THE BELL SYSTEM TECHNICAL JOURNAL
Vol. 61, No. 2, February 1982
Printed in U.S.A.

The Continuing Evolution of the Military Standard 105D Sampling System

By B. S. LIEBESMAN

(Manuscript received June 18, 1981)

Military Standard 105D is the most widely used set of acceptance sampling plans in the world. This paper reviews the early development of the standard and points out the many contributions made by Bell System researchers, such as H. F. Dodge. The paper also reviews recent analyses and indicates areas where the special structure suggested by Dodge, and adopted in the standard, has been extremely valuable. Finally, the paper identifies many questions related to the standard that are still open for investigation.

I. INTRODUCTION

1.1 Genesis of military standard 105D

Lot-by-lot acceptance sampling began just prior to World War II and was given a large boost during the war because of the need to assure the quality of wartime material. Bell Laboratories personnel were heavily involved in the early development of sampling plans. The most prolific Bell Laboratories contributors were G. D. Edwards, H. F. Dodge, and H. G. Romig.

The initial system of acceptance sampling plans was developed to assure wartime material. This system evolved through a number of changes to the current system of plans, Military Standard 105D. This standard is described in Ref. 1. H. F. Dodge was one of the leading contributors to the final development of this system. W. R. Pabst, long-time editor of the Standards Section of the *Journal of Quality Technology*, discussed Dodge's contribution in a paper presented before the 17th annual convention of the American Society for Quality Control (ASQC):[2]

> "Much of the theoretical work underlying the new MIL-STD-105D is directly attributed to David Hill and indirectly to Harold Dodge."

Relatively few changes have occurred since 1963 when the MIL-STD-105D* system was published. What has happened instead has been an in-depth investigation of the properties of the system, with the result that changes have occurred in the way the system is used. Bell Laboratories Quality Assurance Center has been active in this investigation. The Center's effort has been in support of the Western Electric Company Purchased Product Inspection organization, which uses MIL-STD-105D almost exclusively to inspect products purchased by the Bell System.

Today, MIL-STD-105D is the most widely used system of acceptance sampling plans in the world as shown in a 1970 Japanese study.[3] It forms the basis for the American National Standards Institute system, ANSI Z1.4; the Japanese system, JIS 29015; the International Standards Organization system ISO 2859; and the British System DEF 131. Saniga and Shirland[4] estimated in 1977 that 76 percent of the quality control organizations in the United States use the system. The Japanese have made extensive use of MIL-STD-105D, a factor which may have contributed to the improved quality of Japanese products since World War II. Finally, it should be noted that the use of the system has spread to many types of items other than manufactured goods or raw materials. These include data records, maintenance operations, financial records, and administrative procedures.

II. DESCRIPTION OF MIL-STD-105D

2.1 Basic definitions

A number of terms are introduced in this section. These terms are important to the description of the system in Section 2.2 and of the issues, past and present, which are discussed in Sections III and IV. Definitions have been included in the glossary for handy reference.

First of all, we define a lot as a set of items under control of the inspection organization for which an acceptance or rejection must be made. The items forming the lot must be similar in nature. A random sample is a subset of the lot which is selected in a manner which makes it representative of the lot. In a truly random sample, each unit in the lot has the same probability of being included in the sample.

Inspection by attributes classifies individual samples as either "non-defective" or "defective" (good or bad, go or no go, etc.), depending upon whether they pass or fail certain tests of characteristics. The quality of a product is measured in terms of the percent defective or defects per hundred units. Inspection by attributes provides an estimate of the quality which is used for lot acceptance or rejection.

---

* Military Standard 105D is commonly abbreviated using MIL-STD-105D or just 105D. See the glossary for this and other abbreviations and definitions.

The operating characteristics (OC) is a curve of the probability of acceptance as a function of quality for a given sampling plan. Figure 1 is a typical OC curve. Note that there are two parameters marked on the curve, a good quality, AQL, and a poor quality, LTPD. Lots of quality equal to the AQL value have a high probability of acceptance, while lots of quality equal to the LTPD value have low probability of acceptance.

The acceptable quality level (AQL) is the index for the plans of MIL-STD-105D. It is defined in Ref. 1 as "the maximum percent defective (or the maximum number of defects per hundred units) that, for the purposes of sampling inspection, can be considered satisfactory as a process average." It is the AQL value at which the producer generally aims the quality of his process. If he produces at this level, he has a high probability that most of the lots will be accepted.

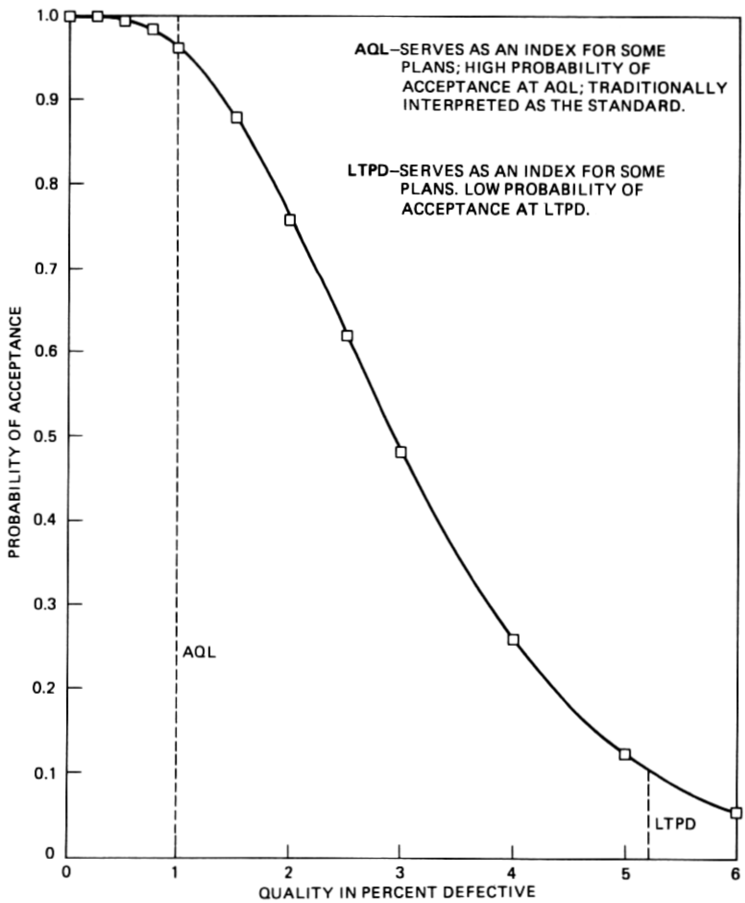

Fig. 1—Operating characteristic curve.

