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Frequency-Hopped Single-Sideband Modulation for Mobile Radio

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A frequency-hopped single-sideband (SSB) modulation with pilot tone (FH-SSB-PT) transmission scheme is described for mobile radio. The single-sideband signal changes its carrier frequency every τ seconds, and the sequence of carrier frequencies is controlled by a suitable scrambling code. Co-channel cells using the same frequency band in the mobile radio system use different scrambling codes with the result that co-channel interference is altered from cross-talk to random noise if τ is small. In the presence of Rayleigh fading, frequency hopping improves reception for stationary vehicles, and does not degrade the performance for moving vehicles. Further, the technique of frequency hopping may offer communications privacy. Expressions for s/n are derived for SSB, SSB plus pilot tone, and FH-SSB-PT in the presence of Rayleigh fading. Co-channel interference effects are also considered.

I. INTRODUCTION

Mobile radio telephone systems are required to operate in a hostile environment compared to radio systems using terrestrial and satellite links. The transmission channel is time varying in a manner dependent on the vehicle speed and location, distribution of buildings and terrain, siting of fixed antennas relative to the mobile, the effect of other mobiles, to mention but a few. Paramount of the design objectives is a high user density with good uninterrupted communication at a reasonable cost. To achieve these goals, a number of system concepts have been advocated,[1,2] and a developmental system to provide Advanced Mobile Phone Service (AMPS) is currently being operated by Illinois Bell Telephone Company in the Chicago Area.[3]

Unlike conventional telephony where extra demand on communication capacity can be accommodated by additional links, the mobile radio spectrum is limited. To increase capacity, the area must be divided into cells and the RF band must be reused. Although mobiles in adjacent cells can be arranged not to interfere, co-channel interference caused by frequency reuse must be controlled in these systems. Thus, the mobile radio engineer increases capacity, not by using extra frequency assignments but by re-using the spectrum and taking care to reduce co-channel interference by adequate spacing between cells using the same frequencies.

For urban mobile radio, the hexagonal cell structure giving complete coverage of an area has been extensively studied and is the one currently in vogue. In a cell there is either an omnidirectional antenna at the center of the cell site (start-up mode), or three directional antennas at alternate corners (fully operational mode). Assuming that mobiles in a cell do not interfere with others in the same cell, the main source of signal impairment results from fading of the received signal because of multipath effects present in an urban environment. The fading is statistically described by the Rayleigh distribution,[3] and is not instantaneously correlated across the frequency band of interest—an effect known as selective fading. Some other sources of signal degradation, in addition to co-channel interference, are ignition and receiver noise.

The choice of type of modulation is very important if we are to attain our design objectives. Various proposals have been made, and can be separated into two basic groups, those advocating digital modulation methods, and others arguing for the relatively well-known analog techniques.[4–7] In this paper, we specifically consider the use of single-sideband modulation (SSB) for mobile radio telephony. We do this because SSB does not result in any expansion of the occupied bandwidth, merely in its linear frequency translation. Single-sideband modulation has been exhaustively investigated at HF, and some proposals[8,9] have been made for its introduction in mobile radio. However, no complete work appears to have been done in connection with its use in cellular structures. The AMPS system opted for frequency modulation (FM) rather than SSB, exploiting the ability of FM to capture the wanted signal in the presence of an interfering signal, provided the latter is below a certain threshold level. There is no capture effect inherent in SSB, and for a given annoyance in cross-talk the interfering signal level must be significantly reduced compared to FM. With its nonconstant envelope, SSB is also more vulnerable to fading than FM. However, FM requires significantly more bandwidth than SSB, and it is this fact that has prompted us to have a closer look at SSB to see if we can mitigate its inherent weaknesses.

We describe quantitatively how well SSB performs when subjected to frequency selective fading and co-channel interference in a mobile radio environment. The effect of using pilot carriers to combat fading is considered in detail.

We then propose a new SSB system which we have called frequency-hopped SSB (FH-SSB). We attempt to ensure that a particular channel is never continuously in a fade, and that two co-channel signals (i.e., two signals occupying the same band in a single duration τ) being scrambled by different codes will produce an effect on each other equivalent to that of additive noise. This technique compensates for some deficiencies in SSB, and, being a scrambling method, may offer communication privacy.* The effect of applying space diversity reception to frequency-hopped SSB with pilot tone is then considered. We conclude with a discussion that contains a summary and suggestions for further research.

II. PERFORMANCE OF FREQUENCY-HOPPED MODULATION IN MOBILE RADIO ENVIRONMENT

To mitigate the effects of multipath and co-channel interference in the mobile radio environment, we propose a frequency-hopping system that ensures that adjacent voice signal segments are in noncontiguous RF bands after modulation. The basic scheme of frequency hopping can be applied to any modulation as illustrated in this section. Its operation and performance using SSB are treated in succeeding sections of this paper.

Consider the transmission of a set of N single-channel voice signals over an RF bandwidth of B Hz. The RF bandwidth occupancy B_c of each voice channel is, therefore,

$$B_c = \frac{B}{N}. \tag{1}$$

Let the complex envelope of the kth speech signal $s_k(t)$, be represented by $m_k(t)$, namely,

$$m_k(t) = s_k(t) + j\hat{s}_k(t), \tag{2}$$

where $\hat{s}_k$ is the Hilbert transform of $s_k(t)$,

$$s_k(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{s_k(x)}{t-x}\, dx. \tag{3}$$

The speech signals are band-limited to W Hz and divided into a set of nonoverlapping intervals of τ seconds. Parameter τ is determined by switching transients, vehicular speeds and available technology, and

* It may be argued that to offer communication privacy, τ has to be small compared to typical syllabic durations and scrambling codes should be changed frequently.

may therefore differ considerably depending on whether the environment is urban or rural. The kth speech signal is represented by

$$m_k(t) = m_k(t) \sum_{l=-\infty}^{\infty} \text{rect}\left(\frac{t - l\tau}{\tau}\right)$$
$$= \sum_{l=-\infty}^{\infty} m_k(t)\text{rect}\left(\frac{t - l\tau}{\tau}\right). \tag{4}$$

In eq. (4), rect(·), the rectangular window function, is defined by

$$\text{rect}(x) = \begin{cases} 1 & -\tfrac{1}{2} < x \le \tfrac{1}{2} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Suppose that in any time slot $(l - 1)\tau < t \le l\tau$, a segment of the speech signal $m_k(t)$ is modulated on a carrier to occupy an RF bandwidth of B_c and that this frequency band is directed to a subband p, $1 \le p \le N$. The subband p changes its location in the spectrum every τ seconds according to a scrambling code.[10]

The kth modulated voice channel can, therefore, be represented as

$$y_k(t) = \sum_{l=-\infty}^{\infty} F\{m_k(t),\ f_k(l, \omega)\}\text{rect}\left(\frac{t - l\tau}{\tau}\right), \tag{6}$$

where $F\{\cdot\}$ denotes the modulator output, $f_k(l, \omega)$ represents the carrier frequency ω during the lth time slot chosen for the kth speech channel by the scrambling code.

