present. The value of A_n in eq. (43) is that of the ideal reference in the training mode and is the receiver's output decision in the decision-directed mode of operation. Note that ϵ_n in eq. (43) is a linear function of the tap weights $\{C_k, W_k\}$ so that it is possible in theory to jointly adapt the tap weights to achieve a unique mse.

It can be shown that the gradients of $|\epsilon_n|^2$ with respect to the tap weights are given by

$$\frac{\partial|\epsilon_n|^2}{\partial C_k} = -2\epsilon_n^* \hat{A}_{n-k}, \tag{46}$$

and

$$\frac{\partial|\epsilon_n|^2}{\partial W_k} = 2\exp[-j(\omega_0 nT + \hat{\theta}_n)]\epsilon_n^* y_{2n-k}. \tag{47}$$

Thus, the adjustment algorithms are

$$C_k(n+1) = C_k(n) + \beta\epsilon_n^* \hat{A}_{n-k} \qquad \begin{cases} k = -M, \cdots, M \\ k \neq 0, \end{cases} \tag{48}$$

and

$$W_k(n+1) = W_k(n) - \beta\exp[-j(\omega_0 nT + \hat{\theta}_n)]\epsilon_n^* y_{2n-k},$$

$$k = L, \cdots, L-1. \quad (49)$$

The step size β is chosen to obtain reasonably fast tap convergence without the algorithm becoming unstable. The problem is similar to that addressed in Ref. 10 for adaptive equalizers where it is proved that the step size needs to satisfy the constraint

$$0 < \beta < \frac{2}{L\langle\chi^2\rangle},$$

where L is the total number of taps, $(2L + 2M)$ in our case, and $\langle\chi^2\rangle$ is the average input signal power. The fastest initial convergence is achieved with

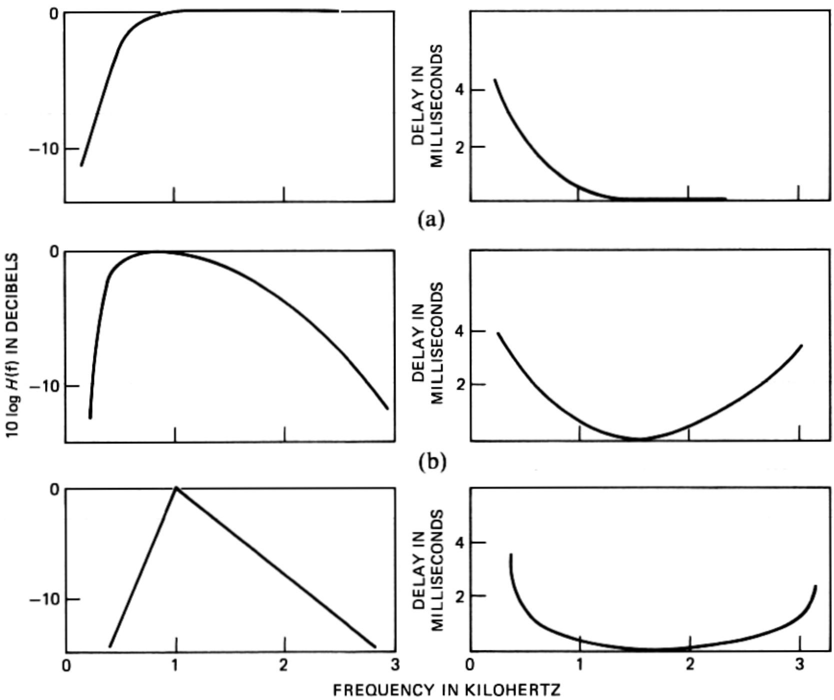
$$\beta = \frac{1}{L\langle\chi^2\rangle}. \quad (50)$$

VI. COMPUTER SIMULATION

The performance of the system over three channel models is evaluated. Channel 1 has a flat amplitude response over the entire voiceband frequency with little delay distortion, except near the lower band edge. Channel 2 has moderate amplitude and delay distortion, which just meets the private-line conditioning, and Channel 3 has severe amplitude distortion. These characteristics are shown in Fig. 6. Pseudorandom digital data at levels ±1 and ±3 are used to modulate the quadrature amplitude modulation (QAM) transmitter at a rate of 9.6 kb/s and the resulting signal is sent over one of the channels. Additive Gaussian noise and, whenever desired, phase jitter are introduced into the channel. The input signal-to-channel noise ratio (s/n_i) will be taken to be either 30, 24, or 20 dB.