The lot tolerance percent defective (LTPD) is a quality level at which most lots will be rejected. Because of sampling error, some lots at this quality will be accepted.

The average outgoing quality (AOQ) is the average quality of all lots that are shipped. The average outgoing quality limit (AOQL) is the upper bound on AOQ when using a sampling plan that requires all units in all rejected lots be inspected and all defectives removed before shipment. The LTPD and AOQL values are not used directly in MIL-STD-105D. However, it is worth noting that there are plans based on these quantities as indices. The most commonly used set of LTPD and AOQL plans was developed by H. F. Dodge and H. G. Romig of Bell Laboratories.[5]

The final definitions introduced in this section are based on the fact that MIL-STD-105D is a system of plans with a feedback mechanism. This mechanism consists of four phases of operation and the switching rules for transferring between phases. The normal phase is used when there is no evidence that the quality being submitted is poorer or better than the specified quality level. The tightened phase is used when there is evidence of poorer quality, while the reduced phase is used when there is evidence of better quality. The discontinue phase is entered when the producer has not been successful in improving the poor quality of his product during the tightened phase.

2.2 Procedures of MIL-STD-105D

An outline of the basic MIL-STD-105D procedures is given here to further introduce important terminology and to give the reader an understanding of acceptance sampling plans. For more details, refer to the military standard,[1] and its accompanying handbook,[6] or to a quality control text, such as that by Duncan[7] or Grant and Leavenworth.[8]

There are five steps used in the selection of a sampling plan. First of all, the lot to be inspected must be formed and the lot size determined. The lot should be as homogeneous as possible. Next, the sample size code letter must be found in a table as a function of the lot size. The third step is to determine the AQL value to use based on the quality requirements of the product. The fourth step is to select the sampling plan from another table based on the sample size code letter and the AQL value. Finally, a random sample is selected, tested, and the lot accepted or rejected based on the allowable number of defectives for the plan.

These procedures are used to select an individual sampling plan for inspection of an individual lot. In reality, the system of plans operates over a sequence of lots using the phases and switching rules to provide consumer protection and control over the economy of inspection.

Figure 2 illustrates the basic switching rules. Generally, the inspection of a product starts in the normal phase (N), where quality is assumed to be at the desired level. When two out of five consecutive lots are rejected, this is taken to be evidence of poor quality. The next lot is inspected under the tightened phase (T). This results in using a plan that will reject a higher percentage of lots at a given quality than the corresponding plan under the normal phase.

The acceptance of five lots in a row during the tightened phase is evidence that the quality is back to the desired level. When this occurs, the next lots are inspected under the normal phase. However, if this does not occur before ten lots have been inspected under the tightened phase, sampling ceases and the discontinue phase (D) is entered.

When a product has been inspected under the normal phase for a number of consecutive lots, it is eligible for reduced inspection. The reduced phase (R) is entered after ten lots in a row are accepted and a defect limit number criterion is met. The product remains in the reduced phase until either a lot is rejected, or is accepted but the number of defectives exceeds a specified number (*Ac*).

Thus, we see that MIL-STD-105D is a sampling scheme with a feedback mechanism to reward good quality and improve poor quality. The manner in which this mechanism operates is not mathematically precise, but relies on the nature of human reaction to reward and punishment. This was recognized by the developers:

> "Whereas sampling plans are mathematically precise, the study of sampling schemes is not limited to pure mathematical considerations. In fact, many of the decisions related to the development of a sampling scheme are more a matter of art, opinion, esthetics, appeal, practical considerations and compromise."[9]

Often these words are lost on the practitioners who concentrate on the individual sampling plans rather than on the system. The author recently attended the 1981 ASQC Quality Congress where he had many discussions with users of MIL-STD-105D who were ignoring important aspects of the system. In essence, MIL-STD-105D was developed as a

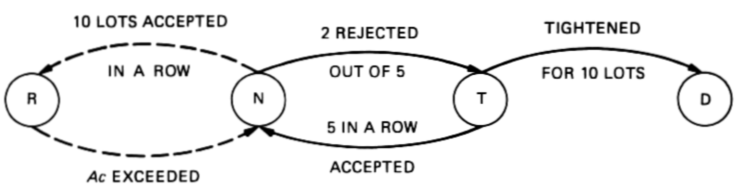

Fig. 2—Dynamics of the switching rules.

system of plans and any abrogation of its features destroys its effectiveness.

III. DEVELOPMENT OF MIL-STD-105D

We will now discuss the historical development of MIL-STD-105D. It is important to review this development because the current structure of the standard is a result of compromises that arose over issues relating to the standard. In fact, MIL-STD-105D was created because of the need to provide industry with a system of sampled plans indexed on AQL values.

3.1 Need for an AQL system

Two other types of sampling plans were developed just prior to MIL-STD-105D and served as competitors of 105D through much of its early history. These are plans indexed by the lot tolerance percent defective (LTPD) and the average outgoing quality limit (AOQL).[5]

I. D. Hill discusses the reasons why industry became disenchanted with LTPD and AOQL plans and pressured instead for AQL plans of the 105D type. He states that:

> "In normal situations, the process average corresponds to a high point on the OC curve; it has to, if the producer is going to make a profit. So it is really a high point rather than a low point which is the primary concern of both the producer and consumer."[10]

In addition, Hill states that an LTPD plan is not cost efficient:

> "This system [LTPD] leads then in general, to the producer making, and the consumer receiving, a quality considerably better than is really necessary, and the price must reflect this."[10]

To compare MIL-STD-105D to an AOQL system, we must first expand on the definitions of average outgoing quality (AOQ) and average outgoing quality limit (AOQL) given in Section 2.1. The AOQ is just the average quality of all lots that are shipped. In an AOQL system, a rejected lot must be inspected 100 percent and all defectives removed. Thus, a rejected lot is made perfect and shipped in sequence with other accepted lots. It can be shown[7,8] that the AOQ has an upper bound called the AOQL. Figure 3 illustrated this. The AOQL value is generally used as the "guaranteed" quality to be provided by a supplier.