Because the scrambling code is known at the receiver, we can detect $m_k(t)$, $(l - 1)\tau < t \le l\tau$.

Assuming the modulated RF bandwidth B_c is much smaller than the "coherence bandwidth"[1] of the multipath interference, we denote the kth received RF signal as

$$z_k(t) = \sum_{l=-\infty}^{\infty} r_l(t)\exp[j\phi_l(t)]F\{m_k(t),\ f_k(l, \omega)\}\text{rect}\left(\frac{t - l\tau}{\tau}\right), \tag{7}$$

where $r_l(t)\exp[j\phi_l(t)]$ is the complex envelope of the Rayleigh fading imposed by the transmission medium. This is the only type of multipath interference considered here; we justify this on the grounds that it is usually the major source of degradation in mobile radio systems.[1]

III. FREQUENCY-HOPPED SSB IN MOBILE RADIO ENVIRONMENT

We now consider SSB in a frequency-hopping system. The kth modulated voice channel can be represented as

$$v_k(t) = \sum_{l=-\infty}^{\infty} \left\{ \frac{1}{2} s_k(t)\cos[f_k(l, \omega)t]\text{rect}\left(\frac{t - l\tau}{\tau}\right) - \frac{1}{2} \hat{s}_k(t)\sin[f_k(l, \omega)t]\text{rect}\left(\frac{t - l\tau}{\tau}\right)\right\}, \quad (8)$$

where $s_k(t)$ is the speech signal in the kth channel. Further, the complex envelope of the kth modulated voice channel in the lth time slot is

$$u_k(t) = m_k(t)\exp[jf_k(l, \omega)t]. \quad (9)$$

Note that the carrier frequency changes every τ seconds according to a scrambling code and $f_k(l, \omega)$ represents the carrier frequency for the kth channel during the lth time slot. Since the RF bandwidth occupancy B_c of each voice channel in SSB is γW, where γ is slightly higher than unity, (typically 1.1), the total number N of radio channels that can be accommodated in the RF bandwidth B is

$$N = \frac{B}{\gamma W}. \quad (10)$$

As in Section II, we assume that the scrambling code is such that successive modulated voice signal elements are in noncontiguous RF bands, no two voice channels are in the same band, and all channels are accommodated in a given band B. A typical time-frequency characteristic of the kth voice channel is shown in Fig. 1.

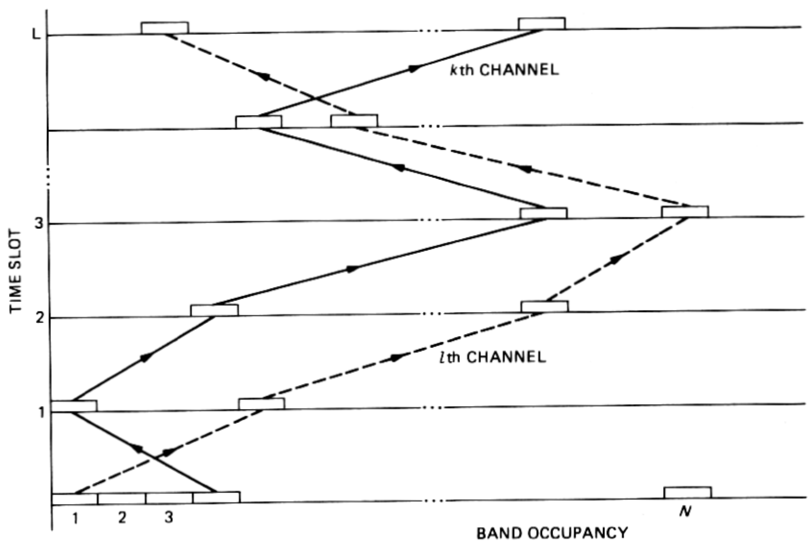

Fig. 1—Frequency hopping of kth and lth channels during successive time slots.

Our intention is to demonstrate that by frequency hopping the speech channels as described, the performance of SSB will not be degraded and its co-channel interference characteristics will be improved. As a prelude, we calculate the s/n of single channel SSB signal, that is *not* accompanied by a pilot tone, and is subject to Rayleigh fading. We show that its performance is too low to be of any practical value. The s/n of SSB with pilot carrier is next examined and a significant improvement observed. We further show that frequency hopping does not degrade the performance of SSB with pilot carrier.

3.1 Effects of Rayleigh fading on a SSB signal

Consider a SSB signal subject to rapid Rayleigh fading of the type found in mobile radio systems. No pilot carrier transmission or frequency hopping is used. Assuming that the modulation bandwidth B_c is much smaller than the coherence bandwidth (typically 250 kHz in an urban environment), the fading of the SSB signal is frequency flat over the signal band, and the envelope of recovered speech signal impaired by fading is

$$\mu(t) = r(t)s(t). \tag{11}$$

As $r(t)$ is Rayleigh distributed, the probability density function (pdf) of r is given by

$$p_r(x) = \begin{cases} \dfrac{x}{\sigma^2} e^{-\frac{x^2}{2\sigma^2}}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases}, \tag{12}$$

where the parameter σ^2 is a function of the transmission medium.

We now proceed to determine the s/n of (11). In Ref. 9, s/n is defined as

$$\text{s/n} = \frac{\langle s^2(t) \rangle_t}{\langle (\mu(t) - s(t))^2 \rangle_{r,t}}, \tag{13}$$

where $\langle \cdot \rangle_r$ represents the average with respect to r and

$$\langle x^2(t) \rangle_t \triangleq \lim_{T \to \infty} \frac{1}{2T} \int_{-T}^{T} x^2(t) dt. \tag{14}$$

However, Ref. 1 uses

$$\text{s/n} = \frac{\langle (\langle \mu(t) \rangle_r)^2 \rangle_t}{\langle (\mu(t) - \langle \mu(t) \rangle_r)^2 \rangle_{r,t}}. \tag{15}$$

The cross-correlation of the noise and signal as defined by eq. (13) is nonzero, although the fading and signal are statistically independent. In (15), the signal is defined as

$$\alpha(t) = \langle \mu(t) \rangle_r \tag{16}$$

and the noise as

$$n(t) = \mu(t) - \langle \mu(t) \rangle_r, \tag{17}$$

yielding the desired result that the signal and noise are uncorrelated. Consequently, we opt for the definition of s/n given by eq. (15).