In the simulations, the LC filters $\{C_k\}$ and $\{W_k\}$ are allowed to start adapting after 1000 iterations to ensure that the LE has converged sufficiently and is able to provide good tentative decisions for the LC. The performance of the LC is measured by the output s/n (s/n_o). This is defined as the ratio of the average data symbol power to the output noise power, which is taken to be the time average of the squared error at the LC output. We exhibit plots of s/n_o as a function of time, expressed in number of data symbol intervals, for both the LE and LC. Scatter plots of the output signal constellation are also provided. In the absence of any impairments, the plot would show 16 points with the x and y coordinates at ±1, ±3.

In the simulations, both LC filters span 31 data symbols so that the matching filter has 62 $T/2$ taps, while the canceller has 31 taps. The



(c)

Fig. 6—Channel characteristics. (a) Channel 1. (b) Channel 2. (c) Channel 3.

LE has 64 $T/2$ taps to sufficiently span the channel-impulse response. An initial step size of 0.0005 is chosen for updating all the tap coefficients and it is reduced to 0.00005, when the taps have nearly converged, to get a small final mse.

We first compare the performance of the LC and LE over the three channels described earlier using an s/n_i of 24 dB. The receiver switches from the ideal reference to the decision directed mode of operation after 3000 iterations. Figures 7, 8, and 9 show the receiver s/n_o as a function of time for the three channels.

Note that in the absence of significant distortion, as in the case of the Channel 1, the LE and LC perform equally well, and we see that s/n_o is approximately s/n_i. However, the LE degrades more than the LC when the channel is more severely distorted in amplitude.

Table I summarizes the results of several simulation runs, for the LE, DFE, and LC, over Channels 2 and 3 and with various s/n_i. Results for Channel 1 are not presented because all three schemes do not suffer any significant degradation. These results are based on the use of an ideal carrier demodulating phase. Note that for Channel 2, the

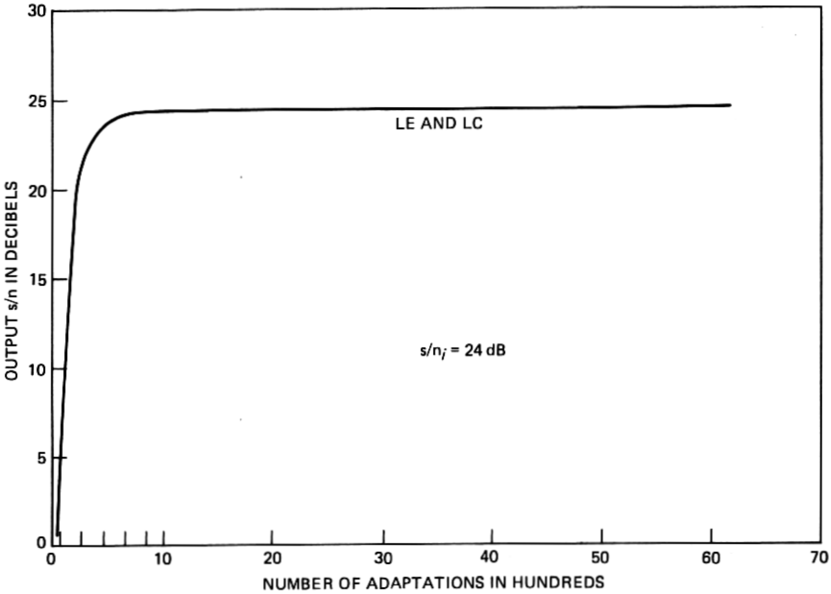


Fig. 7—Performance of LE and LC over Channel 1.