The use of AOQL plans decreased when it was found that 100 percent inspection did not guarantee perfection. Hill states that:

> "Now it is well known in practice that 100 percent sorting is

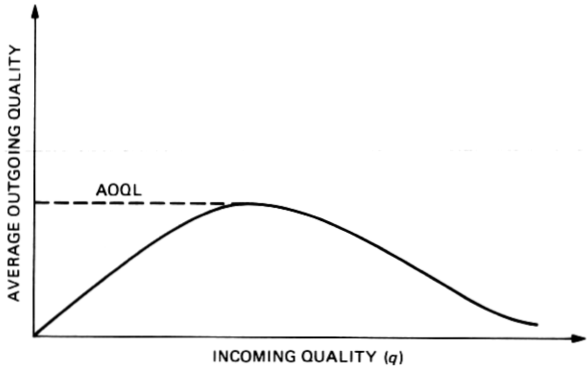

Fig. 3—Average outgoing quality limit (AOQL).

unlikely to be properly done ... In stressing, therefore, that the AOQL concept requires perfection in the inspection operation, I am not claiming that other methods do not require this. It is merely that the lack of such perfection seems to matter more in the case of AOQL."[10]

He then proceeds to show that because of inspection error, the AOQ curve tends to have the shape of the dashed curve in Fig. 4 rather than the solid curve shown in Fig. 3. Note that the maximum value of the AOQ curve in Fig. 4 depends on *r*, the probability that an inspector will call a bad unit good. The inspection error has an effect on the portion of the curve to the left of the dashed curve, but this effect is so small that it cannot be shown in Fig. 4.

The end result of an AOQL plan, according to Hill, is:

"This Table (Table 1 in Ref. 10) shows that the use of an AOQL

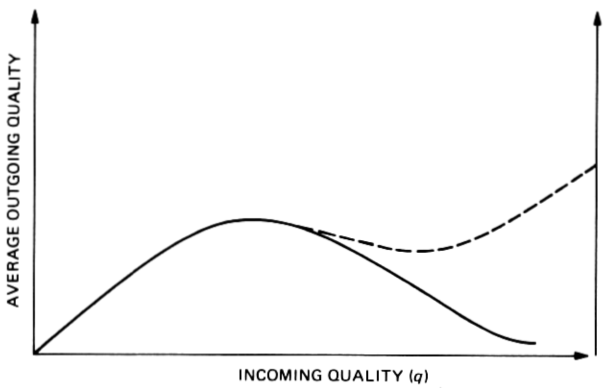

Fig. 4—Effect of inspection error on outgoing quality under an AOQL plan.

plan will, in general, force the producer to offer a quality a good deal better than the AOQL value, although not to such an extent as in the LTPD approach."[10]

And, to summarize the reason for taking the AQL approach over the LTPD and AOQL approaches, Hill states:

"Both the LTPD and AOQL concepts are capable of giving good protection against poor quality, provided they are used efficiently, but both do so at the cost of rejecting a good deal of satisfactory production."[10]

Thus, AQL sampling plans were developed to be indexed on quality values having a high probability of acceptance. This provided the producer with good protection against rejection of satisfactory lots. Protection for the consumer was provided by adoption of switching rules which recognize poor quality over a series of lots and take actions which put pressure on the supplier to improve.

3.2 Important issues during the development of MIL-STD-105D

A number of issues surfaced during the period leading to the 1963 publication of MIL-STD-105D. Most dealt with the practical application of the theoretical framework of the sampling system. Dodge was a prolific contributor during this period because he had close ties to both the theoretical academic world and the practical world of Bell Laboratories and Western Electric.

A change to the interpretation of the AQL is a prime example of Dodge's influence. The early tables were designed to have the probability of acceptance equal to 0.95 for quality equal to the AQL value. Dodge proposed a special structure which resulted in the probability of acceptance for quality equal to the AQL value varying between 0.88 and 0.99. This structure led to plans which were much easier to use by the general quality practitioner, because it consisted of a fixed set of sample sizes and a fixed set of AQL values for the entire range of lot sizes.

Dodge suggested that the values of AQL and sample size both follow the same geometric progression based on multiples of $\sqrt[5]{10} = 1.585$.[11] This resulted in the sequence of AQL values currently used: $\cdots$, 1, 1.5, 2.5, 4, 6.5, $\cdots$; and the sequence of sample sizes (n) currently used: 2, 3, 5, 8, 13, 20, $\cdots$. Furthermore, the structure in the table of sampling plans was enhanced, because along any diagonal, the product of AQL and n is essentially constant and the acceptance number is a constant. (See Fig. 5.) Recently, this structure has been quite useful in a number of analyses which will be described in Section IV.

SINGLE SAMPLING PLANS FOR NORMAL INSPECTION

SAMPLE SIZE	ACCEPTABLE QUALITY LEVELS (NORMAL INSPECTION)																	
	0.25		0.40		0.65		1.0		1.5		2.5		4.0		6.5		10	
	Ac	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*	*Ac*	*Re*
2	⇩		⇩		⇩		⇩		⇩		⇩		⇩		0	1	⇩	
3	⇩		⇩		⇩		⇩		⇩		⇩		0	1	⇧		⇩	
5	⇩		⇩		⇩		⇩		⇩		0	1	⇧		⇩		1	2
8	⇩		⇩		⇩		⇩		0	1	⇧		⇩		1	2	2	3
13	⇩		⇩		⇩		0	1	⇧		⇩		1	2	2	3	3	4
20	⇩		⇩		0	1	⇧		⇩		1	2	2	3	3	4	5	6
32	⇩		0	1	⇧		⇩		1	2	2	3	3	4	5	6	7	8
50	0	1	⇧		⇩		1	2	2	3	3	4	5	6	7	8	10	11
80	⇧		⇩		1	2	2	3	3	4	5	6	7	8	10	11	14	15
125	⇩		1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22
200	1	2	2	3	3	4	5	6	7	8	10	11	14	15	21	22	⇧	
315	2	3	3	4	5	6	7	8	10	11	14	15	21	22	⇧		⇧	
500	3	4	5	6	7	8	10	11	14	15	21	22	⇧		⇧		⇧	
800	5	6	7	8	10	11	14	15	21	22	⇧		⇧		⇧		⇧	
1250	7	8	10	11	14	15	21	22	⇧		⇧		⇧		⇧		⇧	
2000	10	11	14	15	21	22	⇧		⇧		⇧		⇧		⇧		⇧	

⇩ USE FIRST SAMPLING PLAN BELOW ARROW

⇧ USE FIRST SAMPLING PLAN ABOVE ARROW

Ac ACCEPTANCE NUMBER

Re REJECTION NUMBER

Fig. 5—Part of a table of sampling plans from MIL-STD-105D.