From (11) and (16) the recovered signal is

$$\alpha(t) = \langle \mu(t) \rangle_r = \langle r(t) \rangle_r s(t) \tag{18}$$

and

$$n(t) = [r(t) - \langle r(t) \rangle] s(t), \tag{19}$$

$$\langle n(t) \rangle_r = 0 \tag{20}$$

and

$$\langle n^2(t) \rangle_r = \langle s^2(t) \rangle \{ \langle r^2(t) \rangle_r - \langle r(t) \rangle_r^2 \}. \tag{21}$$

With the aid of eq. (12),

$$\langle r^2(t) \rangle_r = 2\sigma^2 \tag{22}$$

and

$$\langle r(t) \rangle_r^2 = \frac{\pi}{2} \sigma^2, \tag{23}$$

resulting in a noise power of

$$\langle n^2(t) \rangle_r = \langle s^2(t) \rangle (2 - \pi/2) \sigma^2. \tag{24}$$

The s/n becomes

$$\text{s/n} = 1 \bigg/ \left(\frac{4}{\pi} - 1 \right) = 3.66 \textit{ or } 5.63 \text{ dB}. \tag{25}$$

We note that this s/n of 5.63 dB is too low to be viable in a mobile radio environment, unless we mitigate the effects of fading.

3.2 *Effects of Rayleigh fading on a SSB signal with pilot carrier (SSB + PT)*

The first step to reduce the distortion in the recovered speech, caused by fading of the SSB signal, is to add a pilot tone $p(t)$ of known magnitude and frequency f_p.[1,9] The spectral arrangement is shown in Fig. 2. The frequency f_p is selected to reside outside the bandwidth B_c of the SSB signal, but sufficiently close for it to experience similar fading conditions. The receiver, on examining the magnitude of the pilot tone, can estimate the degree of fading of the SSB signal, make appropriate compensation, and thereby improve s/n.

The received SSB signal is not the desired $m(t)$, but $r(t)m(t)$, where $r(t)$ is Rayleigh-distributed. The pilot tone is used to correct the distorted demodulated signal to a value

$$\hat{\mu}(t) = \frac{r(t)m(t)}{r_p(t)} \exp\{j[\phi(t) - \phi_p(t)]\}, \tag{26}$$

where ϕ and ϕ_p are the phase angles of the complex fading signals $r(t)\exp[j\phi(t)]$ and $r_p(t)\exp[j\phi_p(t)]$, respectively. Clearly the pdf of

$$A(t) = \frac{r(t)}{r_p(t)} \tag{27}$$

is of importance in determining how closely $\hat{\mu}(t)$ approaches the original complex envelope of the speech signal $m(t)$. It has been shown[1] that the pdf of $A(t)$ is

$$p_A(x) = \frac{2x(1-\lambda^2)(1+x^2)}{[(1+x^2)^2 - 4\lambda^2x^2]^{3/2}}, \tag{28}$$

where λ, the correlation between $r(t)$ and $r_p(t)$, is

$$\lambda = \frac{1}{[1 + (2\pi f_d T_\sigma)^2]^{1/2}}. \tag{29}$$

In eq. (29), T_σ is a measure of the time-delay spread caused by multipath effects in the transmission medium. Parameter f_d is the frequency separation between the pilot tone f_p and a frequency f in the SSB signal. Hence, it follows that there is a minimum and a maximum value of f_d for a given pilot tone f_p (see Fig. 2).

As the noise signal is

$$n(t) = \hat{\mu}(t) - \langle \hat{\mu}(t) \rangle_A, \tag{30}$$

where

$$\langle \hat{\mu}(t) \rangle_A = \langle A(t) \rangle m(t) \exp\{j[\phi(t) - \phi_p(t)]\} \tag{31}$$

and has infinite variance, unless $\lambda = 1$, it is possible for $\langle \hat{\mu}(t) \rangle_A$ to swamp the average output signal as $r_p(t)$ can be zero. This situation is similar to that for AM,[1] and can be prevented by ensuring that the compensating term $r_p(t)$ cannot fall beyond a value r_o. A block diagram of such a system is shown in Fig. 3. The demodulated signal is now

$$\hat{\mu}(t) = A_L(t) m(t) \exp\{j[\phi(t) - \phi_p(t)]\}, \tag{32}$$

where

$$A_L(t) = \begin{cases} \dfrac{r}{r_p} & \text{if} \quad r_p \geq r_o \\ \dfrac{r}{r_o} & \text{if} \quad r_p < r_o \end{cases} = r \min\left(\frac{1}{r_p}, \frac{1}{r_o}\right). \tag{33}$$

Parameter r_o can be selected to maximize the system performance. By imposing this limit on r_p, the signal $\langle \hat{\mu}(t) \rangle_{A_L}$ can be shown to be[1]

$$\langle \hat{\mu}(t) \rangle_{A_L} = \langle A_L(t) \rangle m(t) \exp\{j[\phi(t) - \phi_p(t)]\}, \tag{34}$$

where

$$\begin{aligned}
\langle A_L(t) \rangle = & \int_0^\infty dr \int_{r_o}^\infty dr_p \frac{r^2}{\sigma^4(1-\lambda^2)} \exp\left[-\frac{r^2 + r_p^2}{2\sigma^2(1-\lambda^2)} \right] \\
& \cdot I_o \left(\frac{r r_p}{\sigma^2} \frac{\lambda}{1-\lambda^2} \right) \\
& + \int_0^\infty dr \int_0^{r_o} dr_p \frac{r^2 r_p}{r_o \sigma^4 (1-\lambda^2)} \exp\left[-\frac{r^2 + r_p^2}{2\sigma^2(1-\lambda^2)} \right] \\
& \cdot I_o \left(\frac{r r_p}{\sigma^2} \frac{\lambda}{1-\lambda^2} \right) \\
& = E(\lambda) - \sqrt{\pi} \frac{r_o}{\sqrt{2\sigma^2(1-\lambda^2)}} \exp\left[\left(\frac{\lambda^2}{2} - 1 \right) \frac{r_o^2}{2\sigma^2(1-\lambda^2)} \right] \\
& \cdot \left[\frac{\lambda^2}{2} I_1 \left(\frac{\lambda^2 r_o^2}{4\sigma^2(1-\lambda^2)} \right) + \left(1 - \frac{\lambda^2}{2} \right) I_o \left(\frac{\lambda^2 r_o^2}{4\sigma^2(1-\lambda^2)} \right) \right] \\
& + \frac{\sqrt{\pi}}{\lambda^2} \sum_{k=0}^{\infty} (2^k k!)^{-2} \\
& \cdot \left\{ \frac{\sqrt{2\sigma^2(1-\lambda^2)}\, \Gamma\left(2k+1, \left(\frac{\lambda^2}{2} - 1 \right) \left\{ r_o^2 / [2\sigma^2(1-\lambda^2)] \right\} \right)}{r_o \left(\frac{2}{\lambda^2} - 1 \right)^{2k+1}} \right. \\
& \left. - \frac{2^{3/2}(1-\lambda^2) \Gamma\left\{ 2k + \frac{3}{2}, \left(\frac{\lambda^2}{2} - 1 \right) [\Gamma_o^2 / \{2\sigma^2(1-\lambda^2)\}] \right\}}{\lambda \left(\frac{2}{\lambda^2} - 1 \right)^{2k+3/2}} \right\}
\end{aligned} \tag{35}$$