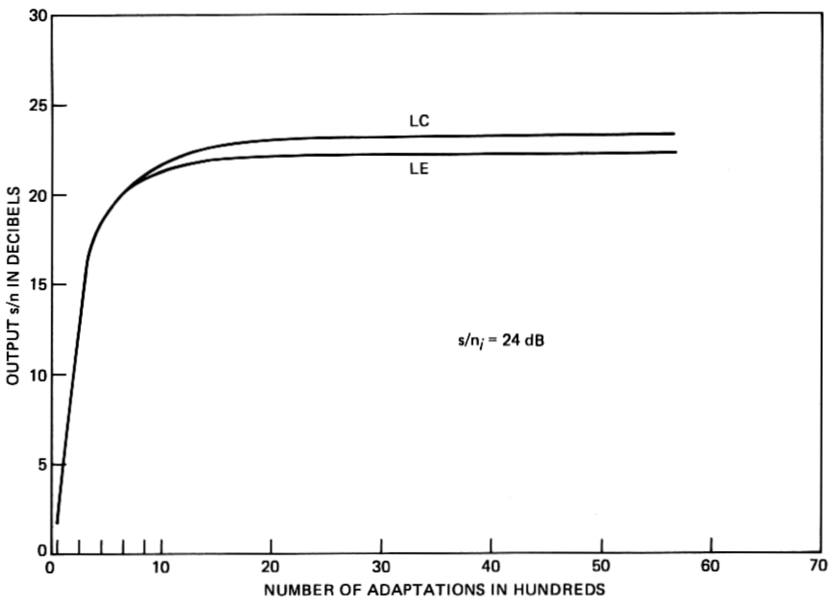


Fig. 8—Performance of LC and LE over Channel 2.

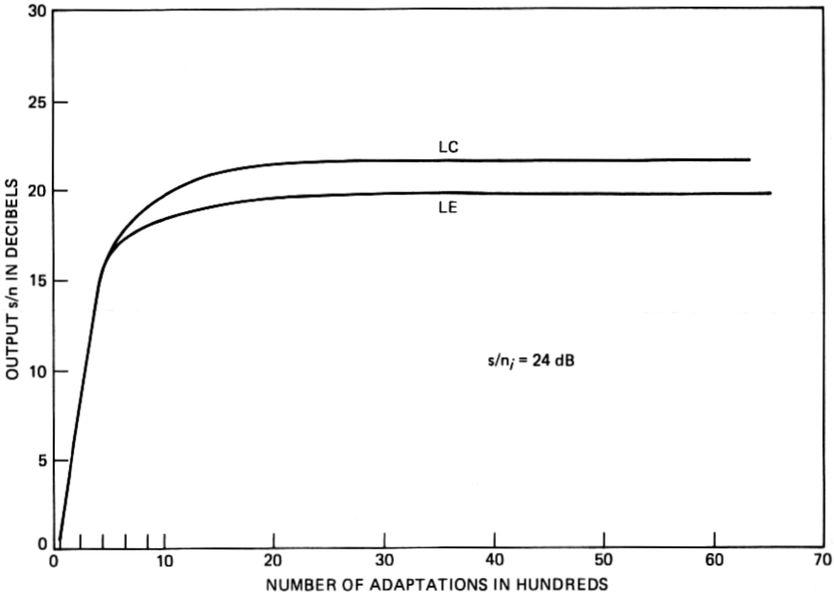


Fig. 9—Performance of LC and LE over Channel 3.

LC is able to attain s/n_o's which are very close to s/n_i, for all three cases. The gain is about 2 dB over the LE and about 1 dB over the DFE. If we can use the rule of thumb that every 1-db gain corresponds to an order of magnitude reduction in error rate, then the LC would have an error rate two orders of magnitude less than the LE and one order less than the DFE. The results for Channel 3, again, show the improvement over both LE and DFE. We see that the LE performance degrades significantly when the channel has considerable slope distortion, and on the average, it suffers losses of about 4 dB from the input s/n. The LC shows improvements of about 3 db over the LE and 1 dB over the DFE in all three cases.

We conclude this section by comparing the scatter plots of the receiver outputs after processing by an LE and an LC. In this example, Channel 3 is used and the s/n_i is 24 dB. Figure 10 is the result of linear

Table I—Comparison of linear equalizer, decision feedback equalizer, and canceller performances

	Channel 2			Channel 3		
s/n_i (dB)	20	24	30	20	24	30
Linear equalizer s/n_o	18.0	22.1	26.8	15.4	20.4	25.5
Decision feedback equalizer s/n_o	19.1	23.0	27.9	17.3	22.0	27.1
Canceller s/n_o	19.7	23.8	29.2	18.5	23.1	28.7

equalization. There is a considerable point spread because of severe amplitude distortion. Figure 11 shows the result of cancellation, which considerably tightens the spread and provides a larger noise margin than equalization.