The structure devised by Dodge had very strong practical implications. The fixed set of AQL values and sample sizes were easy to understand, interpret, and use by the general quality practitioner.

Dodge also provided inputs concerning tightened inspection and the switching rules. He suggested[11] that a switch to tightened inspection be done when two out of five consecutive lots fail. This was a change from the process average criteria used in early versions of MIL-STD-105D and was based on Dodge's experience with users.[12] For tightened plans, he suggested the use of the same sample size as for normal plans (adopted), so that inspection costs would not increase, and the use of an acceptance number one less than the normal acceptance number (partially adopted). The tightened plans adopted as part of 105D do have AOQL values that are close to AQL values of the corresponding normal plans, as Dodge had suggested.[11]

There were also questions concerning the sample size during the reduced phase, the sample sizes during double and multiple sequential sampling, and the relationships between lot size and sample size. Compromises were reached which resulted in the following:

(*i*) The reduced sample size was set at 40 percent of the normal sample size:

(*ii*) Sequential samples were set to be the same size as the first sample;

(*iii*) Empirical relationships for relating lot size to sample size were used; and

(*iv*) Multiple sampling levels were added to give the user a variety of sampling options.

Finally, there was controversy over analytical tools published to help the practitioner analyze his plans. The major tool that eventually was added to MIL-STD-105D was a set of charts and tables defining the OC curve for each plan. Discussions arose over the distributions to be used to calculate the probabilities, and a compromise was made:[13]

(*i*) The Poisson distribution would be used for AQL > 10;

(*ii*) The Poisson approximation to the binomial distribution would be used for AQL ≤ 10 and sample size ≥ 50; and

(*iii*) The binomial distribution would be used for all other plans.

Some analysts felt that different measures of plan capability would be more useful. For one, Professor Barnard of the Royal Statistical Society, in his comments on Hill's paper,[10] suggested the use of the average run length (ARL) needed to detect shifts in quality as a better measure of plan capability.

The current version of MIL-STD-105D published in 1963 includes other analytical tools. These are tables to determine (*i*) the AOQL and LTPD values corresponding to each plan, and (*ii*) charts showing the average sample size.

We are now ready to examine the most recent period in the life of MIL-STD-105D. During this period, only minor changes have been made to the procedures. This stability has allowed researchers time to observe the use of the standard and to analyze the impact of the total system. Prior to this time, the research had concentrated on the properties of individual plans.

An interest developed to provide tools to aid users in selecting sets of plans from 105D and to analyze the effectiveness of the combination of these plans under specified conditions. Some of this work will be discussed in the next four sections.

IV. ANALYSIS OF MIL-STD-105D

4.1 System of OC curves

The development of OC curves for the system of plans represented a breakthrough in the use of MIL-STD-105D because it recognized the effect of the total system and not just individual plans. This section describes the evolution of these curves, which took approximately thirteen years, and illustrates the difficulty of changing aspects of widely used procedures such as MIL-STD-105D.

The MIL-STD-105D was developed under the assumption that the normal, tightened, and discontinue phases would be strictly adhered to.[1] Use of the reduced phase is considered optional. The need to follow all procedures was recognized by Dodge,[14] Hald and Thyregod,[15] and Stephens and Larson.[16]

Stephens and Larson of Western Electric Company were the first to explore the system OC concept. They constructed a Markov model of MIL-STD-105D considering two cases: (*i*) tightened, normal, and reduced phases combined, and (*ii*) tightened and normal phases combined. The discontinue phase was not included in either case.

The end result of their model was a composite operating characteristic curve for the system:

$$P_{ac}(q) = r_T P_{aT}(q) + r_N P_{aN}(q) + r_R P_{aR}(q), \quad (1)$$

where

P_{ac} = system OC curve
r_T, r_N, r_R = expected proportion of lots inspected during tightened, normal, and reduced inspection
P_{aT}, P_{aN}, P_{aR} = tightened, normal, and reduced OC curves
q = quality.

In addition, they used the Markov model to find the expected number of units sampled per lot or the average sample number (ASN):

$$\text{ASN} = r_T n_T + r_N n_N + r_R n_R, \quad (2)$$

where

n_T, n_N and n_R = sample sizes under tightened, normal, and reduced inspection.

More recently, Schilling and Sheesley[17,18] used the Markov Model of Stephens and Larson to develop tables of system values for the AOQL, the limiting quality, the operating characteristics curve, the average sample number, the average outgoing quality, and the average total inspection. They have suggested incorporation of these curves in future revisions of MIL-STD-105D.

Schilling and Sheesley[17] made a number of observations. Three of these will now be discussed. First of all they state that:

"Unfortunately, the standard [MIL-STD-105D] is frequently misused, particularly in nonmilitary applications, through the selection and use of normal plans only—disregarding the tightened and reduced plans and the switching rules."[17]

Perhaps the major objective of their papers was to show the value of using all of the rules, which results in "enhanced protection for both the producer and consumer."[17]

They also note that the limiting quality, which is synonymous with the LTPD value, "is for use with isolated lots and does not reflect the limiting quality afforded by the MIL-STD-105D sampling system."[17]

Finally, they state that "reduced inspection provides an obvious reward to the producer of a good quality product in terms of lower sample size and slightly higher probability of acceptance."[17] But, they indicate that part of the switching procedures (the use of reduced limit numbers) have a minimal effect on the system OC curves.

Although the work cited in this section represented a breakthrough in the use of MIL-STD-105D, Hald and Thyregod,[15] Stephens and Larson,[16] and others recognized a shortcoming in their own approaches. The main criticism is that they considered a static situation with a fixed level of quality as input to the inspection system. As noted by W. R. Pabst in the discussion of Ref. 15:

"What would also be interesting, and perhaps more difficult to explore is the dynamic effect of these switching rules on the production process."[15]

Stephens and Larson agreed and stated that:

"Hence, the actual behavior of the process under the influence of the sampling procedure may thus be very dynamic."[16]

These comments influenced some of the work described in the next section and led the author to investigate the dynamic effects of the switching rules.