In this equation, $I_n(\cdot)$ is the modified Bessel function of the first kind and of order n, $E(\lambda)$ is the complete elliptic integral of the second kind given by

$$E(\lambda) = \frac{\pi}{2} \left[1 - \sum_{n=1}^{\infty} \left\{ \frac{(2n-1)!!}{2^n n!} \right\}^2 \frac{\lambda^{2n}}{2n-1} \right], \tag{36}$$

$$(2n-1)!! \triangleq 1.3.5 \cdots (2n-1), \tag{37}$$

and $\Gamma(p, x)$ is the incomplete gamma function,[7]

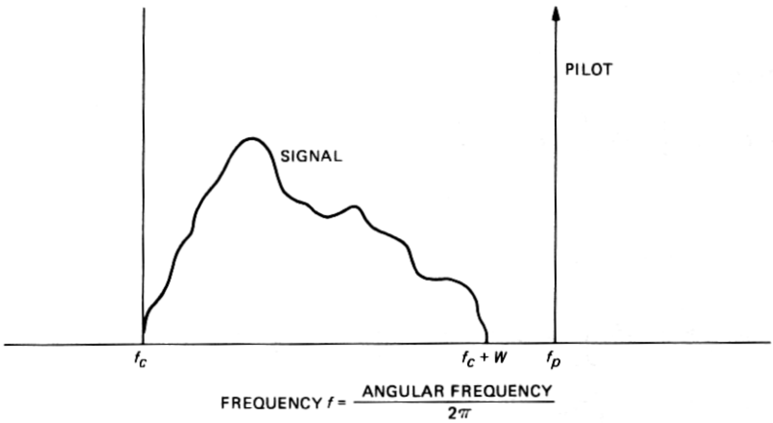

Fig. 2—Spectral arrangement of SSB signal with pilot tone.

$$\Gamma(p, x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{p+n}}{n!(p+n)}. \tag{38}$$

Knowing the average output signal $\langle \hat{\mu}(t) \rangle_{A_L}$ we determine the variance of the noise $n(t)$, namely,

$$\langle n^2(t) \rangle = \{\langle A_L^2 \rangle - \langle A_L \rangle^2\} |m(t)|^2, \tag{39}$$

where $\langle A_L \rangle$ is given in eq. (35) and

$$\begin{aligned}\langle A_L^2 \rangle &= \int_0^\infty dr \int_{r_o}^\infty dr_p \frac{r^3}{r_p \sigma^4 (1-\lambda^2)} \exp\left[-\frac{r^2 + r_p^2}{2\sigma^2(1-\lambda^2)}\right] \\ &\quad \cdot I_o\left(\frac{rr_p}{\sigma^2} \frac{\lambda}{1-\lambda^2}\right) \\ &\quad + \int_0^\infty dr \int_0^{r_o} dr_p \frac{r^3 r_p}{r_o^2 \sigma^4 (1-\lambda^2)} \exp\left[-\frac{r^2 + r_p^2}{2\sigma^2(1-\lambda^2)}\right] \\ &\quad \cdot I_o\left(\frac{rr_p}{\sigma^2} \frac{\lambda}{1-\lambda^2}\right) \\ &= (1-\lambda^2) E_1\left(\frac{r_o^2}{2\sigma^2}\right) + \frac{1 - \exp\left(-\frac{r_o^2}{2\sigma^2}\right)}{r_o^2/(2\sigma^2)}.\end{aligned} \tag{40}$$

In eq. (40), $E_1(x)$ is the exponential integral[11]

$$E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt \qquad x > 0. \tag{41}$$

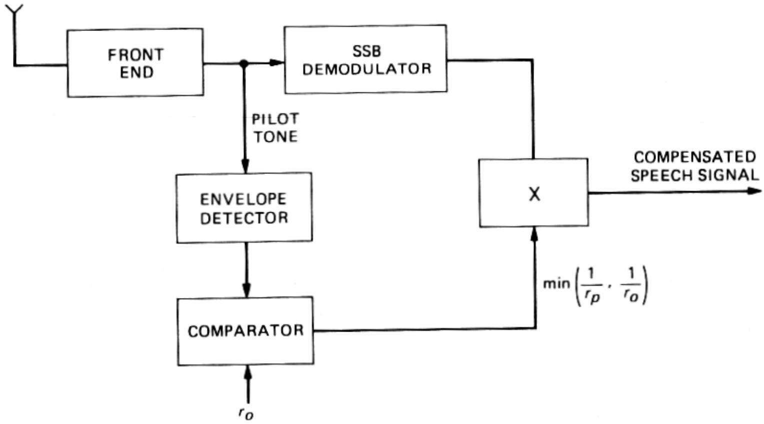

Fig. 3—Pilot correction scheme.

From eqs. (32), (34), (35), and (39), the s/n is given by

$$\text{s/n} = \frac{\langle A_L \rangle^2}{\langle A_L^2 \rangle - \langle A_L \rangle^2}, \tag{42}$$

where $\langle A_L \rangle$ and $\langle A_L^2 \rangle$ are given by eqs. (35) and (40), respectively.

The s/n of eq. (42) is plotted in Fig. 4 as a function of $(r_o^2/2\sigma^2)$, where r_o is the smallest value of $r_p(t)$ and $2\sigma^2$ is the variance of $r(t)$ [or of $r_p(t)$] for various correlation amplitudes λ. This figure is similar to Fig. 4.1-23 of Ref. 1. When r_o is too small, the speech signal may occasionally be over-corrected, resulting in poor s/n. If r_o is too large, the signal compensation may be insufficient, even for high values of λ, a condition of false pilot fade. Thus, for a given σ^2 and λ, there is a broad optimum for r_o, the limit of permissible gain control. The dotted curve in Fig. 4 is the locus of $r_o^2/2\sigma^2$ for optimum gain limiting to ensure peak s/n for a given λ.

The figure also shows that λ needs to be large if the noise is to be low and high s/n achieved. For example, with optimum gain limiting and $\lambda = 0.999$, the output s/n is approximately 20 dB. For a time-delay spread T_d of 1 μs and $\lambda = 0.999$, the maximum frequency separation of the pilot from any part of the SSB signal must be less than

$$f_d = \frac{\sqrt{1-\lambda^2}}{2\pi T_d \lambda} = 7.12 \text{ kHz}, \tag{43}$$

a value that is attainable in SSB speech systems. Thus, an s/n of better than 20 dB can be achieved with the transmission of a pilot carrier and fast-acting automated gain control.