VII. CONCLUSION

Cancellation is a powerful alternative approach to linear equalization that strives to mitigate the effects of slope distortion. This technique is based on cancelling the sidelobes of the overall channel-impulse response, unlike equalization, which attempts to invert the overall channel.

Simulation results of QAM transmission at 9.6 kb/s have shown that the canceller performs impressively, especially for severely amplitude-

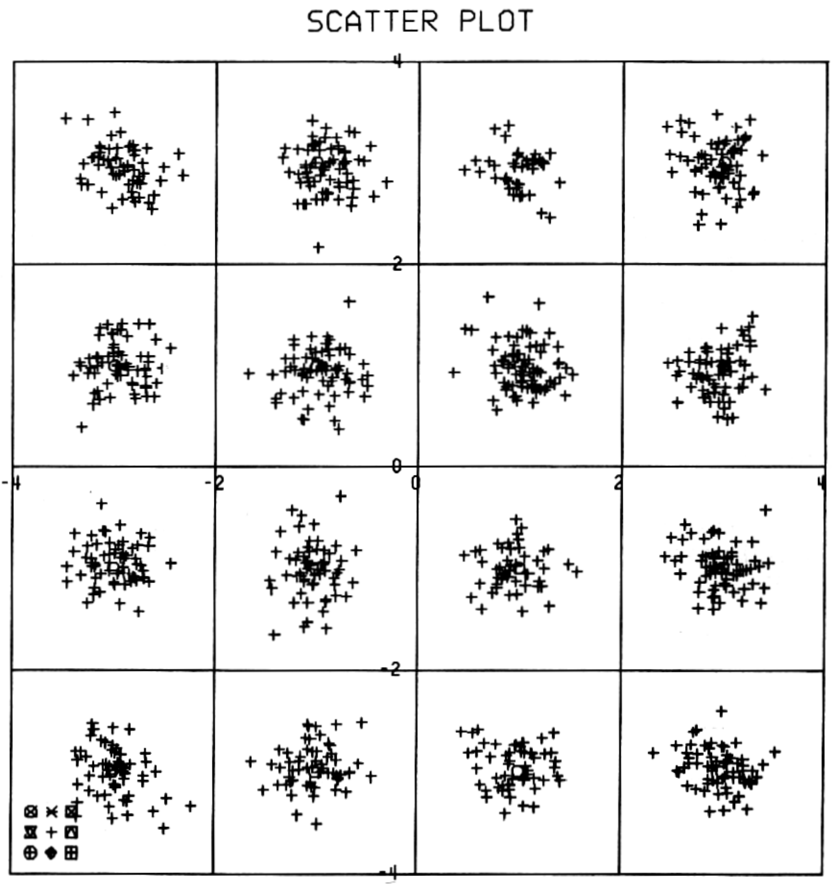


Fig. 10—Linear equalizer output signal constellation.