4.2 Controlling average outgoing quality

The average outgoing quality (AOQ) was chosen as the parameter of interest in the analysis of the dynamic effects of the switching rules. This is because it is a measure of the quality of the product supplied to the customer. Hence, AOQ is the "bottom line" for any sampling plan. The investigation in Ref. 19 centered on measurement of the effects of switching rule feedback to the supplier and the resultant quality provided to the customer. It was reasoned that the switching rules were included in 105D to cause the supplier to take actions resulting in improved quality when the tightened phase is entered:

"When the quality of a product degrades, the tightened inspection and discontinue features of MIL-STD-105D can be used to motivate the producer to improve."[19]

This is because fewer lots will be accepted under tightened inspection if quality does not improve. This may be observed in Fig. 6, which is a typical comparison of OC curves for a normal and tightened pair of plans. The difference between the probabilities of acceptance is shown in the curve at the bottom. For example, when quality equals 2.5 percent, this difference is about 25 percent. Hence, if quality remains at 2.5 percent defective during the tightened phase, about 25 percent more lots will be rejected than during the normal phase. This should act as a strong stimulus to the supplier to improve his quality. In addition, the discontinue phase and its consequences will be the end results of no improvement during the tightened phase.

The approach described in Ref. 19 was based on three assumptions which differed from those used in Refs. 16 to 18. First, the discontinue phase was included in the analysis because the threat of discontinue (when enforced) strengthens the effect of the tightened phase on the supplier. The premise was that the switching rules were meant to provide feedback to the supplier causing him to improve his quality. Secondly, it was assumed that a reasonable supplier does respond and that the average outgoing quality will be based on the level of improvement. Thus, two levels of quality were considered: (*i*) q_N, the quality during the normal phase, and (*ii*) q_T, the quality during the tightened phase. This is in marked contrast to the assumption of a single quality level in Refs. 16 to 18. Finally, only accepted lots were included in the analysis, whereas the previous work had been based on all inspected lots. Note that only accepted lots affect the quality received by the customer.

Under these assumptions, the equation for the average outgoing quality (AOQ) for the system of plans is

$$\text{AOQ} = \frac{q_N E_{AN}}{E_{AN} + E_{AT}} + \frac{q_T E_{AT}}{E_{AN} + E_{AT}}, \tag{3}$$

where

q_N, q_T = quality during normal, tightened phase

E_{AN}, E_{AT} = expected number of lots accepted during normal, and tightened phases.

If the level of response (q_T) is fixed and q_N is varied, the result is a curve that is very similar to the AOQL curve (Fig. 3). Then, if different levels of response are analyzed, we obtain a set of curves similar to those shown in Fig. 7. This figure gives an example of the AOQ for the system of 105D plans. The curves are for an AQL of 0.65 percent, a

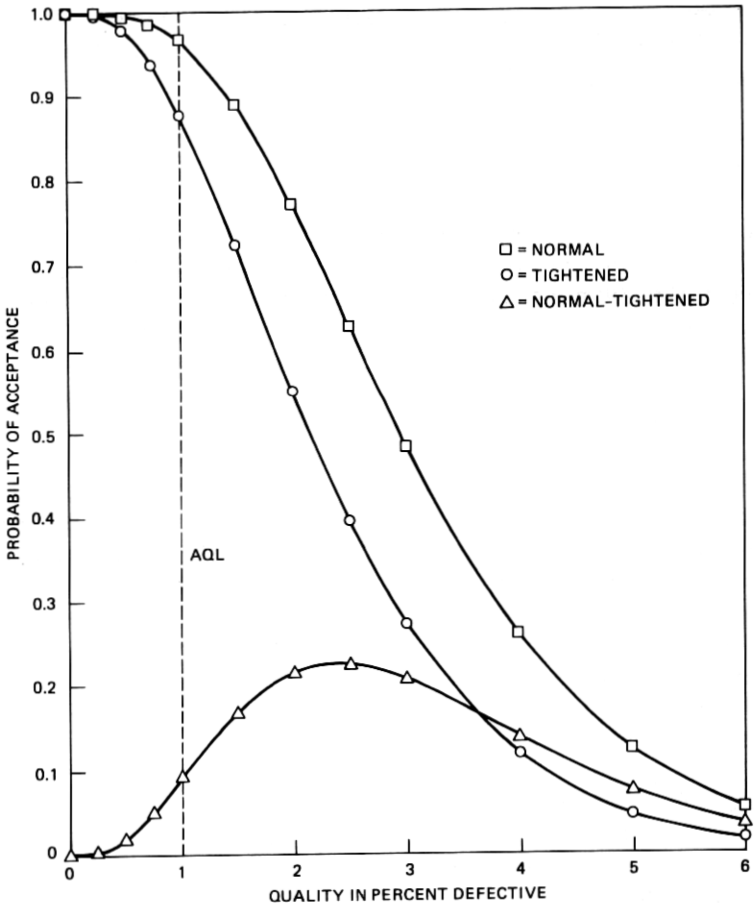

Fig. 6—Effects of tightened inspection.

sample size of 125, and an *Ac* of 2. There is one curve for each set of values of quality during the tightened phase (q_T). The maximum value on each curve is called the maximum average outgoing quality (AOQM) value.