3.3 Effects of Rayleigh fading on a single frequency-hopped SSB signal transmitted with a pilot carrier

After considering the effects of Rayleigh fading on a SSB signal and

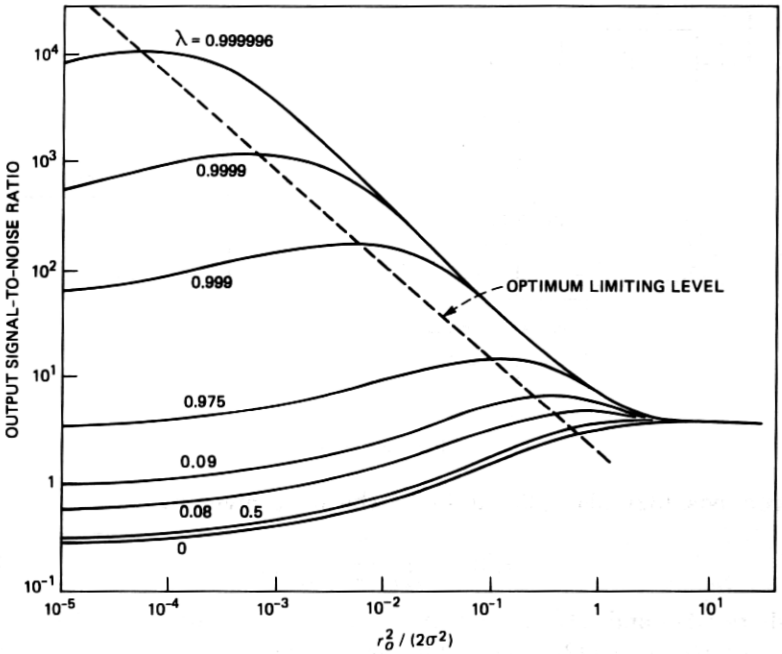

Fig. 4—Signal-to-noise ratio as a function of $r_o^2/2\sigma^2$ for various frequency correlation values.

how its reception is improved with the aid of a pilot carrier, we now examine the effect of frequency-hopping the SSB, plus pilot tone signal in the presence of Rayleigh fading. The frequency-hopping strategy has already been outlined in Section II.

The recovered speech signal in frequency-hopped SSB with pilot tone (FH-SSB-PT) is found by proceeding as in Section 3.2, but modifying eq. (26) to allow for the time segmentation, namely,

$$\hat{\mu}(t) = \sum_{l=-\infty}^{\infty} \hat{\mu}_l(t)\text{rect}\left(\frac{t - l\tau}{\tau}\right), \tag{44}$$

where $\hat{\mu}_l(t)$ from eqs. (32) and (33) is

$$\hat{\mu}_l(t) = r_l(t)\min\left\{\frac{1}{r_{pl}(t)}, \frac{1}{r_o}\right\}\exp\{j[\phi_l(t) - \phi_{pl}(t)]\}m_l(t). \tag{45}$$

The subscript l refers to the lth time slot, and $\phi_l(t)$ and $\phi_{pl}(t)$ are the phase angles of the complex fading signals $r_l(t)\exp[j\phi_l(t)]$ and $r_{pl}(t)\exp[j\phi_{pl}(t)]$ associated with the signal and pilot tone, respectively. Quantity $m_l(t)$ is the segment of the complex envelope of the original speech in the lth time slot. Equation (44) can be expressed as

$$\hat{\mu}(t) = \sum_{l=-\infty}^{\infty} R_l(t) m_l(t) \exp\{j[\phi_l(t) - \phi_{pl}(t)]\}, \tag{46}$$

where $R_l(t)$ is the composite fading envelope because of the mobile radio environment. Notice that $R_l(t)$, given by

$$R_l(t) = r_l(t) \min\left\{\frac{1}{r_{pl}(t)}, \frac{1}{r_o}\right\} \text{rect}\left(\frac{t - l\tau}{\tau}\right), \tag{47}$$

has similar characteristics to those of $A_L(t)$ [see eq. (33)], except that $R_l(t)$ applies for the lth slot of τ seconds duration.

Before proceeding, we make the following assumptions.

(*i*) Values of $R_l(t)$ for different l, i.e., for different time slots of duration τ, are statistically independent. This implies that the scrambling algorithm controlling the frequency hopping ensures that the carrier frequencies in two contiguous time slots are separated by a frequency band that is larger than the coherence bandwidth of the mobile radio channel.

(*ii*) Over a long period of time, each of the N channel frequency bands is likely to be occupied by each of the N voice signals.

(*iii*) The fading characteristics of all the radio channels are identically distributed.

If we consider any two angular frequencies ω_1 and ω_2, where $\omega_2 - \omega_1$ exceeds the coherence bandwidth, their fading patterns will in general be very different. Figure 5 shows a typical example. Even if

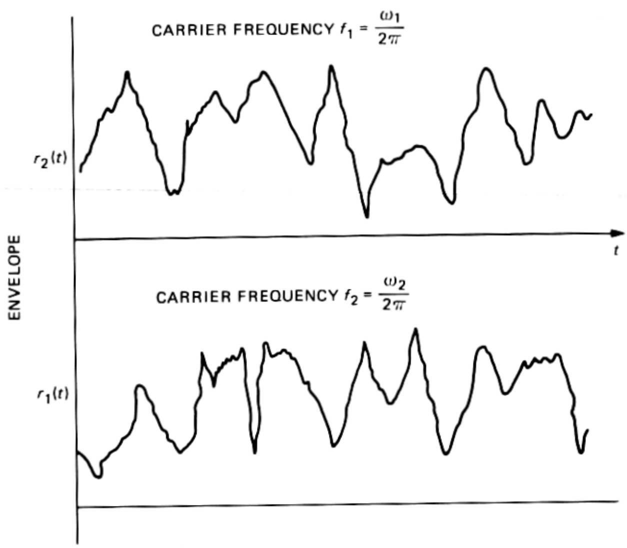

Fig. 5—Fading patterns for angular frequencies ω_1 and ω_2; $\omega_1 - \omega_2$ exceeds the coherence bandwidth.

two mobiles use the same carrier frequency their fading characteristics will generally be different if the vehicles are separated by several wavelengths of the carrier frequency. In our analysis, we assume all fading characteristics are statistically independent.

The recovered signal is found by averaging $\hat{\mu}(t)$ of eq. (46) with respect to the set $\{R_l\} = \{\cdots, R_o, R_1, \cdots, R_l, \cdots\}$

$$\langle \hat{\mu}(t) \rangle_{\{R_l\}} = \sum_{l=-\infty}^{\infty} \langle R_l(t) \rangle m_l'(t)$$

$$= \langle R(t) \rangle \sum_{l=-\infty}^{\infty} m_l'(t),$$

$$m_l'(t) = m_l(t)\exp\{j[\phi_l(t) - \phi_{pl}(t)]\}, \tag{48}$$

where $\langle R_l(t) \rangle$ is assumed to be independent of the time slot l, and is represented by $\langle R(t) \rangle$.