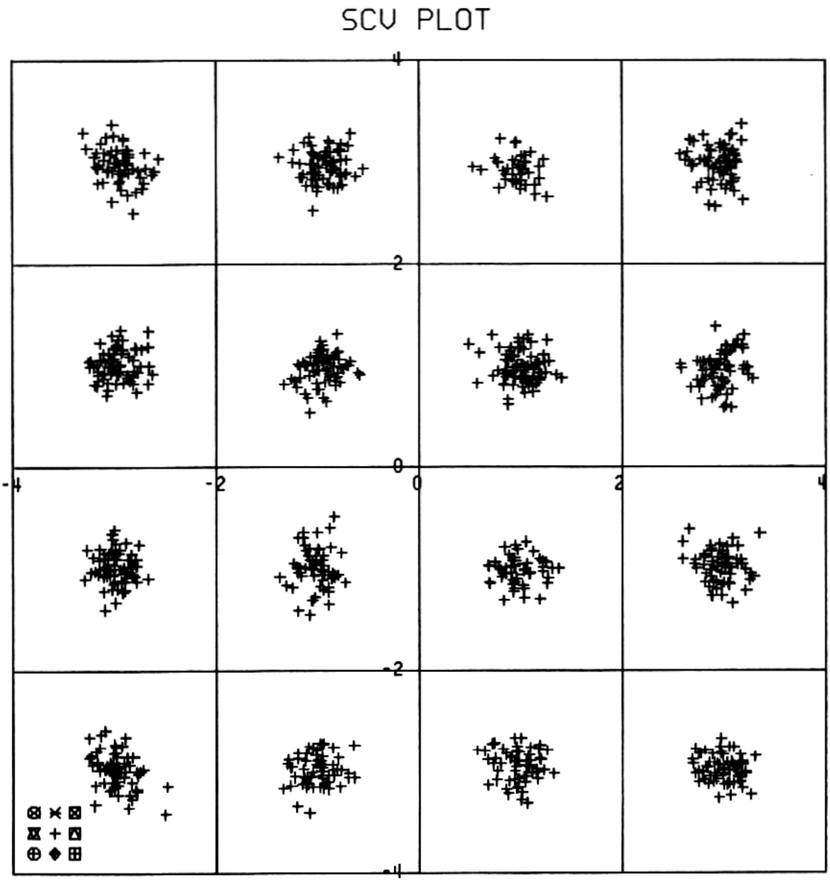


Fig. 11—Linear canceller output signal constellation.

distorted channels, where neither the LE nor the DFE is satisfactory. The LC needs an LE to provide it with reasonably good tentative decisions to perform the cancellation. It is then able to provide a final output which has a lower mse than the tentative output. In addition, the canceller is insensitive to phase distortion, provided the matching filter has an input sampling rate at least twice the data symbol rate. Hence, the LC accommodates the task of a fractionally spaced LE as well.

In summary, we have introduced a fundamentally new approach that performs better than either the linear or decision-feedback equalization, as confirmed by simulation results. Moreover, a recent theoretical study[11] applicable to this cancellation method further confirmed the effectiveness of our approach.

VIII. ACKNOWLEDGMENT

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APPENDIX

We derive the optimum minimum mse LC filters $\underline{C}$ and $\underline{W}$ under the assumption of (i) uncorrelated noise samples, and (ii) correlated noise samples.

Case 1. Uncorrelated noise samples

In Case 1

$$E[\alpha(lT/2)\alpha^*(kT/2)] = \sigma^2\delta_{lk},$$

where σ^2 is the noise variance. Consequently, we have, from eqs. (10), (11), and (15),

$$E(U_n^* A_{n-m}) = \sum_k W_k \exp(j\omega_0 kT/2) \quad h^*[(2m-k)T/2], \tag{51}$$

$$E(V_n^* A_{n-m}) = C_m, \qquad m \neq 0, \tag{52}$$

$$\begin{aligned} E(y_{2n-k}^* y_{2n-m}) &= \exp[j\omega_0(k-m)T/2] \sum_p \sum_q E\{A_p^* h^*[(2n-k)T/2 \\ &\qquad -pT] + \alpha_1^*[(2n-k)T/2]\} \\ &\qquad \times \{A_q h[(2n-m)T/2 - qT] \\ &\qquad + \alpha_1[(2n-m)T/2]\} \\ &= \exp[j\omega_0(k-m)T/2] \sum_p h^*(pT - kT/2) \\ &\qquad \times h(pT - mT/2) + \sigma^2\delta_{km} \\ &= R_h^k(k-m) \qquad + \sigma^2\delta_{km}, \end{aligned} \tag{53}$$

where

$$R_h^l(\tau) = \begin{cases} \exp(j\omega_0\tau) \sum_k h^*(kT + T/2) h(kT + T/2 + \tau), & l \text{ odd} \\ \exp(j\omega_0\tau) \sum_k h^*(kT) h(kT + \tau) & 1 \text{ even} \end{cases} \tag{54}$$

and we also define another term,

$$R_h(l) = \sum_j h^*(jT/2) h(jT/2 + lT/2), \tag{55}$$

which will be used later. Equation (55) is the autocorrelation function of the $T/2$-sampled impulse response $h(jT/2)$, whereas eq. (54) is T-spaced. So, from eq. (11)

$$\begin{aligned} E(U_n^* y_{2n-m}) &= \exp(j\omega_0 nT) \sum_k W_k E(y_{2n-k}^* y_{2n-m}) \\ &= \left[\sum_k W_k R_h^k(k-m) + \sigma^2 W_m \right] \exp(j\omega_0 nT). \end{aligned} \tag{56}$$

Also,

$$E(A^*_{n-k}y_{2n-m}) = \exp[j\omega_0(2n-m)T/2]\sum_p E(A^*_{n-k}A_p)$$
$$\times h[(2n-m)T/2 - pT]$$
$$= \exp[j\omega_0(2n-m)T/2]h[(2k-m)T/2]. \quad (57)$$