Figure 8 is a sample plot of AOQM as a function of the level of response (q_T). The AOQM value is analogous to the AOQL value and if q_T can be estimated, AOQM represents a limit on the quality provided to the customer. The result of this analysis is a demonstration that "MIL-STD-105D has feedback properties which tend to limit the worst average outgoing quality without relying on the mechanism of screening rejected lots."[19]

4.3 *Selection of specific plans from MIL-STD-105D*

One of the main results of the AOQM analysis was a more comprehensive look at the properties of the 105D sampling plans. An under-

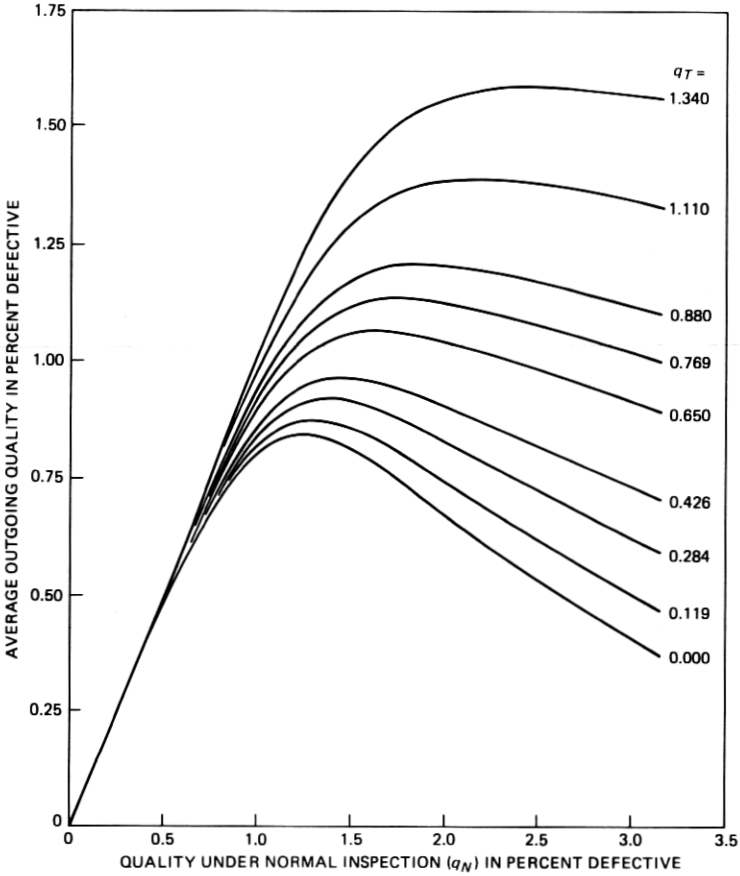

Fig. 7—Example of average outgoing quality for the system of MIL-STD-105D plans.

standing of these properties generated a desire to use them in the selection of specific plans from the 105D system.

The main result in Ref. 19 was a measure of the average outgoing quality based on the responsiveness of the supplier. Other measures of the plan capability were based on the ability of the 105D system to detect changes in quality. These were developed from the standpoint of control engineering. Professor Barnard's comments on Hill's paper point out that some of the early developers recognized the need for this type of analysis:

> "One particular thing which would have come out of this [view the scheme in terms of control engineering] would have been a description of the scheme, not in terms of the OC curve ··· but in terms of the average amount of production passed after quality has deteriorated before the deterioration is picked up by the inspection."[10]

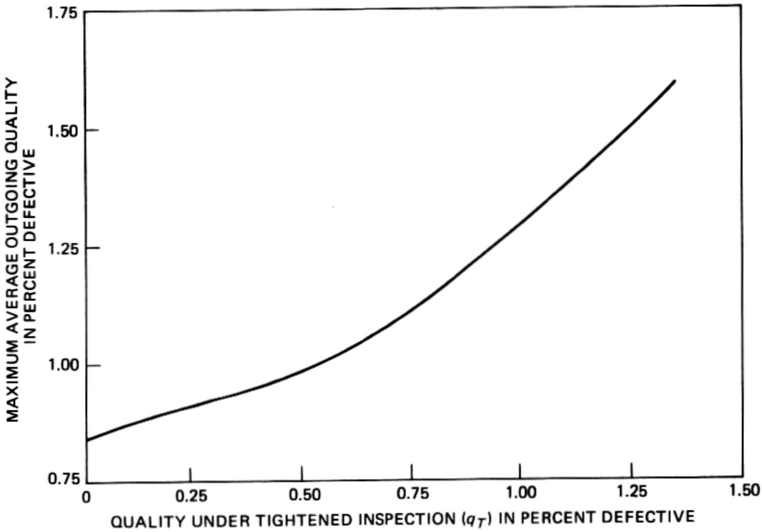

Fig. 8—Maximum average outgoing quality (AOQM).

The results in Ref. 19 led to the development of a preliminary set of criteria for evaluating MIL-STD-105D plans. Three quantities were considered in Ref. 20. First, the expected number of lots accepted during the normal phase prior to switching to tightened phase (E_{AN}) estimates both the speed of detecting a change to poor quality and the false alarm jeopardy when quality remains good. Secondly, the expected number of lots accepted during the reduced phase prior to switching back to the normal phase (E_{RN}) estimates the same quantities under reduced inspection. Both E_{AN} and E_{RN} are functions of quality during their respective phases. Finally, AOQM estimates the limiting quality leaving the inspection system. The value of AOQM chosen for this estimate assumes that the supplier will respond to a tightened quality level, q_T^*, which will have only a small chance of causing a switch to the discontinue phase.

Table I is a summary of the decision parameters developed to select sets of 105D plans. The value q_G in the table represents a good quality level while q_B represents a poor quality level. The value of q_B is typically three times that of q_G.

The computation of the parameters in Table I for the plans of MIL-STD-105D is a large task. However, the structure proposed by Dodge[11] and adopted in MIL-STD-105D (see Section 3.2) simplified this problem a great deal.

A set of normalized tables and curves were presented in Ref. 20 that reduces the required information by a factor of 14. When quality is normalized by the AQL value, a single curve is needed for each acceptance number rather than for each sampling plan. For example, the 152

Table I—Decision parameters for selection of 105D plans

Parameter	Phase	Purpose
1. $E_{AN}(q_G)$	Normal	Estimate false alarm rate during the normal phase.
2. $E_{AN}(q_B)$	Normal	Estimate response of a plan to a shift to bad quality during the normal phase.
3. AOQM(q_T^*)	Tightened	Estimate maximum average outgoing quality for a supplier's change to a safe quality q_T^*.
4. $E_{RN}(q_G)$	Reduced	Estimate false alarm rate during the reduced phase.
5. $E_{RN}(q_B)$	Reduced	Estimate response to a shift to bad quality during the reduced phase.

OC curves under normal inspection can be reduced to the eleven normalized curves in Fig. 9. These curves give the probability of acceptance as a function of the normalized quality. The units of normalized quality are multiples of the AQL value. Other curves and tables are provided in Ref. 20 which may be used to facilitate analysis of 105D sampling plans and help in the selection of appropriate AQL values.