The noise $n(t)$ becomes for the frequency-hopped system

$$n(t) = \sum_{l=-\infty}^{\infty} [R_l(t) - \langle R_l(t) \rangle] m_l'(t), \tag{49}$$

and its variance

$$\langle n_s^2(t) \rangle = \{\langle R^2(t) \rangle - \langle R(t) \rangle^2\} \sum_{l=-\infty}^{\infty} |m_l(t)|^2 \tag{50}$$

as $R_l(t)$ for $-\infty < l < \infty$ are assumed to be independent.

The s/n is, from eqs. (48) and (50),

$$\text{s/n} = \frac{\langle R(t) \rangle^2}{\langle R^2(t) \rangle - \langle R(t) \rangle^2} \frac{\left\langle \left(\left| \sum_{l=-\infty}^{\infty} m_l(t) \right| \right)^2 \right\rangle_t}{< \sum_{l=-\infty}^{\infty} |m_l(t)|^2 >_t}, \tag{51}$$

where

$$\left\langle \left(\left| \sum_{l=-\infty}^{\infty} m_l(t) \right| \right)^2 \right\rangle_t = \langle s^2(t) \rangle_t$$

is the average power in the original speech signal, and

$$\left\langle \sum_{l=-\infty}^{\infty} |m_l^2(t)|^2 \right\rangle_t$$

is the sum of the average powers in the segments of the speech signal. For Gaussian signals, it has been shown[12] that

$$\frac{\left\langle \left(\left| \sum_{l=-\infty}^{\infty} m_l(t) \right| \right)^2 \right\rangle_t}{\left\langle \sum_{l=-\infty}^{\infty} |m_l(t)|^2 \right\rangle_t} = 1. \tag{52}$$

Assuming that speech signals have approximately Gaussian characteristics, this result is also true for speech. Hence,

$$\text{s/n} = \frac{\langle R(t) \rangle^2}{\langle R^2(t) \rangle - \langle R(t) \rangle^2}. \tag{53}$$

Since this s/n is the same as eq. (42), frequency hopping does not lead to any degradation in performance of the system. Note further that Fig. 4 can be used to determine the s/n of an FH-SSB-PT system.

IV. EFFECTS OF CO-CHANNEL INTERFERENCE ON A FH-SSB-PT SIGNAL

Consider a mobile radio system which has been allocated an RF bandwidth of B_T *Hz*, where each cell is assigned a bandwidth of B Hz. Cells B_T/B can use different parts of the B_T spectrum; but if more than B_T/B cells are required to accommodate user demand, some frequency bands must be reused. Let us consider two such cells which are spaced apart as far as possible to restrict mutual interference. Vehicles in each cell communicate via the cell site station using FH-SSB-PT, and let us concentrate on a vehicle in one cell which is subjected to interference, called co-channel interference, from mobiles in the co-channel cell. By arranging for each cell to frequency-hop using different scrambling codes, the co-channel interference will be incoherent. Observe that FH-SSB-PT operates on the interfering signal with the incorrect descrambling code and adds to the wanted speech signal short segments of numerous speech signals in the co-channel cell. The co-channel signal is, therefore, more noise-like and perceptually more acceptable than a single coherent interfering speech signal, provided τ is suitably small. This is the main advantage of using frequency hopping.

An analysis to determine the subjective nature of interference is very difficult and, therefore, we characterize the interference in terms of its mean square value. We proceed as follows.

Since the voice signal associated with a particular frequency slot of width B_c is different from one time slot to the next, we will evaluate the effect of co-channel interference in a single time slot of duration τ. The total input to the receiver consists of the sum of the wanted and interfering components and is given by

$$\alpha_R = r(t)m(t)\exp[j(\omega_c t + \phi)] + r_p(t)\exp[j(\omega_p t + \phi_p)]$$
$$+ \delta\{r_i(t)m_i(t)\exp[j(\omega_i t + \phi_i)] + r_{pi}(t)\exp[j(\omega_{pi} t + \phi_{pi})]\}, \quad (54)$$

where the subscript i refers to the interfering or co-channel signal, subscript p refers to the pilot tone, ω_c and ω_i are the carrier frequencies of the wanted and unwanted signals, and the remaining symbols have their usual meaning. Carrier frequencies, ω_c and ω_i, are equal as eq. (54) applies for one time slot. The relative amplitude δ of the interference determined by antenna and propagation characteristics, and in a nonfading environment the signal-to-interference ratio at RF is $1/\delta^2$.

Suppose that the pilots associated with the wanted and co-channel signals are off-set (say, one above and one below the SSB signal frequency band) with the result that the desired pilot is unaffected by the unwanted pilot. The demodulated speech signal is, therefore,

$$\beta(t) = \frac{r(t)}{r_p(t)} m(t)\exp[j(\phi - \phi_p)] + \delta \frac{r_i(t)}{r_p(t)} m_i(t)\exp[j(\phi_i - \phi_p)]. \quad (55)$$

Hence,

$$\beta(t) = \hat{\mu}(t) + \delta \frac{r_i(t)}{r_p(t)} m_i(t)\exp[j(\phi_i - \phi_p)], \quad (56)$$

where $\hat{\mu}(t)$ is given by eq. (26). Averaging with respect to $r(t)$ and $r_i(t)$, we compute the average signal $\langle\beta(t)\rangle$. The noise signal is $\beta(t) - \langle\beta(t)\rangle$. Hence,

$$\text{s/n} = \frac{\langle A\rangle^2}{\langle A^2\rangle - \langle A\rangle^2}, \quad (57)$$

where

$$\langle A\rangle = \langle A_L\rangle \quad (58)$$

$$\langle A^2\rangle = \langle A_L^2\rangle + \delta^2\langle A_i^2\rangle \quad (59)$$

and

$$\langle A_i^2\rangle = \left\langle\left\{\min\left(\frac{r_i}{r_p}, \frac{r_i}{r_o}\right)\right\}^2\right\rangle_{r_i, r_p}. \quad (60)$$

Note that $\langle A_L\rangle$ and $\langle A_L^2\rangle$ are given by eqs. (35) and (40), respectively.