Therefore,

$$E(V^*_n y_{2n-m}) = \exp[j\omega_0(2n-m)T/2]\sum_{i\neq 0} C_i h[(2i-m)T/2], \quad (58)$$

and

$$E(A^*_k y_{2n-m}) = \exp[j\omega_0(2n-m)T/2]h(-mT/2). \quad (59)$$

By substituting eqs. (51) to (59) into eqs. (17) and (18), we have

$$C_m = \sum_k W_k \exp(j\omega_0 kT/2)h^*[(2m-k)T/2], \qquad m \neq 0, \quad (60)$$

and

$$\sum_k W_k R^k_h(k-m) + \sigma^2_n W_m = \exp(-j\omega_0 mT/2)$$
$$\times \sum_{i=0} C_i h[(2i-m)T/2] + \exp(-j\omega_0 mT/2)h(-mT/2), \quad \text{all } m. \quad (61)$$

We next solve for $\{C_i\}$ and $\{W_i\}$ from this pair of equations. Substituting eq. (60) into eq. (61) gives

$$\sum_k W_k R^k_h(k-m) + \sigma^2_n W_m = \exp(-j\omega_0 mT/2)\sum_{i\neq 0}\sum_k W_k$$
$$\times \exp(j\omega_0 kT/2)h^*[(2i-k)T/2]$$
$$\times h[(2i-m)T/2]$$
$$+ \exp(-j\omega_0 mT/2)h(-mT/2)$$
$$= \sum_k W_k[\exp j\omega_0(k-m)T/2]$$
$$\times \sum_i h^*[(2i-k)T/2]$$
$$\times h[(2i-m)T/2] + \exp(-j\omega_0 mT/2)$$
$$\times h(-mT/2)[1 - \sum_k W_k \exp$$
$$\times (j\omega_0 kT/2)h^*(-kT/2)]$$
$$= \sum_k W_k R^k_h(k-m) + \exp(-j\omega_0 mT/2)$$
$$\times h(-mT/2)(1-\beta), \quad (62)$$

using eq. (55), and where

$$\beta = \sum_k W_k \exp(j\omega_0 kT/2) h^*(-kT/2). \tag{63}$$

Therefore, we obtain the equality

$$\sigma^2 W_m = \exp(-j\omega_0 T/2) h(-mT/2)(1-\beta), \qquad \text{all } m, \tag{64}$$

or

$$W_m = \exp(-j\omega_0 mT/2) h(-mT/2) \frac{(1-\beta)}{\sigma^2}. \tag{65}$$

On substituting eq. (65) into eq. (63), we can show that

$$\beta = \frac{E_h}{E_h + \sigma^2}, \tag{66}$$

where, using eq. (55),

$$\begin{aligned} E_h &= \sum_j \left| h\left(j\frac{T}{2}\right)\right|^2 \\ &= R_h(0). \end{aligned} \tag{67}$$

From eqs. (66) and (65), we see that the matching filter has $T/2$-spaced tap weights

$$W_m = \exp(-j\omega_0 mT/2) \frac{h(-mT/2)}{E_h + \sigma^2}, \qquad \text{all } m, \tag{68}$$

which is clearly proportional to a matched-filter impulse response.

Case 2. Correlated noise samples

As described in Section IV, Case 2 corresponds to the voiceband telephone channel where the noise has approximately the same bandwidth as the signal so that noise samples at $T/2$ spacing are correlated.

With noise correlation

$$E[(\alpha kT/2)\alpha^*(lT/2)] = R_n(k-l),$$

and eq. (53) becomes

$$E(y^*_{2n-k} y_{2n-m}) = R^k_h(k-m) + R_n(k-m). \tag{69}$$

Then, in place of eq. (56), we have

$$E(U^*_n\, y_{2n-m}) = \sum_k W_k[R^k_h(k-m) + R_n(k-m)]. \tag{70}$$

Consequently, eq. (62) becomes

$$\sum_k W_k R_n(k-m) = \exp(-j\omega_0 mT/2) h(-mT/2)(1-\beta), \qquad \text{all } m. \tag{71}$$

We have to solve a set of linear equations for the $\underline{W}$ taps, and the solution in the time domain is not obvious. Instead, we examine the results in the frequency domain. Transforming both sides of eq. (70) gives

$$W(\omega)S_n(-\omega) = (1-\beta)\tilde{H}(-\omega-\omega_0);$$

that is,

$$W(\omega) = (1-\beta)\tilde{H}(-\omega-\omega_0)/S_n(-\omega), \tag{72}$$

where $W(\omega)$ is the Fourier transform of the $\underline{W}$ tap weights, $S_n(\omega)$ is the sampled noise spectrum, and $\tilde{H}(\omega)$ is the Fourier transform of the channel-sampled impulse response. Again, $W(\omega)$ takes the form of a filter matched to the channel with additive band-limited noise.