4.4 Other results based on the structure of MIL-STD-105D

Analysts in the Bell Laboratories Quality Assurance Center recently investigated three questions which arose during the normal use of MIL-STD-105D by Western Electric inspectors. The first of these was concerned with the distribution of proportion defective in outgoing lots, assuming a distribution of quality in the incoming lots. Brush et al.* assume a beta distribution for incoming lot quality, and using the set of sampling plans from 105D, they find the mean, variance, and equivalent 0.90 beta quantile for the outgoing distribution.[21] Rejected lots are assumed to be scrapped. The special structure of 105D leads to a small set of normalized curves for the three outputs. These results provide the analyst with a powerful tool for determining the effect of a single sampling plan on the outgoing quality.

The second question was also resolved with the aid of the special structure of 105D. This is the multiple group situation.[22] Questions arose over the use of 105D when the set of inspection characteristics is divided into groups each with its own sampling plan. The MIL-STD-105D encourages this situation.[1,6] A conflict may arise because the supplier views quality in terms of each individual group, whereas the customer views quality in terms of collections of groups called categories. The ratio, k, of category AQL to the individual group AQL was determined in Ref. 22, assuming each group uses the same sampling plan. The special structure of 105D led to the development of tables of k as a function of the acceptance number and the number of groups in the category.

The third problem area was that of developing limiting quality or LTPD plans which are compatible with 105D plans. Again, the special

* Members of Bell Laboratories Quality Assurance Center.

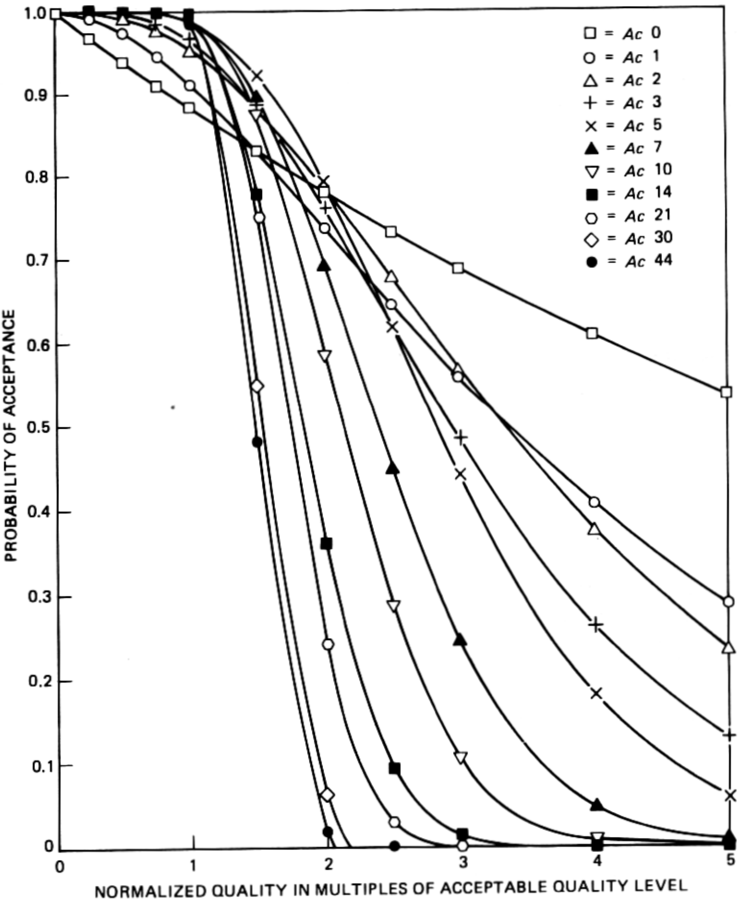

Fig. 9—Normalized OC curves.

structure of 105D aided in the resolution of this problem. The plans developed by Duncan, Mundel, Godfrey,* and Partridge* use the same lot size-sample size relationship as 105D.[23] In addition, the sequence of limiting quality levels that indexes the plans and the sample sizes follow the same geometric progression as 105D. The authors have proposed a table of these plans for inclusion in the next revision of ISO 2859.[24]

V. FUTURE RESEARCH

5.1 Open questions

The selection of the "best" sampling plans is certainly an open question. The analysis described in Section 3.3 gives the user a meth-

---

* Members of Bell Laboratories Quality Assurance Center.

odology for selection of an AQL value when costs are unknown. When costs are known, or partially known, the problem becomes one of including the effects of MIL-STD-105D on total cost. The most recent work in this area was given in a paper presented at the 1981 ASQC Quality Congress.[25] In that paper, a cost model and simplified procedures were developed for selecting AQL values. The analysis should be extended to include sample size variation and its effect on total cost.

A second open question is the amount of switching desirable during plan operation. Most of the interest in this question has centered in Japan, where researchers have developed a modification to 105D that includes different switching rules.[3] Little attention has been given to this modification in the United States.

A third open question is a comparison of the procedures of MIL-STD-105D with the procedures used by the Bell Laboratories Quality Assurance Center to audit Western Electric Company's manufactured product. A new reporting system for the audit, QMP, has recently been developed.[26] Future work should compare the cost basis and the feedback properties of the two systems.

Finally, the three results discussed in Section 4.4 should each be extended. First, the study of outgoing distributions should be extended to include the effects of the system of plans. The original study encompassed only individual plans. Secondly, the multiple group situation should be extended to groups using a mixture of sampling plans. And, finally, LQL plans should be incorporated in a system similar to 105D, or these plans should be combined with 105D plans into a complete attribute acceptance system.

Much of the early 105D development work is still open to review because of the nature of the system and its basis in compromise. In a continually changing environment, the 105D system may have to change. Good data are needed to evaluate many of the effects of 105D, while good analysts are needed to understand and extend the features of 105D.

GLOSSARY

Ac	Acceptance number; if this number is exceeded, either a lot is rejected or a switching rule is applied.
AOQ	The average outgoing quality; the average quality of all lots shipped.
AOQL	Upper bound of AOQ when all rejected lots are inspected 100 percent and all defectives are removed.
AOQM	Maximum average outgoing quality.
AQL	Acceptable quality level (good quality).
ARL	Average run length.
ASN	Average sample number.