Assuming r_i and r_p are statistically independent and both are Rayleigh-distributed with the probability density function of eq. (13),

$$\langle A_i^2\rangle = \langle r_i^2\rangle_{r_i}\left\langle\left\{\min\left(\frac{1}{r_p}, \frac{1}{r_o}\right)\right\}^2\right\rangle_{r_p}. \quad (61)$$

Further,

$$\langle r_i^2\rangle_{r_i} = 2\sigma^2 \quad (62)$$

and

$$\left\langle\left\{\min\left(\frac{1}{r_p}, \frac{1}{r_o}\right)\right\}^2\right\rangle_{r_p} = \frac{1}{r_o^2}\int_0^{r_o} \frac{r}{\sigma^2}\, e^{-r^2/2\sigma^2} dr + \int_{r_o}^{\infty} \frac{1}{r^2}\cdot\frac{r}{\sigma^2}\, e^{-r^2/2\sigma^2} dr$$

$$= \frac{1 - e^{-r_o^2/2\sigma^2}}{r_o^2} + \frac{1}{2\sigma^2} E_1\left(\frac{r_o^2}{2\sigma^2}\right), \tag{63}$$

where $E_1(\cdot)$ is given by eq. (41). Hence,

$$\text{s/n} = \frac{\langle A_L \rangle^2}{\langle A_L^2 \rangle + \delta^2 \langle A_i^2 \rangle - \langle A_L \rangle^2}, \tag{64}$$

where $\langle A_i^2 \rangle$ is given in eqs. (61) to (63). Notice that if $\delta = 0$, i.e., no co-channel interference, eqs. (64) and (42) are identical. For interference from M co-channel cells, the s/n becomes

$$\text{s/n} = \frac{\langle A_L \rangle^2}{\langle A_L^2 \rangle + \sum_{j=1}^{M} \delta_j^2 \langle A_{ij}^2 \rangle - \langle A_L \rangle^2}, \tag{65}$$

where δ_j is the relative amplitude, and $\langle A_{i,j}^2 \rangle$ is the appropriate value of $\langle A_i^2 \rangle$ given by eqs. (61) to (63).

V. FH-SSB-PT WITH SPACE DIVERSITY RECEPTION

The impairments associated with FH-SSB-PT in a mobile radio environment can be alleviated by using diversity techniques. As we would like to maximize the number of users in a given RF band, the diversity methods most appropriate are space-diversity techniques.

The simulation analysis provided in Ref. 5 of an equal gain, N branch diversity receiver for conventional SSB-PT is applicable to the FH-SSB-PT system presented here. In the receiver the individual branch pilot signals are phase-corrected and then averaged to provide the envelope correction signal. Amplitude correction is then applied as in the nondiversity case described in Section 3.2.

The improvements in s/n obtained through space-diversity reception are substantial. For example, a two-branch space-diversity receiver with 20 dB ($=10 \log r_o^2/2\sigma^2$) of correction gives 18-dB improvement in s/n when the delay spread T_d is as high as 3 μs. A four-branch diversity receiver provides another 13-dB gain in s/n. Those improvements are obtained in FH-SSB+PT when space-diversity techniques are applied.

VI. COMPARISON OF FH-SSB-PT WITH FM

In this section, we shall present a comparison of user densities obtained with FH-SSB-PT and wide-index FM. Assuming that the use of space diversity mitigates the effects of fading, we consider only degradation from co-channel interference. Consider a mobile radio system based on a cellular structure,[13] where each cell is hexagonal in shape. Figure 6 shows two such cells; cell A is the center of a cluster of U cells that have been allocated an RF bandwidth of B_T Hz and each cell has a bandwidth of B Hz. The value of U is

$$U = i^2 + ij + j^2, \tag{66}$$

where i and j are marked on Fig. 6. There are co-channel cells arranged symmetrically around cell A that will cause interference with mobiles travelling inside this cell. One of these co-channel cells is cell B, spaced a distance D from cell A such that

$$\frac{D}{R} = \sqrt{3U}, \tag{67}$$

where R is the cell radius.

Suppose a mobile in cell A is at the cell boundary on a line connecting the center of cell A to the center of cell B, and that the cell site

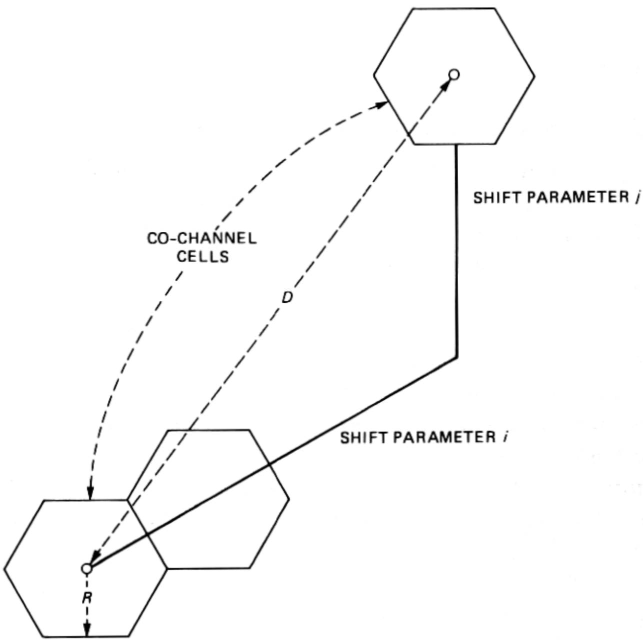

Fig. 6—Co-channel cells for hexagonal cell structure.[8]

antennas are at the cell centers. The ratio δ^2 [see eq. (54)] of the received radiated powers by the mobile from the antennas of cells A and B is approximated by

$$\delta^2 = \left(\frac{R}{D - R}\right)^n, \tag{68}$$

where n varies between 2 and 4. We assume $n = 3.5$. However, there are six co-channel cells[2] at a distance D and other cells spaced kD from A, where $k = 2, 3, \cdots$. We consider only the closest six cells, as they cause the dominant co-channel interference, and assuming each co-channel cell radiates the same power and $D \gg R$,

$$\frac{\text{RF co-channel interference power}}{\text{RF signal power}} = 6\delta^2 = 6[1/(D/R - 1)]^{3.5}. \tag{69}$$

Following demodulation, the s/n values for the SSB and FM systems are

$$\text{s/n}_1 = \frac{1}{6\delta_1^2} \tag{70}$$

and[1]

$$\text{s/n}_2 = \left(\frac{1}{6\delta_2^2}\right)\sqrt{\frac{\pi}{3}}\frac{\chi^3}{0.46}, \tag{71}$$

where the subscripts 1 and 2 refer to the SSB and FM systems, respectively, and χ is the FM modulation index.