The constant β can be written as

$$\beta = \frac{T}{4\pi}\int_{\frac{-2\pi}{T}}^{\frac{2\pi}{T}} \tilde{H}^*(-\omega-\omega_0)W(\omega)d\omega \tag{73}$$

On substituting eq. (72) into eq. (73), we can solve for β as

$$\beta = \frac{\xi}{1+\xi},$$

where

$$\xi = \frac{T}{4\pi}\int_{\frac{-2\pi}{T}}^{\frac{2\pi}{T}} \frac{|\tilde{H}(-\omega-\omega_0)|^2}{S_n(-\omega)}d\omega. \tag{74}$$

Consequently, from eq. (72),

$$W(\omega) = \frac{\tilde{H}(-\omega-\omega_0)}{(1+\xi)S_n(-\omega)}. \tag{75}$$

Even though the $\underline{C}$ and $\underline{W}$ filters are jointly adapting, the $\underline{C}$ taps are, in fact, slaved to the $\underline{W}$ taps. On substituting eq. (68) into eq. (60), we obtain, using eq. (55),

$$\begin{aligned} C_m &= \frac{1}{(E_h+\sigma^2)}\sum_k h(-kT/2)h^*[(2m-k)T/2] \\ &= \frac{1}{(E_h+\sigma^2)}R_h(2m), \qquad m \neq 0. \end{aligned} \tag{76}$$

Thus, the canceller-impulse response (for $m \neq 0$) is that of the overall T-spaced impulse response of the channel and matching filter; that is, the canceller recreates the entire ISI component present in the matching filter output signal as long as the correct data symbols are applied to the canceller.

The C taps corresponding to eq. (75) can similarly be obtained. From eq. (71), we have

$$h[(2m-k)T/2] = \frac{1}{(1-\beta)} \exp[j\omega_0(k-2m)T/2] \sum_l W_l R_n(l-k+2m). \quad (77)$$

On substituting into eq. (60), we have

$$C_m = \frac{\exp(j\omega_0 mT)}{(1-\beta)} \sum_{k,l} W_k W_l^* R_n^*(2m+l-k), \qquad m \neq 0. \quad (78)$$

If, in addition, we define

$$C_0 = \frac{1}{(1-\beta)} \sum_{k,l} W_k W_l^* R_n^*(l-k),$$

we can transform the sequence $\{C_m\}$ to obtain

$$\begin{aligned} C(\omega) &= \sum_m C_m \exp(-j\omega mT) \\ &= \frac{1}{(1-\beta)} \sum_k W_k \exp[-j(\omega-\omega_0)kT/2] \\ &\quad \times \sum_l W_l^* \exp[j(\omega-\omega_0)lT/2] \\ &\quad \times \sum_m R_n^*(2m+l-k) \\ &\quad \times \exp[-j(\omega-\omega_0)(2m+l-k)T/2] \end{aligned} \quad (79)$$

Since both indices k and l run from $-\infty$ to ∞, we can replace the summation over m as

$$\begin{aligned} \sum_p R_n^*(p) \exp[-j(\omega-\omega_0)pT/2] &= S_n^*(-\omega+\omega_0) \\ &= S_n(-\omega+\omega_0). \end{aligned}$$

Therefore, eq. (79) can be simplified to

$$C(\omega) = \frac{1}{(1-\beta)} |W(\omega-\omega_0)|^2 S_n(-\omega+\omega_0). \quad (80)$$

Substituting for $W(\omega)$ from eq. (72) gives

$$C(\omega) = \frac{(1-\beta)|\tilde{H}(-\omega)|^2}{S_n(-\omega+\omega_0)}, \quad (81)$$

or, from eq. (74),

$$C(\omega) = \frac{1}{1+\xi} \frac{|\tilde{H}(-\omega)|^2}{S_n(-\omega+\omega_0)}. \quad (82)$$

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