Attribute	A characteristic of a product which is classified as nondefective or defective, good or bad, yes or no, etc.
Discontinue	The phase entered from the tightened phase when the supplier does not respond and improve his quality.
E_{AN}, E_{AT}, E_{RN}	Expected number of lots accepted during the normal, tightened, and reduced phases.
IQ	Incoming quality.
Lot	A set of items under control of the inspection organization for which an acceptance or rejection decision must be made.
Lot quality (q)	Percent defective or defects per hundred units in the lot.
LTPD	Lot tolerance percent defective (poor quality level). Also called the limit quality level (LQL) or limit quality (LQ).
MIL-STD-105D	Military Standard 105D.
n	Sample Size.
Normal phase	Entered when there is no evidence of better or poorer quality than desired.
Operating characteristic (OC) curve	Probability of acceptance as a function of quality.
OQ	Outgoing quality.
P_{ac}	System OC curve.
P_{aT}, P_{aN}, P_{aR}	Tightened, normal and reduced OC curves.
q_N, q_T	Quality level during normal, tightened phase.
r	The probability that an inspector will call a bad unit good.
r_N, r_T, r_R	The expected portion of all lots inspected during normal, tightened and reduced phases.
Random sample	A truly representative subset of a lot; each unit in the lot has the same probability of being included in the sample.
Reduced phase	Entered when there is evidence of good quality.
Screening	100 percent inspection of a rejected lot.
Sequential sampling	A sequence of samples are used during which the decisions are to accept the lot, reject the lot or take the next sample; the average total sample size is smaller than for comparable single sampling plans.
Switching rules	Rules for switching between phases.

Tightened phase	Entered when there is evidence of poor quality.
105D	Military Standard 105D.

REFERENCES

1. "Sampling Procedures and Tables for Inspection by Attributes," MIL-STD-105D, April, 1963, Department of Defense, Washington, D.C.
2. W. R. Pabst, Jr., "Some Background Consideration of MIL-STD-105D," 17th Annual ASQC Convention and Exhibit, May 20, 1963, Chicago, Ill.
3. T. Koyama et al., "MIL-STD-105D and the Japanese Modified Standard," *J. Quality Tech.*, , No. 2 (April 1970), pp. 99–108.
4. E. M. Saniga and L. E. Shirland, "Quality Control in Practice—A Survey," Quality Progress, *10*, No. 4 (May 1977), pp. 30–3.
5. H. F. Dodge and H. G. Romig, *Sampling Inspection Tables, 2nd Edition*, Wiley, New York, 1959.
6. "Guide for Sampling Inspection," Handbook H-53, June 30, 1965, Office of the Assistant Secretary of Defense (Installations and Logistics), Washington, D.C. 20301.
7. A. J. Duncan, *Quality Control and Industrial Statistics*, 4th ed., Homewood, Illinois: Richard D. Irwin, Inc., 1974.
8. E. L. Grant and R. W. Leavenworth, *Statistical Quality Control*, 4th ed., New York: McGraw Hill Book Co., 1972.
9. O. A. Cocca, "MIL-STD-105D, An International Standard for Attribute Sampling," Ind. Quality Control, *21*, No. 5 (November 1964), pp. 249–53.
10. I. D. Hill, "Sampling Inspection and Defense Specification DEF-131," J. Royal Stat. Soc., Series A, *125*, Part 1, pp. 31–73, 1962.
11. H. F. Dodge, "A General Procedure for Sampling Inspection by Attributes—Based on the AQL Concept," Rutgers Statistics Center Technical Report No. 10, December 1959.
12. H. F. Dodge, "Notes on the Evolution of Acceptance Sampling Plans, Part III," J. Quality Tech. *1*, No. 4 (October, 1969), pp. 225–32.
13. H. F. Dodge, "Notes on the Evolution of Acceptance Sampling Plans, Part II," J. Quality Tech., *1*, No. 3 (July 1969), pp. 155–62.
14. H. F. Dodge, "Evaluation of a Sampling Inspection Having Rules for Switching Between Normal and Tightened Inspection," Technical Report No. 14, Rutgers University, August 1965.
15. A. Hald and P. Thyregod, "The Composite Operating Characteristic Under Normal and Tightened Sampling Inspection by Attributes," 35th Session Int. Stat. Inst., Beograd, 1965.
16. K. S. Stephens and K. E. Larson, "An Evaluation of the MIL-STD-105D System of Sampling Plans," Ind. Quality Control, *23*, No. 7 (January 1967), pp. 310–9.
17. E. G. Schilling and J. H. Sheesley, "The Performance of MIL-STD-105D Under Switching Rules, Part 1: Evaluation," J. Quality Tech., *10*, No. 2 (April 1978), pp. 76–83.
18. E. G. Schilling and J. H. Sheesley, "The Performance of MIL-STD-105D Under Switching Rules, Part 2: Tables," J. Quality Tech., *10*, No. 3 (April 1978), pp. 104–24.
19. B. S. Liebesman, "The Use of MIL-STD-105D to Control Average Outgoing Quality," J. Quality Tech., *11*, No. 1 (January 1979), pp. 36–43.
20. B. S. Liebesman, "The Characterization of MIL-STD-105D Sampling Plans Using Normalized OC Curves," Workshop on Statistical Quality Control, West Berlin, June 11–13, 1980.
21. G. G. Brush, H. Cautin, and B. R. Lewin. "Outgoing Quality Distributions for MIL-STD-105D Sampling Plans," J. Quality Tech., *13*, No. 4 (October 1981), pp. 254–63.
22. B. S. Liebesman, "The Quality Standard for Multiple Group Categories," 1980 ASQC Technical Conf. Trans., pp. 468–74.
23. A. J. Duncan et al., "LQL Indexed Plans that are Compatible with the Structure of MIL-STD-105D," J. Quality Tech., *12*, No. 2 (January 1980), pp. 40–6.
24. International Organization for Standardization, "Draft Collection of Sampling Plans and Tables to Complement ISO 2859," ISO/TC69/SC5, N69, London, September 1973.
25. B. S. Liebesman, "Selection of MIL-STD-105D Plans Based on Costs," 1981 ASQC Quality Congress Trans., San Francisco, California, May 1981, pp. 475–84.
26. A. B. Hoadley, "The Quality Measurement Plan (QMP)," B.S.T.J.., *60*, No. 2 (February 1981), pp. 215–73.