Let the comparison be made on the basis that

$$\text{s/n}_1 = \text{s/n}_2, \tag{72}$$

which results in

$$\frac{\delta_1^2}{\delta_2^2} = \sqrt{\frac{3}{\pi}}\frac{0.46}{\chi^3}. \tag{73}$$

Equations (67), (68), and (73) yield

$$\left(\frac{\sqrt{3U_2} - 1}{\sqrt{3U_1} - 1}\right)^{3.5} = \frac{0.45}{\chi^3}. \tag{74}$$

The RF bandwidths allocated to the entire system, to each cell and to each voice channel are B_T, B, and B_c, respectively. The number of available channels is B_T/B_c, and the number of users in each cell is

$$N = \frac{B_T}{B_c U} = \frac{B}{\gamma W} = \frac{B}{B_c}. \tag{75}$$

If the cell sizes are the same in the SSB and FM systems, the user density ρ, i.e., the number of users per unit area, is

$$\rho = \frac{1}{\pi} \frac{B_T}{D^2 B_c}, \tag{76}$$

where D is the co-channel distance [see eq. (67)]; i.e., we approximate the cluster of non-cochannel cells by a circle of radius D. The ratio of user densities in the SSB and FM systems is, therefore,

$$\frac{\rho_1}{\rho_2} = \left(\frac{D_2}{D_1}\right)^2 \left(\frac{B_{c,2}}{B_{c,1}}\right). \tag{77}$$

As an example, let the number of cells in the FM system be 12, and we will assume $\chi = 2.5$. From eq. (74), the number of cells in the SSB scheme becomes 73. For $B_T = 20$ MHz, $B_{c,1} = 4$ kHz, $B_{c,2} = 30$ kHz, $D_1 = R_1\sqrt{3U_1} = 14.8R_1$, $D_2 = R_2\sqrt{3U_2} = 6R_2$, and as $R_1 = R_2$, $D_2/D_1 = 0.405$. Hence from eq. (76), $\rho_1/\rho_2 = 1.23$, i.e., the user density of SSB is 1.23 times higher than that for FM. From eqs. (69) and (70), the s/n is

$$\text{s/n} = \frac{1}{6}(\sqrt{3U} - 1)^{3.5} = 32 \text{ dB},$$

a value that is more than adequate.

VII. DISCUSSION

Users of a mobile radio system want good service, being completely indifferent to the type of modulation employed. The designer of a mobile radio system, however, is faced with a hostile communications environment. If digital modulation methods are used the speech signals have to be encoded into digital waveforms that have significantly more bandwidth (typically a factor of 6) than that of the speech signal. Unfortunately, narrow-band FM with its small signal expansion is difficult to use for mobile radio because of its vulnerability to co-channel interference, while wideband FM with its acceptable performance requires a bandwidth that is comparable to the bandwidth of digitized speech. However, the bandwidth of the SSB signal is the same as the original speech, and this has encouraged us to innovate new techniques to overcome inability of SSB to perform well in the presence of co-channel interference and fading. We were aware at the inception of this study that many people have analog radio receivers and, therefore, the capability to eavesdrop on mobile radio conversations using analog modulation methods. The outcome of our deliberations is the frequency-hopped SSB with pilot tone (FH-SSB-PT) scheme described in the previous sections.

Although the concept of SSB and its compensation by a pilot tone is well known, we presented in Sections 3.1 and 3.2 new analytical results. For the case of pilot tone compensation of the SSB signal in the presence of Rayleigh fading, we used an alternative definition of s/n from that used in Ref. 9, and further, we expressed it as a function of λ instead of limiting it to the special case of $\lambda = 1$. Our results show that pilot compensation can achieve an s/n of more than 20 dB if $\lambda > 0.999$ and $f_d = 7.1$ kHz. A more relevant value of f_d for speech is probably 3 kHz, and T_d is typically 1 μs, then from eq. (29) $\lambda = 0.9998$, resulting in s/n = 27 dB.

When the SSB signal, plus pilot tone is frequency-hopped, we show that no degradation in s/n occurs. Thus, although frequency hopping does not alter the s/n [compare eqs. (42) and (53)], it does have important perceptual advantages. For example, consider a stationary mobile (e.g. parked, or in congested traffic) operating in an SSB system that does not employ frequency hopping. Suppose the mobile is subjected to a deep fade that defies pilot correction, resulting in a complete disruption of communications. Now imagine that frequency hopping is applied; the probability that communication will be restored increases as the probability of repeatedly hopping into a deep fade condition is small. Of course, if the mobile were not in a deep fade but receiving excellent communications, the effect of frequency hopping would degrade the recovered speech, although the degradation would probably be marginal. What frequency hopping ensures is that no mobile is ever in a permanent deep fade, provided the vehicle is not in a completely hostile environment, such as a tunnel, etc.

Generally, pilot correction can be performed, and the recovered speech will have only occasional segments of duration τ that have been obliterated by the fade. As they are likely to be in juxtaposition with good quality segments, speech enhancement procedures can be used on the degraded segments, particularly if τ is of the order of a pitch period. If τ is small (<5 ms), the model of Rayleigh fading is not applicable and the accuracy of our s/n analysis is unknown. Note that the short-time statistics of fading are also unknown. The choice of τ, therefore, depends on many factors, ease of switching, aid to enhancement, etc. Duration τ also determines how noise-like the co-channel interference will be. From these considerations, a suitable choice of τ is between 10 to 50 ms.

We have derived and discussed in detail the problem of frequency selective fading. By employing space diversity reception (see Section V) the signal deterioration caused by fading can be further reduced. This means that the factor controlling user density is co-channel interference, and it is here that frequency hopping makes its most significant contribution by changing the nature of co-channel interference from cross-talk to almost band-limited white noise. Comparing eqs. (64) and (53), we see that the effect of co-channel noise is to introduce the term $\delta^2\langle A_i^2\rangle$, where $\langle A_i^2\rangle$ is noise-like, composed of segments of τ seconds, where adjacent segments contain speech from different speakers in the interfering cell. By frequency hopping we have circumvented, in perceptual terms, the inability of the SSB receiver to capture the wanted signal. What we are left with is the speech signal corrupted by noise, and we envisage that the next phase of the research will involve the development of techniques to strip the noise from the speech signal. By doing this, we will be able to operate with high s/n, or alternatively move the cells closer together, thereby raising the user density.

Frequency hopping is not confined to SSB. When applied to FM, eq. (53) applies with the same advantages as for SSB. In terms of co-channel interference, there would be no need to rely on the capture effect of FM, and further, a measure of privacy could be obtained.

Summarizing: SSB can combat fading if pilot tones are used, and has enhanced quality if space diversity reception is added. By using frequency hopping under the auspices of a scrambling code, improvements occur for stationary and slow-moving vehicles. The most significant advantages accrue from combating co-channel interference, where crosstalk is perceived as random noise. Scrambling codes to satisfy the requirements of the system are trivial to generate and have not been discussed here. When compared to FM, the user density of FH-SSB-PT is marginally higher. Frequency hopping is not confined to SSB, can be applied to FM with advantage, and offers privacy in mobile radio where eavesdropping may be a problem.

Finally, we comment that since SSB demodulation requires recovery of a carrier, linearity of its modulators, etc., it is still possible that SSB user density may not be higher than that of wide-index FM in all situations. Also, since the availability and economics of SSB are not discussed in this paper, we do not imply that current FM modulation used in the AMPS system is not technologically appropriate at this time. We present a new analysis of SSB and point out some directions of research to resolve these questions.

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